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A Level Set based Regularization Framework for EIT Image Reconstruction

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Abstract. Electrical Impedance Tomography (EIT) reconstructs the conductivity distribution within a medium from electrical stimulation and measurements at the medium surface. Level set based reconstruction method (LSRM) has gained attention during the last decade as an effective solution to address the need of reconstructing structures with limited amount of available data. The classical LSRM is based on the quadratic formulations (L2 norms); however, the L2 norms are not robust to outliers and spatial noise. The L1 norm is a more solid alternative to produce high robustness against outliers and noise. The L1 norm is minimized by Primal dual-interior point method (PDIPM). In this paper, we derive a novel level set (LS) based regularization framework for using the L1 norm independently on the data and the regularization term of an inverse problem. The proposed LS based regularization method, called LS based PDIPM (LS-PDIPM), applies the PDIPM to minimize the L1 norms. We use the LS-PDIPM to reconstruct 2D images from EIT simulated data. The proposed LS-PDIPM with the L1 norms provides sharper and less noisy images, when comparing with the L2 norm based regularization method.

Keywords: Level set, Inverse problem, Primal-Dual, Electrical Impedance Tomography

1. Introduction

Electrical Impedance Tomography (EIT) is an imaging modality to reconstruct the conductivity distribution within a medium by solving an inverse problem. EIT image reconstruction is an ill-posed problem. To provide robustness to measurement noise, regularization techniques are applied. The level set based regularization method (LSRM) is desirable because of its topological based representation of unknown structures using level zero (a front) of a higher dimension function (level set function) (Dorn & Lesselier 2009). The LSRM reconstructs the unknown topological information of structures using the evolution of a level set function which minimizes a predefined cost functional. We (Rahmati et al. 2012) showed the first clinical results of applying the LSRM to reconstruct EIT lung images. Borsic and Adler (2012) discuss the minimization of the L1 norm based regularization problem. They minimize the L1 norms using primal-dual interior point method (PDIPM) and show the L1 norms provide higher robustness against outliers and noise when compared with L2 norm based regularization method, such as Gauss-Newton (GN) method. In this paper, we derive a framework to solve a L1 norm based inverse problem using the LSRM, hereinafter called LS-PDIPM. There are two main advantages to applying the LSRM with the L1 norms: 1) level set techniques preserve the edges (Rahmati et al. 2012), 2) the L1 norms on the data mismatch term and the regularization term of an inverse problem provide robust estimations against outliers and spatial noise, respectively (Borsic & Adler 2012).

2. Generalized PDIPM

A general primal problem can be written as follows

$$(P) = \operatorname{argmin} \left\{ \sum_{i=1}^{D_1} |f_{d_i}(m)| + \sum_{j=1}^{D_2} |f_{p_j}(m)| + \|g_d(m)\|^2 + \|g_p(m)\|^2 \right\} \quad (1)$$

where $f_d(m)$ is a L1 norm based data mismatch term, $f_p(m)$ is a L1 norm based regularization term, $g_d(m)$ is a L2 norm based data mismatch term, and $g_p(m)$ is a L2 norm based regularization term. A primal minimization problem can be formed through any combination of the error terms defined in (1). The non-differentiability of L1 norm is resolved through using a centering condition which is the replacement of the L1 norm by a quadratic norm using an auxiliary variable, called β . The Primal-Dual (PD) framework can be formulated as

$$C_d(m) = f_{d_i}(m) - x_{d_i} \sqrt{f_{d_i}(m)^2 + \beta} = 0, \quad \forall i \quad (2)$$

$$C_p(m) = f_{p_j}(m) - x_{p_j} \sqrt{f_{p_j}(m)^2 + \beta} = 0, \quad \forall j \quad (3)$$

$$|x_{d_i}| \leq 1, |x_{p_j}| \leq 1 \quad (4)$$

$$F_c(m) = \frac{\partial}{\partial m}(f_d(m))x_d + \frac{\partial}{\partial m}(f_p(m))x_p + \frac{\partial}{\partial m}(\|g_d(m)\|^2) + \frac{\partial}{\partial m}(\|g_p(m)\|^2) = 0 \quad (5)$$

