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Gravitational strings beyond quantum theory: Electron as a closed heterotic string

Alexander Burinskii

Lab. of Theor. Phys. , NSI, Russian Academy of Sciences, B. Tulskaya 52 Moscow 115191 Russia

E-mail: `bur@ibrae.ac.ru`

Abstract. The observable parameters of the electron indicate unambiguously that its gravitational background should be the Kerr-Newman solution without horizons. This background is not flat and has a non-trivial topology created by the Kerr singular ring. This ring was identified with a closed gravitational string. We discuss the relation of this string to the closed heterotic string of the low energy string theory and show that travelling waves along the KN string give rise to the Dirac theory of electron. Gravitational strings form a bridge between gravity and quantum theory, indicating a new way to consistent Quantum Gravity. We explain the pointlike experimental exhibition of the electron and argue that the predicted closed string may be observed by the novel experimental method of the "nonforward" Compton scattering.

1. Introduction

Modern physics is based on Quantum theory and Gravity. Both theories are confirmed experimentally with great precision. Nevertheless, they are conflicting and cannot be unified in a whole theory. One of the principal contradictions between Quantum theory and Gravity is the question on the shape and size of an electron. Quantum theory states that electron is pointlike and structureless [1, 2], and it is experimentally supported by the high energy scattering. In the same time, gravity based on the Kerr-Newman (KN) solution indicates unambiguously that electron should form a closed string of the Compton size. Contrary to the Schwarzschild solution, which displays the 'range' of a gravitational field (radius of the horizon $r_g = 2m$) proportional to the mass of the source [1], the KN geometry of the rotating bodies presents a new dimensional parameter $a = J/m$, which grows with angular momentum J and has the reverse mass-dependence. As a result, the zone of gravitational interaction is determined by parameter a, which increases for the large angular momentum and small masses, and therefore, it turns out to be essential for elementary particles. The reason for that is a specific structure of the KN gravitational field, which concentrates near the Kerr singular ring, forming a closed gravitational waveguide – a type of the closed gravitational string [3].

In 1968 Carter obtained that the KN solution for the charged and rotating black holes has $g = 2$ as that of the Dirac electron, [4, 5], which allowed one to consider KN solution as consistent with gravity electron model, [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Mass of the electron in the considered units is $m \approx 10^{-22}$, while $a = J/m \approx 10^{22}$. Therefore, $a >> m$, and the black hole horizons disappear, opening a two-sheeted spacetime with a nontrivial topological

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[1] We use the units $\hbar = G = c = 1$

defect in the form of the naked Kerr singular ring. The Kerr ring takes the Compton radius, corresponding to the size of a "dressed" electron in QED and to the limit of localization of the electron in the Dirac theory.

There appear two questions: 1) How does the KN gravity know about one of the principal parameters of Quantum theory? and 2) Why does Quantum theory work successfully on the flat spacetime, ignoring the stringlike peculiarity of the background gravitational field? A small and slowly varying gravitational field could be ignored; however, the stringlike KN singularity forms a branch line of the spacetime and creates a two-sheeted topology, ignorance of which cannot be justified. A simple answer to these questions is to assume that there is a general underlying theory providing the consistency of quantum theory and gravity. In this paper we suggest an approach which allows one to resolve this puzzle from a rather unexpected suggestion, that the underlying theory is the Einstein-Maxwell gravity as a fundamental part of the theory of superstrings. In this case, quantum theory should follow from the theory of superstring, and we make here first steps in this direction. Starting from description of the structure of the KN spacetime in Section II, we show in Section III the relationships of the Kerr singular ring to a heterotic string of the low energy string theory, and then, in Section III we show 'emergence' of the Dirac equation and the corresponding wave functions from the physical model of the lightlike travelling waves, propagating along the KN circular ring. In Section IV we describe a model for regularization of the KN solution by the Higgs fields, which allows us to regularize the KN background, retaining its asymptotical KN form. It justifies the use of flat background in the Dirac theory and QED, answering the second above-mentioned question.

So far as gravity predicts the existence of the closed Kerr string on the boundary of the Compton area, such a string, if it really exists, should be experimentally observable, and there appears the question while it was not obtained earlier by the high energy scattering. In the Conclusion, we give an explanation for this fact and argue that the KN string may apparently be detected by the novel experimental method based on the theory Generalized Parton Distributions (GPD) [19, 20] which represents a new regime for probing the transverse shape of the particles by the "nonforward Compton scattering" [21].

2. The Kerr-Newman background

The Kerr-Newman solution in the Kerr-Schild form has the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2Hk_\mu k_\nu, \tag{1}$$

where

$$H = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}, \tag{2}$$

and $\eta_{\mu\nu}$ is metric of auxiliary Minkowski space in the Cartesian coordinates (t, x, y, z), and the Kerr coordinates r and θ are the spheroidal oblate coordinates, which are related to the Cartesian coordinates as follows

$$\begin{aligned} x + iy &= (r + ia)e^{i\phi} \sin \theta \\ z &= r \cos \theta. \end{aligned} \tag{3}$$

The function H is singular at $r = 0$, $\cos \theta = 0$, corresponding to the Kerr singular ring. The KN electromagnetic potential has the form

$$\alpha^\mu_{KN} = Re \frac{e}{r + ia \cos \theta} k^\mu. \tag{4}$$

The KN potential as well as the KN metric are aligned with the null direction k^μ, which forms the Kerr principal null congruence (PNC) determined by the differential form [5]

$$k_\mu dx^\mu = dt + \frac{z}{r}dz + \frac{r(xdx + ydy)}{r^2 + a^2} - \frac{a(xdy - ydx)}{r^2 + a^2}. \tag{5}$$

The potential and metric are singular at the Kerr ring, which forms a branch line of the Kerr spacetime in two sheets, corresponding to $r > 0$ and $r < 0$ in the Kerr oblate coordinate system. Vector field k^μ forms Principal Null Congruence (PNC) of KN space, which is determined by the Kerr theorem in twistor terms, [12, 13]. The Kerr PNC is smoothly propagated via the Kerr disk $r = 0$ from the 'negative' sheet ($r < 0$) of spacetime to the 'positive' one ($r > 0$) (see Fig.1), and therefore, it covers the KN space twice: $k^{\mu(+)}$ for $r > 0$ and $k^{\mu(-)}$ for $r < 0$, leading to different metrics and different electromagnetic field on the 'positive' and 'negative' sheets [22]. Therefore, the Kerr ring creates a two-sheeted background topology.
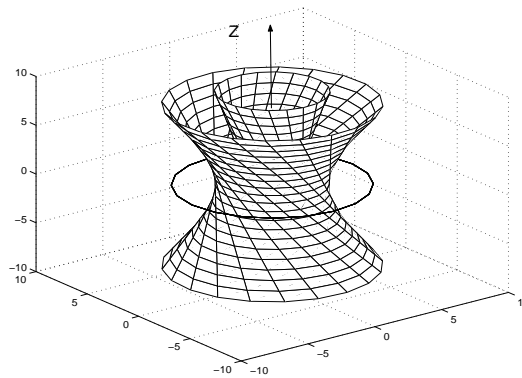
Figure 1. Vortex of the Kerr congruence. Twistor null lines are focused on the Kerr singular ring, forming a circular gravitational waveguide, or string with lightlike excitations.

This two-sheetedness forms a principal puzzle of the Kerr geometry over a period of four decades. In in 1967 Keres [23] and then Israel [6] truncated negative KN sheet, $r < 0$,, replacing it by the *rotating disklike source* at $r = 0$, spanned by the Kerr singular ring of the Compton radius $a = \hbar/2m$. Then Hamity assumed in [24] that the disk is to be rigidly rotating, which led to a reasonable interpretation of the matter of the source as an exotic stuff with the zero energy density and negative pressure. The matter distribution was singular at the disk boundary, forming an additional closed string source, and López suggested in [9] to regularize this source, covering the Kerr singular ring by a disk-like ellipsoidal surface. As a result the KN source was turned into a rotating and charged oblate bubble with a flat interior, and further, the bubble source was realized as a regular soliton-like model [17], formed by a domain wall interpolating between the external KN solution and a flat pseudo-vacuum state inside the bubble.

Alternatively, from the very beginning, there were considered the string-like models of the KN source, which retained Kerr's two-sheeted topology, forming a closed 'Alice' string [3, 13, 11, 25]. The Kerr singular ring was considered as a waveguide for electromagnetic travelling waves generating the spin and mass of the KN solution in accordance with the old Wheeler's "geon" model of 'mass without mass' [7, 8, 26, 27].

3. The Kerr singular ring as a closed string
Exact *non-stationary* solutions for electromagnetic excitations on the Kerr-Schild background, [11, 22, 44], showed that there are no smooth harmonic solutions. The typical exact

electromagnetic solutions on the KN background take the form of singular beams propagating along the rays of PNC, contrary to smooth angular dependence of the wave solutions used in perturbative approach!

Position of the horizon for the excited KS black holes solutions is determined by function H which has for the exact KS solutions the form, [5],

$$H = \frac{mr - |\psi|^2/2}{r^2 + a^2 \cos^2 \theta} \, ,$$ (6)

where $\psi(x)$ is related to the vector potential of the electromagnetic field

$$\alpha = \alpha_\mu dx^\mu = -\frac{1}{2} Re \, [(\frac{\psi}{r + ia \cos \theta})e^3 + \chi d\bar{Y}], \, ,$$ (7)

where $\quad \chi = 2 \int (1 + Y\bar{Y})^{-2} \psi dY$, and the vector field α satisfies the alignment condition

$$\alpha_\mu k^\mu = 0.$$ (8)

The equations (1)and (6) display compliance and elasticity of the horizon with respect to the electromagnetic field.

The Kerr-Newman solution corresponds to $\psi = q = const..$. However, any nonconstant holomorphic function $\psi(Y)$ yields also an exact KS solution, [5]. On the other hand, any nonconstant holomorphic functions on sphere acquire at least one pole. A single pole at $Y = Y_i$

$$\psi_i(Y) = q_i/(Y - Y_i)$$ (9)

produces the beam in angular directions

$$Y_i = e^{i\phi_i} \tan \frac{\theta_i}{2}.$$ (10)

The function $\psi(Y)$ acts immediately on the function H which determines the metric and the position of the horizon. The analysis showed, [22], that electromagnetic beams have very strong back reaction to metric and deform topologically the horizon, forming the holes which allows matter to escape interior (see fig.3).

The exact KS solutions may have arbitrary number of beams in different angular directions $Y_i = e^{i\phi_i} \tan \frac{\theta_i}{2}$. The corresponding function

$$\psi(Y) = \sum_i \frac{q_i}{Y - Y_i},$$ (11)

leads to the horizon with many holes. For the parameters of an electron, the BH horizons disappear, however the radiating singular beams appear unavoidable, and in the far zone the beams tend to the known exact singular pp-wave solutions. The considered in [28] multi-center KS solutions showed that the beams are extended up to the other matter sources positioned at infinity.