The derived general Primal-Dual (PD) framework above is solved iteratively using an iterative method, such as Newton method as follows

$$\begin{bmatrix} \frac{\partial}{\partial m} F_c(m) & \frac{\partial}{\partial x_d} F_c(m) & \frac{\partial}{\partial x_p} F_c(m) \\ \frac{\partial}{\partial m} C_d(m) & \frac{\partial}{\partial x_d} C_d(m) & \frac{\partial}{\partial x_p} C_d(m) \\ \frac{\partial}{\partial m} C_p(m) & \frac{\partial}{\partial x_d} C_p(m) & \frac{\partial}{\partial x_p} C_p(m) \end{bmatrix} \begin{bmatrix} \delta m \\ \delta x_d \\ \delta x_p \end{bmatrix} = - \begin{bmatrix} F_c(m) \\ f_d(m) - (\sqrt{f_d(m)^2 + \beta})x_d \\ f_p(m) - (\sqrt{f_p(m)^2 + \beta})x_p \end{bmatrix} \quad (6)$$

where two auxiliary variables x_d and x_p are in the range $[-1, 1]$ depending on the absolute value of $f_d(m)$ and $f_p(m)$; respectively. The primal variables (m) are updated with a line search procedure, as $m^{(k+1)} = m^{(k)} + \lambda_m \delta m^{(k)}$, where k is the iteration number, δm is the update value with a descend direction to the optimal point, and λ is the step length (Nocedal & Wright 1999). In a similar manner, the dual variables (x_d and x_p) are also updated using *scaling rule* as follows $x^{(k+1)} = x^{(k)} + \min(1, \varphi^*) \delta x^{(k)}$ (Andersen et al. 2000), where φ^* is a scalar value such that $\varphi^* = \sup \left\{ \varphi : \left| x_i^{(k)} + \varphi \delta x_i^{(k)} \right| \leq 1, \quad i = 1, \dots, n \right\}$.

3. Level Set based Primal Dual - Interior Point Framework

In this section, we formulate the solution of a system defined as (1) using the proposed LS-PDIPM. The solution of the L1L2 problem is derived using the LS-PDIPM. The solutions for the L2L1 and L1L1 problems are analogous.

3.1. LS-PDIPM for the primal problem with L1 norm based data mismatch term and L2 norm based regularization term

The level set function (Ψ) is a signed distance function which is zero at the optimal solution and nonzero otherwise. The minimum distance from the optimal solution is achieved at zero level set function. The evolution of the level set function according to the minimization of a functional objective function (primal problem), which can be a standard least square error function, results in the optimal solution of an inverse problem. A mapping function (Φ) is used to project the level set function onto finite element mesh (FEM). The level set evolution function is as follows

$\Psi_{k+1} = \Psi_k + \lambda(\Delta\Psi)$, where Ψ_{k+1} is the updated level set function, Ψ_k is the current level set function, $\Delta\Psi$ is the update, Φ is the mapping function, λ is the step size. The primal formulation (P) for the L1L2 problem is a special state of the general primal problem in (1) when $|f_p(m)| = \|g_d(m)\|^2 = 0$ and can be written as

$$(P) = \operatorname{argmin}_{\Phi(\Psi)} \left[\sum_i W_i |h_i(\Phi(\Psi)) - d_i| + \alpha \|L(\Phi(\Psi) - \Phi(\Psi^0))\|^2 \right] \quad (7)$$