The stationary KS beam-like solutions may be generalized to the time-dependent wave pulses, [11].

Due to the factor $\frac{1}{r+ia\cos\theta}$ in the vector potential α, any electromagnetic excitation of the KN geometry generates the travelling waves along the Kerr singular ring (at $r = \cos \theta = 0$), and simultaneously, the pole in Y creates an 'axial' singular beam, which is topologically coupled with the Kerr ring, see Fig.2. Therefore, the EM excitations on the KN background have a paired character, creating simultaneously a 'circular' travelling wave and the coupled with it a

propagating outward 'axial' travelling wave.[2] The vector field k^μ is constant along the 'axial' beams, and asymptotically (by $r \to \infty$) the beams tend to the well known pp-wave (plane fronted) solutions, for which k_μ is a covariantly constant Killing direction. Adapting the z-axis of the coordinate system along the asymptotic pp-wave direction, and using the light-cone Cartesian coordinates $u = (t+z)/\sqrt{2}, \quad v = (z-t)/\sqrt{2}$ and $\zeta = (x+iy)/\sqrt{2}, \quad \bar\zeta = (x-iy)/\sqrt{2}$, one writes the outgoing pp-wave metric in the form

$$ds^2 = 2dudv + 2d\zeta d\bar\zeta + 2H(v, \zeta, \bar\zeta)dv^2. \tag{12}$$
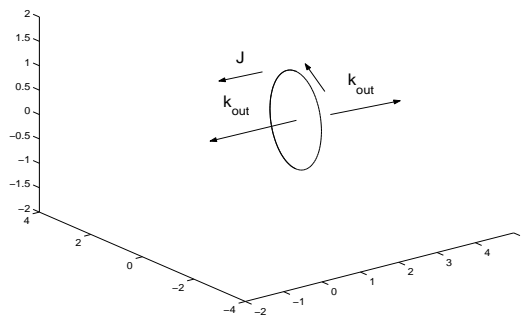
Figure 2. Skeleton of the Kerr geometry [11] formed by the topologically coupled 'circular' and 'axial' strings.

The pp-waves take important role in superstring theory. In the nonperturbative approach based on analogues between the strings and solitons, the pp-wave solutions are considered as fundamental strings forming fundamental classical solutions to the low-energy string theory [30, 31, 32, 33, 34]. The pp-waves may carry traveling electromagnetic and gravitational waves which represent propagating modes of the fundamental string [35]. In particular, the generalized pp-waves represent the singular strings with travelling electromagnetic waves [11, 37].

The string solutions are compactified to four dimensions, and the singular pp-waves are regarded as the massless fields around a lightlike singular source of the fundamental heterotic string. Indeed, metric of the fundamental string solution for travelling waves in z+ direction takes the form [34],

$$ds^2_{str} = e^{2\phi}(2dudv + 2H(v, \zeta, \bar\zeta)dv^2) + 2d\zeta d\bar\zeta, \tag{13}$$

which differs from (12) only by factor $e^{2\phi}$ determined by the extra dilaton field ϕ, which deforms the stringy metric in the longitudinal null direction (note, that the used in string theory 'stringy' metric ds^2_{str} differs from the used in gravity 'Einstein' metric ds^2_E by the conformal rescaling $ds^2_{str} = e^{2\phi}ds^2_E$). In particular, for $\phi = 0$ the both metrics are equivalent. The "solution-generating transforms" [38] allowed Sen to get corresponding charged string with the lightlike moving current and oscillations, which was interpreted as a charged (and superconducting) heterotic string [34]. There appears also an extra axion field and the resulting dilaton field turns out to be singular at the string core.

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[2] Similar spinor solutions were obtained on the KN background in the frame of the coupled Einstein-Maxwell-Dirac equations, [36]. The resulting massless spinor solutions turn out to be singular at the Kerr ring and the fermionic wave excitations generate simultaneously the travelling waves along the Kerr ring and the coupled 'axial' singular pp-waves.

It is suspected that the singular source of the string will be smoothed out in the full string theory which should take into account all orders in α'. Meanwhile, it was shown that the pp-waves have remarkable property that all the α'- corrections in the string equation of motion are automatically zero [31].

It has been noticed that the field structure of the Kerr singular ring is lightlike [8], similarly to the closed fundamental or the heterotic string [39]. The twisted Kerr congruence (Fig. 1.) represents a "hedgehog" defocused by the rotation. The null lines of the Kerr congruence are focused only in equatorial plane ($z = \cos\theta = 0$). The Eq. (5) shows that PNC takes near the ring the form

$$\tilde{k} = k|_{r=\cos\theta=0} = dt - (xdy - ydx)/a = dt - ad\phi, \tag{14}$$

and therefore, the lightlike vector field k_μ near the ring is tangent to the Kerr string world sheet, and the Kerr ring is sliding along itself with the speed of the light. As a consequence, the vector potential (4) and the metric (1) near the Kerr ring are aligned with the local direction of the Kerr string reproducing the structure of the closed pp-string. Therefore, the electromagnetic excitations of the KN background create the circular travelling waves along the Kerr ring coupled with the 'axial' outgoing lightlike pp-beams.

This similarity of the KN ring with the closed string is not incidental, since many solutions to the Einstein-Maxwell theory turn out to be particular solutions to the low energy string theory with a zero (or constant) axion and dilaton fields. Indeed, after compactification the bosonic part of the action for the four-dimensional low-energy string theory takes the form, [40],

$$S = \int d^4x \sqrt{-g}(R - 2(\partial\phi)^2 - e^{-2\phi}F^2 - \frac{1}{2}e^{4\phi}(\partial a)^2 - aF\tilde{F}), \tag{15}$$

and contains the usual Einstein term

$$S_g = \int d^4x \sqrt{-g}R \tag{16}$$

completed by the kinetic term for dilaton field $-2(\partial\phi)^2$ and by the scaled by $e^{-2\phi}$ electromagnetic part $e^{-2\phi}F^2$. The last two terms are related with axion field a and represent its nonlinear coupling with dilaton field

$$-\frac{1}{2}e^{4\phi}(\partial a)^2 \tag{17}$$

and interaction of the axion with the dual electromagnetic field

$$\tilde{F}_{\mu\nu} = \epsilon^{\lambda\rho}_{\mu\nu}F_{\lambda\rho}. \tag{18}$$

It follows immediately that *any solution of the Einstein gravity, and in particular the Kerr solution, is to be exact solution of the effective low energy string theory* with a zero (or constant) axion and dilaton fields.[3] The relationship between the classical cosmic superconducting strings and the heterotic superstrings was first mentioned by Witten in [30]. The KN string is closed, charged and has the lightlike current, what are the characteristic features of the heterotic strings.[4]

The stringy analog to the Kerr-Newman solution with nontrivial axion and dilaton fields was obtained by Sen [33], and it was shown in [10], that the field around the singular string in the

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[3] Situation turns out to be more intricate for the Einstein-Maxwell solutions since the electromagnetic invariant F^2 plays the role of the source of dilaton field, while the term $F\tilde{F}$ turns out to be the source of the axion field.

[4] The characteristic peculiarities of the heterotic strings were identified by Witten with fundamental cosmic strings in [30]. There was also mentioned their relation with axion field and positioning at the boundary of a domain wall.

'axidilatonic' Kerr-Sen solution is very similar to the field around the heterotic string. The usual KN metric written in the Kerr-Schild tetrad form is

$$ds_E^2 = 2e^3 e^4 + 2e^1 e^2, \tag{19}$$

where e^3 and e^4 are real, and e^3 is directed along the Kerr congruence, $e^{3\mu} \sim k^\mu$, while e^1 and e^2 are the two complex conjugate null vectors orthogonal to k^μ. The metric of the axidilatonic Kerr-Sen solution has the form (contrary to (13) we use here the usual Einstein metric)

$$ds_E^2 = 2e^3 \tilde{e}^4 + 2e^1 e^2 e^{-2(\Phi - \Phi_0)}, \tag{20}$$

which shows that the KN metric is deformed by the dilaton field $e^{-2(\Phi - \Phi_0)}$ is the orthogonal to the Kerr congruence directions. Near the Kerr ring, the lightlike direction e^3 is tangent to the ring, and the Kerr-Sen metric is to be deformed by dilaton in the orthogonal to the ring direction. The explicit form of the dilaton

$$e^{-2(\Phi - \Phi_0)} = 1 + \frac{Q^2}{M(r^2 + a^2 \cos^2 \theta)} \tag{21}$$

shows that it is singular near the Kerr ring, where $\cos \theta = 0$, $r \to 0$, and therefore, metric of the closed heterotic string of the Kerr-Sen solution is singular.

This indicates that the Kerr singular ring forms really a heterotic string with has to acquire the lightlike traveling modes. The structure of the Lagrangian (15) shows that the axion field involves the dual magnetic field, and therefore, the complex axidilaton combination $\lambda = a + ie^{i\phi}$ may generate the duality rotation creating a twist of the electromagnetic travelling waves. Exact solutions of this type represent especial interest, but so far are unknown. Note that the complex axidilaton field λ was interpreted in F-theory [39] as a complex structure of a torus forming an extra two-dimensional compact space.

The low energy string theory is classical one. Assuming that the lightlike string forms a core of the electron structure, we have to obtain a bridge to quantum theory of electron. The lightlike traveling waves along the KN closed string generate the spin and mass of the KN particle. Physically, it resembles the original Wheeler's idea of a "geon", [27], in which 'mass without mass' emerges from the system of circulating massless particles. It hints that the adequate first quantized quantum theory should be constructed from the initially massless fields, and we show further, that starting from the massless fields one can derive the usual Dirac equation for the massive electron.

4. Travelling waves and 'emergence' of the Dirac equation

The Wheeler's model of "geon" represents a cloud of the lightlike particles, which are held by own gravitational field. A "microgeon" with the Kerr geometry [8] represents a degenerate case of a sole 'photon' (or other lightlike particle) circulating on the lightlike orbit of the Compton size. It may be considered as a corpuscular analogue to travelling waves along the KN gravitational string.

The observable parameters of the electron indicate unambiguously the Kerr-Newman background spacetime and the Compton radius of the corresponding Kerr string. On the other hand, the Compton radius plays also peculiar role in the Dirac theory, as a limit of localization of the wave packet. Localization beyond the Compton zone creates a "zitterbewegung" affecting *"...such paradoxes and dilemmas, which cannot be resolved in frame of the Dirac electron theory..."* (Bjorken and Drell, [41]). Dirac wrote in his Nobel Prize Lecture : *"The variables* α *(velocity operators, AB) also give rise to some rather unexpected phenomena concerning the motion of the electron. .. It is found that an electron which seems to us to be moving slowly,*

must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light."

These words by Dirac may be considered as a direct hint regarding the inner structure of the electron, the core of which should therefore be represented by a circulating lightlike particle, and consequently, its wave function should satisfy the *massless* wave equation, while the mass should appear as an averaged energy of the circular motion. It corresponds also to generation of the mass in stringy models from excitations of the traveling modes.