where $|f_d(m)|$ is replaced by $f_d(\Phi(\Psi)) = W|h(\Phi(\Psi)) - d|$, $|g_p(m)|$ is replaced by $g_p(\Phi(\Psi)) = \alpha \|L(\Phi(\Psi) - \Phi(\Psi^0))\|^2$, W is a weighting diagonal matrix, W_i is the i -th diagonal element, $h_i(\Phi(\Psi))$ is the i -th forward measurement, d_i is the i -th measured data, L is the regularization matrix, $\Phi(\Psi)$ is the model parameter distribution or the primal variables, $\Phi(\Psi^0)$ is a reference model parameter distribution. According to the chain rule, the LS Jacobian matrix (J_{LS}) can be written as $J_{LS} = \frac{\partial d}{\partial \Psi} = \left(\frac{\partial h}{\partial \Phi(\Psi)} \right) \left(\frac{\partial \Phi(\Psi)}{\partial \Psi} \right) = (J_{GN})(M)$. To make the algorithm computationally efficient, we restrict the Jacobian computation within a narrow band containing the data (non-zeros). To construct the narrow band, we define the level set function, or the signed distance function, to be negative inside its boundary and positive outside. Matrix M is non-zero within the narrow band and zero otherwise, which is the notion of Dirac delta function. In every iteration of the level set function (Ψ), we calculate the Jacobian matrix (J_{LS}) for the narrow band. We define a dual variable x_i in the range $[-1, 1]$, depending on the absolute value of $W_i(d - h(\Phi(\Psi)))$. The dual problem can be written as

$$(D) = \operatorname{argmin}_{\Phi(\Psi)} \left[\max_x x^T W(h(\Phi(\Psi)) - d) + \alpha \|L(\Phi(\Psi) - \Phi(\Psi^0))\|^2 \right], \text{ with } |x| \leq 1 \quad (8)$$

The smoothed version of the LS based PD framework can be obtained through applying the *centering condition* which is the replacement of $|W_i(h(\Phi(\Psi)) - d_i)|$ by $\sqrt{(W_i(h(\Phi(\Psi)) - d_i)^2 + \beta}$, with $\beta > 0$. Replacing $|f_p(m)| = 0$ and $\|g_d(m) = 0\|$ in (2-5), the smoothed LS based PD framework is achieved as

$$|x_i| \leq 1, \quad (9)$$

$$F_c(\Phi(\Psi)) = J_{LS}^T(\Phi(\Psi))Wx + 2\alpha M^T L^T L(\Phi(\Psi) - \Phi(\Psi^0)) = 0, \quad (10)$$

$$C_d(\Phi(\Psi)) = (h_i(\Phi(\Psi)) - d_i) - x_i \sqrt{(h_i(\Phi(\Psi)) - d_i)^2 + \beta} = 0, \quad \beta > 0, \quad (11)$$

We define $f = h(\Phi(\Psi)) - d$, $F = \operatorname{diag}(f)$, $X = \operatorname{diag}(x)$, $\kappa = \sqrt{f^2 + \beta}$, $E = \operatorname{diag}(\kappa)$. Applying the general solution in (6), the following newton system to be iteratively solved to calculate the updates for the primal variables (Ψ) and the dual variable (x) can be written

$$\begin{bmatrix} 2\alpha M^T L^T L M & J_{LS}^T W \\ (I - X E^{-1} F) J_{LS} & -E \end{bmatrix} \begin{bmatrix} \delta \Psi \\ \delta x \end{bmatrix} = - \begin{bmatrix} J_{LS}^T W x + 2\alpha M^T L^T L(\Phi(\Psi) - \Phi(\Psi^0)) \\ f - E x \end{bmatrix} \quad (12)$$

the dependency of J_{LS} on Ψ is dropped in the derived LS based PD framework. The derived set of equations in (12) are iteratively solved for the primal variables ($\delta\Psi$) and the dual variables (δx) using an iterative method such as Newton method. A traditional line search procedure (Nocedal & Wright 1999) can be applied to find an appropriate step length λ_Ψ resulting in the update $\Psi^{(k+1)} = \Psi^{(k)} + \lambda_\Psi \delta\Psi^{(k)}$, where k is the iteration number. A *scaling rule* is applied to compute the updates for the dual variables (x) (Andersen et al. 2000).