4.1. Wheeler's microgeon – mass without mass
The local 4-momentum of a massless particle circulating around z-axis has to be lightlike,

$$p_x^2 + p_y^2 + p_z^2 = E^2, \tag{22}$$

while the effective mass-energy has to be related with an averaged orbital motion,

$$< p_x^2 > + < p_y^2 > = \tilde{m}^2. \tag{23}$$

The lightlike particle circulating along the Kerr ring of radius a will generate the angular momentum

$$\tilde{J}_z = Ea \equiv \tilde{m}a, \tag{24}$$

which is in agreement with the Kerr relation $J_z = ma$ providing consistent source of the mass $\tilde{m} = m$ and angular momentum for the KN solution , $\tilde{J}_z = J_z$. The wavelength of the lightlike particle will be

$$\lambda = 2\pi\hbar/E \equiv 2\pi\hbar/m \equiv 2\pi a\hbar/J_z. \tag{25}$$

For the half-integer z-projection of the spin, $J_z = n\hbar/2$, $n = 1, 2, 3..$, the travelling waves along the KN ring have the half-integer wavelengths. The ratio

$$\frac{2\pi a}{\lambda} = J/\hbar \tag{26}$$

shows the number of the wave-length trapped by the KN ring. For $J_z = \hbar/2$, we obtain that travelling mode should have one-half of the wavelength corresponding to a two-valued wave function, or to be treated on a covering two-sheeted manifold.

Averaging (22) under the condition (23) yields the relation

$$< p_x^2 + p_y^2 + p_z^2 > = \tilde{m}^2 + p_z^2 = E^2. \tag{27}$$

The simplest quantum analogue of this model corresponds to the wave equations obtained from the operator version of these relations: $\vec{p} \to \hat{\vec{p}} = -i\hbar\nabla, \quad \hat{E} = i\hbar\partial_t$.

4.2. Massive wave solutions of the scalar massless equation
We start from the solutions of the scalar massless equation

$$\partial_\mu\partial^\mu\phi = 0, \tag{28}$$

which should satisfy the wave analogue of the constraint (23),

$$(\partial_x^2 + \partial_y^2)\phi = -(\tilde{m}/\hbar)^2\phi. \tag{29}$$

For the considered orientation of spin, the system of the equations (28) and (29) is equivalent to the system

$$(\partial_x^2 + \partial_y^2)\phi = -(\tilde{m}/\hbar)^2\phi = (\partial_t^2 - \partial_z^2)\phi, \tag{30}$$

which shows explicitly that the variables may be separated by the ansatz

$$\phi = \mathcal{M}(x, y)\Phi_0(z, t), \tag{31}$$

where Φ_0 is the wave function of a massive particle which satisfies $(\partial_t^2 - \partial_z^2)\Phi_0 = -(\tilde{m}/\hbar)^2\Phi_0$ or equivalently

$$(\partial_\mu\partial^\mu - (\tilde{m}/\hbar)^2)\Phi_0(x, y) = 0. \tag{32}$$

The corresponding plane wave solution

$$\Phi_0(z, t) = \exp\frac{i}{\hbar}(zp_z - Et), \tag{33}$$

has the usual de Broglie periodicity. In the same time, the l.h.s. of (30) yields the equation

$$(\partial_x^2 + \partial_y^2)\mathcal{M}(x, y) = -(\tilde{m}/\hbar)^2\mathcal{M}(x, y), \tag{34}$$

which determines a factor of "internal vortex structure" $\mathcal{M}(x, y)$. The corresponding cylindrical solutions may be obtained in the polar coordinates ρ, θ. From the relation $x + iy = \rho e^{i\theta}$ we have

$$\rho = \sqrt{(x + iy)(x - iy)}, \quad e^{i\theta} = \sqrt{\frac{x + iy}{x - iy}}, \tag{35}$$

and the solutions of (34) are expressed via the Hankel functions of order ν, $\mathcal{H}_\nu(\frac{\tilde{m}}{\hbar}\rho)$

$$\mathcal{M}_\nu = \mathcal{H}_\nu(\frac{\tilde{m}}{\hbar}\rho)\exp\{i\nu\theta\}, \tag{36}$$

which are eigenfunctions of the operator $\hat{J}_z = \frac{\hbar}{i}\partial_\theta$ with eigenvalues $J_z = \nu\hbar$. For electron we put $J_z = \pm\hbar/2, \quad \nu = \pm1/2$, and the corresponding functions

$$\mathcal{M}_{\pm1/2} = \rho^{-1/2}\exp\{i(\frac{\tilde{m}}{\hbar}\rho \pm \frac{1}{2}\theta)\} \tag{37}$$

turn out to be anti-periodic in θ. One sees that the wave functions acquire singular ray along the vortex axis z, which forms the branch line of spacetime. The half-integer spin appears here from topological reason.

    We introduce operators

$$\hat{m}_\pm = -i\hbar\partial_\pm \equiv -i\hbar(\partial_x \pm \partial_y) \tag{38}$$

and, using the relations

$$\frac{1}{\rho}(\partial_x + i\partial_y)\rho = \frac{1}{x - iy}, \quad (\partial_x + i\partial_y)\theta = \frac{i}{x - iy}, \tag{39}$$

obtain that the equation (34) may be split into the pair

$$\begin{aligned}\hat{m}_+\mathcal{M}_{-1/2}(x, y) &= \tilde{m}\mathcal{M}_{1/2}(x, y) \\ \hat{m}_-\mathcal{M}_{1/2}(x, y) &= \tilde{m}\mathcal{M}_{-1/2}(x, y),\end{aligned} \tag{40}$$

which is similar to splitting of the Dirac equation.

The wave function ϕ, solution of the massless equation (28) is factorized into the plane wave solution $\Psi_0(z, t)$ of the massive Klein-Gordon equation, and the extra singular factor $\mathcal{M}_\nu$, which turns the wave function into a singular string playing the role of a carrier of the de Broglie plane wave $\Psi_0(z, t)$. The resulting solution reproduces the old de Broglie wave-pilot conjecture.[5]

The obtained solutions may be generalized in diverse directions. First of all, there may be obtained the corresponding wave solutions, which are eigenfunctions of the operator of the total angular momentum $\mathbf{J^2}$ and simultaneously of the spin projection operator $\mathbf{J_z}$. The treatment of the spin in terms of the Pauli-Lubanski pseudovector $W^\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}J^{\nu\rho}p^\sigma$ and the Casimir invariant $W^2 = W_\mu W^\mu$ allows one to consider the above solutions in a Lorentz covariant form for arbitrarily positioned and oriented wave functions. And finally, the corresponding solutions of the massless spinor equation may be obtained.

4.3. Massive wave solutions of the Dirac massless equation

Solutions of the Dirac massless equation

$$\gamma^\mu \partial_\mu \psi = 0 \tag{41}$$

may be constructed from the obtained above scalar wave functions $\mathcal{M}_\nu$. It is well known, that if a function $\phi(x)$ satisfies the equation $\partial_\mu \partial^\mu \phi = 0$, the corresponding spinor solution

$$\psi = \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} \phi(x)$$

with arbitrary coefficients A, B, C, D will satisfy the massless Dirac equation (41). In particular, using the relations 38), one can show that the known two basic plane-wave solutions of the Dirac equation[6]

$$\gamma^\mu \partial_\mu \psi_D^r = m\psi_D^r \tag{42}$$

corresponding to positive energy $E > 0$,

$$\psi_D^r(x) = w^r(p) \exp\{-\frac{i}{\hbar}p_\mu x^\mu\}, \tag{43}$$

where $p_\mu = (E, 0, 0, p_z)$,

$$w^1 = \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ 0 \end{pmatrix}, \quad w^2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -\frac{p_z}{E+m} \end{pmatrix}, \tag{44}$$

being modified by the singular function $\mathcal{M}_\nu(x, y)$, yield the wave functions

$$\begin{aligned}
\psi^1(x) &= \mathcal{M}_{-1/2}(x, y)w^1(p) \exp\{-\frac{i}{\hbar}p_\mu x^\mu\}, \\
\psi^2(x) &= \mathcal{M}_{1/2}(x, y)w^2(p) \exp\{-\frac{i}{\hbar}p_\mu x^\mu\},
\end{aligned} \tag{45}$$

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[5] Note that structure of this solution is absolutely different from the well known Bohm model of the double solution.

[6] We use here notations of the book [41].

satisfying the massless Dirac equation (41)[7]. Like the massless scalar solutions, the wave functions of the massless Dirac equation form a singular string, modulated by the Dirac plane wave solution.

The performed separation of the variables allowed us to obtain the ordinary Dirac theory with the massive Dirac equation and the ordinary Dirac plane wave solutions from the solutions of the corresponding massless underlying theory, which regards zitterbewegung as a corpuscular analogue of the travelling waves. Note also that the considered separation of the variables is exact analogue of the usual Kaluza-Klein compactification. However, there is a principal difference that the "compactification" is performed in 4d spacetime, in which the Kerr ring plays the role of an extra dimension, performing the model of a "compactification without compactification".

4.4. Impact of the KN gravity and topology

Like the usual Dirac theory, the above treatment was performed on the flat background, ignoring Kerr-Newman gravity. There appears important question, how these solutions could be deformed by the KN gravitational field. The result may be elucidated by the known method of complex shift (see [13, 36, 43]). This method, initiated by Appel in 1887 , was used by Newman at al. to obtain KN solution from the Kerr one, and it was shown in [8] that the KN electromagnetic field and the wave excitations of the KN solutions may be obtained by the complex shift from the corresponding Coulomb solutions and spherical harmonics. Application of the complex shift to the empty Minkowski space displays strong deformation of its causal structure. In particular, spinor structure of the light cone, which describes a spherically symmetric hedgehog of the light-like directions, is deformed into a two-sheeted structure of the Kerr Congruence which should be described by the Kerr theorem in twistor terms. It follows that the two deforming effects – influence of the gravitational field and impact of the two-sheeted topology – are indeed separated, and the KN background creates the Kerr string and two-sheeted topology even in the limit of the zero gravitational field (zero mass of the source). On the other hand, the role of the KN gravitational field turns out to be very essential too. The known exact solutions on the Kerr-Schild background show that the necessary condition for the consistency with gravity is alignment of the all fields to the Kerr Principal Null Congruence (PNC) [5, 22, 44]. It puts very hard restrictions on the structure of the'solutions. In fact, the usual plane waves turn out to be inconsistent with the Kerr-Schild gravity and should be replaced by the pp-wave analogues, i.e. by fundamental strings, and, as it was described above, the wave excitations take the form of the coupled excitations of the axial and circular strings. Even the very weak Kerr-Schild gravity has very strong impact on the structure of the wave solutions leading to their strong localization near the fundamental strings. As a result, in the KN gravity appears new dimensional parameter – the Compton wavelength $a = J/m$, and the impact of the very weak gravitational field turns out to be essential on the Compton scale, which is very far from the claimed usually Planck scale.