4. Simulated Data

We used EIT with 16 electrodes on one electrode plane and a circular FEM. The adjacent current stimulation was considered for the evaluation of our simulation. The applied 2D phantom contains two sharp inclusions with the same conductivity of $0.9 S/m$. The background conductivity value is $1 S/m$. The inverse problem used the mesh density of 576 elements, which was different than the mesh density of the forward problem (1024 elements).

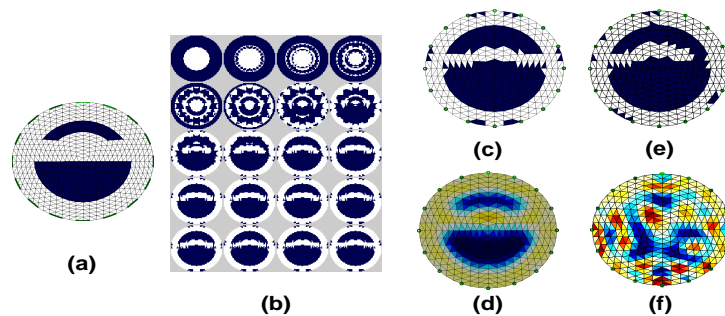


Figure 1. Image reconstruction of EIT simulated data with and without synthetic additive noise. (a) The applied 2D phantom. (b) The iterations of the derived LS-PDIPM with no added noise. (c) The final reconstructed image of the LS-PDIPM with no added noise. (d) Gauss-Newton method with no added noise. (e) The final reconstructed image of the LS-PDIPM with added 60dB Gaussian noise. (f) The Gauss-Newton method with additive 60dB Gaussian noise.

5. Results

In this section, we apply the derived LS-PDIPM to solve a primal problem with L1 norm on both the data mismatch and the regularization term (L1L1 problem). The reason of choosing the L1L1 problem is because the L1 norms has been shown to offer the highest robustness to spatial noise and outliers. The inverse solution is calculated using the proposed LS-PDIPM with $\beta = 1 \times 10^{-12}$. The stopping term to terminate the iterations depends on the value of the primal dual gap computed in every iteration. 20 iterations for the LS-PDIPM were sufficient to reach to convergence. Figure 1(a) shows the applied 2D phantom with two inclusions with conductivity of $0.9 S/m$. The reconstructed images for every iteration of the LS-PDIPM are shown in figure 1(b). The final reconstructed image using the LS-PDIPM is demonstrated in figure 1(c). Figure 1(d) is the reconstructed image of GN method over the same simulated data achieved from the 2D phantom. To account for the possible systematic and random errors occurring in EIT data acquisition process, a 60 dB Gaussian noise was added to the EIT simulated data. Figure 1(e) and figure 1(f) present the final reconstructed image of the LS-PDIPM and the image of GN method in the presence of additive noise, respectively. GN method fails due to the usage of the L2 norms on the regularization term of the inverse problem. The LS-PDIPM is highly robust to the additive noise and still provides sharp image. Unlike the GN method, the LS-PDIPM preserves the edges and is not sensitive to the spatial noise.

6. Conclusion

We derive a level set based regularization method which allows any possible combination of norms (L1 norm or L2 norm) on the data term and the regularization term of an inverse problem. The LS-PDIPM contains the benefit of a shape based reconstruction algorithm in producing sharp edges as well as using the L1 norm in the primal problem, increasing the robustness to outliers and noise. we applied the proposed LS-PDIPM to reconstruct EIT 2D images and showed the LS-PDIPM is highly robust to the additive 60dB Gaussian noise. An extended version of the LS-PDIPM to elaborate the derivations of the LS-PDIPM is currently under way.

7. References

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