5. Regularization: Electron as gravitating KN soliton

'Emergence' of the Dirac equation from gravitational model of the travelling waves circulating along the closed Kerr string indicates a principally new point of view that gravitation and superstring theory may form some more fundamental theory lying beyond the Dirac theory of electron. On the level of the low energy string theory, the Dirac field loses its meaning of the wave function and should be considered as a source of the charge and currents for the Maxwell equations. The fundamental heterotic strings are charged. The charges are distributed freely

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[7] One sees that the new spinor components acquire explicit dependence $e^{i \pm \frac{1}{2}\theta}$ from angle θ. It corresponds to the spinor representation of the Lorentz group for the case when the axis of rotation is given explicitly [42] (Ch.3, sec.23 and App.a).

along closed strings and localized singularly at a core of the strings. The mechanism of such localization is going beyond the low energy string theory and belongs to an (unknown so far) effective theory of a more high level. As we have seen, metric of the KN and Kerr-Sen solution turns out to be singular too, and the raised in introduction question: "why does Quantum theory works successfully on the flat spacetime, ignoring the stringlike peculiarity of the background gravitational field?" has left unanswered. To justify consistency of Quantum theory with the experimentally observable parameters of the electron, there should also be performed a "regularization of the KN or the Kerr-Sen solution" which should retain the asymptotic form of the KN solution. Regularization of the BH solutions represents a very old problem which is close related with the more general problem of the regularization of the black hole singularity [45] and with the old problem of the regular source of the KN solution [14, 46, 47].

It is often mentioned that the heterotic strings are to be superconducting [30, 38, 34], which assumes the presence of the mechanism of broken gauge symmetry realized by the Higgs field. A hint on the important role of the Higgs field is also coming from the standard model. However, the Higgs field is absent in the Dirac theory of electron, in QED and also in the usual version of the low energy string theory. Meanwhile, the $U(1) \times \tilde{U}(1)$ Witten field model for superconducting cosmic strings [48] suggests that the Higgs field may effectively be used for regularization of the singular core of the heterotic strings. A supersymmetric version of the $U(1) \times \tilde{U}(1)$ Witten field model was applied for regularization of the KN solution in the gravitating soliton model [17], which was mentioned in Section II. Detailed treatment of this model goes out of the frame of this work, and in the next Section we consider it briefly, concentrating on its *gravitational aspect* and on the *interaction of the KN electromagnetic (EM) field with the Higgs field*.

The model of gravitating soliton [17] represents a regular version of the Löpez bubble model [9]. The KN source forms a domain wall separating external KN solution from the flat interior of the bubble which replaces the former Kerr singular ring. Therefore, interior of the bubble is flat and the Kerr singularity is removed.

Gravitational aspect of this model is related with a smooth metric, which interpolates between the external KN solution and the internal flat spacetime. The bubble boundary is determined from the equation $H = 0$, (2), which yields

$$r = r_e = e^2/(2m), \tag{46}$$

where r is the Kerr ellipsoidal radial coordinate, (3). As a result, the bubble takes the form an oblate disk of the Compton radius $a = \hbar/(2m)$ with the thickness $r_e = e^2/(2m)$, corresponding to the known 'classical size' of the electron. The KN electromagnetic field is regularized since the maximal value of the vector potential is realized in the equatorial plane $\cos\theta = 0$, on the stringy boundary of the bubble

$$|\alpha_\mu^{(str)}| \leq e/r_e = 2m/e. \tag{47}$$

Note, that radius of the regularized closed string, being shifted to the boundary of the bubble, turns out to be slightly increased. This position of the string confirms the known suggestions that the heterotic strings have to be formed on the boundary of a domain wall [30].

Chiral sector. The domain wall should provide a smooth phase transition from the external KN solution to a flat false vacuum state inside the bubble. This phase transition is formed by a supersymmetric system of the chiral fields Φ^i, [17] and by the potential $V(r)$ generated from the specially adapted superpotential W, which provides the smooth transfer from the external KN 'vacuum state', $V_{ext} = 0$, to a flat internal 'pseudovacuum' state, $V_{int} = 0$. Note, that the considered chiral field model is a supersymmetric version of the suggested by Witten $U(1) \times \tilde{U}(1)$ chiral field model for superconducting strings [48], see details in [17, 14]. Therefore, the assumptions that the heterotic strings are to be superconducting [30] are also confirmed in this model.

Interaction of the Higgs field with the KN EM field is controlled by the system of equations

$$\partial^\nu \partial_\nu \alpha_\mu = I_\mu = e|\Phi|^2(\chi_{,\mu} + e\alpha_\mu), \tag{48}$$

which are equivalent to the equations used by Nielsen and Olesen for the stringlike vortex in a superconductor, [49]. The Higgs field $\Phi = |\Phi|\exp\{i\chi\}$ forms a superconducting condensate inside the bubble, which regularizes the KN EM field by mechanism of broken symmetry. Inside the bubble, the KN EM potential α^μ is eaten by the Higgs field and the current I_μ should vanish there, $I_\mu = 0$. It gives

$$\partial^\nu \partial_\nu \alpha_\mu = 0, \quad \chi_{,\mu} + e\alpha_\mu = 0, \tag{49}$$

which shows that gradient of the Higgs phase $\chi_{,\mu}$ compensates the gauge field α_μ. The field strength and currents are expelled to the string-like boundary of the bubble, and there appear the following two conditions:

$$\omega = \chi_{,0} = -e\alpha_0^{(str)}, \quad \chi_{,\phi} = -e\alpha_\phi^{(str)}. \tag{50}$$

We have seen earlier, that the spacelike component $\alpha^{(str)}$ of the KN potential $\alpha_\mu^{(str)} = (\alpha_0^{(str)}, \alpha^{(str)})$ is tangent to the Kerr string. As a consequence, it forms a regular flow in ϕ-direction, $\alpha_\phi^{(str)}$, and creates a closed Wilson line along the Kerr string. As a result of (50), the KN gravitating soliton exhibits two important peculiarities:

- the quantum Wilson loop $e \oint \alpha_\phi^{(str)} d\phi = -4\pi ma$ leads to quantization of the total angular momentum of the soliton, $J = ma = n/2, \ n = 1, 2, 3, ...,$
- the Higgs field inside the bubble turns out to be oscillating with the frequency $\omega = 2m$, and therefore, it forms a coherent vacuum state, typical for the 'oscillon' bubble models.

The negative sheet of the metric disappears and metric turns out to be regularized and practically flat. The model contains the lightlike heterotic string on the border of the bubble, however, it is axially symmetric and the travelling waves are absent. Therefore, some extra excitations are needed to create the travelling waves.[8] The Compton size of the bubble does not contradict to the "dressed" QED electron, however, there is an essential difference. The dynamics of the virtual particles in QED is chaotic and can be conventionally separated from the "bare" electron, while the vacuum state inside the KN soliton forms a *coherent oscillating state* joined with the closed Kerr string. Therefore, the bubble source of the KN soliton and the Kerr String represent an *integral coherent structure* which cannot be separated from the "bare" particle.

6. Conclusion

We have showed that gravity definitely indicates presence of a closed string of the Compton radius $a = \hbar/(2m)$ in the electron background geometry. This string has gravitational origin and is closely related with the fundamental heterotic string of the low energy string theory. Starting from the corpuscular aspect of the underlying model of the massless travelling modes along the Kerr string, we showed 'emergence' of the usual Dirac equation and the corresponding 'massive' solutions with de Broglie periodicity. The original Dirac theory is modified in this case, and the wave functions acquire the singular stringlike carriers, so that the Dirac plane wave solutions turn out to be propagating along 'axial' singular strings. Contrary to the widespread opinion that quantum theory predominates over all other theories, the considered structure indicates the opposite: gravity, as a basic part of the underlying superstring theory, is to be lying beyond

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[8] This point has interesting consequences which can be interpreted in the terms of quarks, and we expect to considered it elsewhere.

quantum theory. As we have seen, in the KN gravity appears a new dimensional parameter – the Compton wavelength $a = J/m$, and *the impact of the very weak gravitational field turns out to be essential on the Compton scale, which is very far from the usually claimed Planck scale.* The Kerr-Newman solution, together with interpretation of its source as a closed heterotic string, forms a bridge between gravity, superstring theory and the Dirac quantum theory, displaying a new way towards unification of quantum theory and gravity.

The observable parameters of the electron show unambiguously that the background of the electron should be determined by the KN geometry and should contain the Kerr string of the Compton radius. This radius is very big with respect to the modern scale of the experimental resolution, and it seems that this string should be experimentally detected. However, the high-energy scattering detects the pointlike structureless electron down to $10^{-16}cm$. One of the reasons of this fact was considered earlier in [12], where it was argued that the pointlike character of the interactions may be caused by the interactions of the KN particles via the considered above 'axial' pp-strings – carriers of the wave function. There are also diverse alternative explanations, for example, related with a known complex interpretation of the KN solution [8, 10, 13, 36], which shows that the real KN solution is generated by a complex pointlike source shifted in the imaginary direction. Here we present a new explanation of this fact related with the lightlike character of the closed heterotic string. Observation of the lightlike, relativistically rotating closed string should be accompanied by the *Lorentz contraction* for each line element of the closed string, and as a result, to a shrinking of the full image of this string to a point in the ultrarelativistic limit.

Indeed, observation of the lightlike objects is a very nontrivial process which depends essentially on the method of the observation. In particular, it has been shown by Penrose [50] that a *momentary photo-image* of a relativistic sphere shouldn't display the Lorentz contraction at all. Similar effect for a meter stick was obtained still earlier by Terrel [51]. It doesn't mean that the Einstein relativity is wrong, rather, the effect of the Lorentz contraction depends essentially on the method of observation. We note that the relativistic scattering differs absolutely from the *momentary photo-image*, and visibility of the Lorentz contraction by the relativistic scattering on the closed relativistic heterotic string is a-priori unclear.

For an intermediate *real* photon, propagating between the separated points P_1 and P_1, the usual null condition for the lightlike interval $s_{12}^2 = 0$ is valid. Such a scattering corresponds to an 'action-at-a-distance' and should lead to a pointlike 'contact' interaction which doesn't allow one to determine the form of a string or its size. An alternative regime may be related with scattering of a virtual photon, for which the strong lightlike condition $s_{12}^2 = 0$ is broken. Specifically, to recognize the stringlike relativistic object of a big size, there are necessary two special conditions:

a) the scattering should be *deeply virtual*, which means very large $Q^2 = q_{12}^2$, and $p \cdot q_{12}$,

b) the momentum transfer should be *relatively low* to provide the wavelengths comparable with extension of the object.

Both these conditions are satisfied in the novel method of scattering – the "non-forward Compton scattering", or Deeply Virtual Compton Scattering (DVCS), suggested by Radyushkin and Ji [19, 20] and broadly discussed last decade. It is projected now as a new regime of the high energy scattering, which could realize tomography of the elementary particles [21].

Experimental observation of the predicted closed heterotic string in the electron structure could have an extraordinary meaning leading to essential progress in understanding of Quantum theory and development of Quantum Gravity.

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