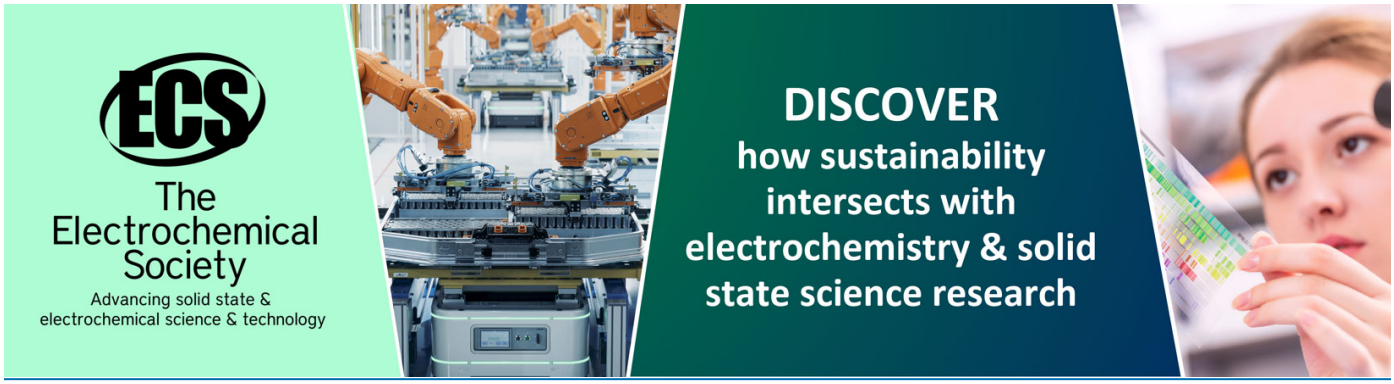
Quantumness beyond quantum mechanics



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Quantumness beyond quantum mechanics

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Abstract. Bohmian mechanics allows us to understand quantum systems in the light of other quantum traits than the well-known ones (coherence, diffraction, interference, tunnelling, discreteness, entanglement, etc.). Here the discussion focusses precisely on two of these interesting aspects, which arise when quantum mechanics is thought within this theoretical framework: the *non-crossing property*, which allows for distinguishability without erasing interference patterns, and the possibility to define *quantum probability tubes*, along which the probability remains constant all the way. Furthermore, taking into account this hydrodynamic-like description as a link, it is also shown how this knowledge (concepts and ideas) can be straightforwardly transferred to other fields of physics (for example, the transmission of light along waveguides).

1. Introduction

At present, there is no doubt that quantum mechanics can be considered the most successful theory ever devised to explain the physical world. Its applicability ranges from very fundamental physical problems to the high-technology applications that are nowadays an important part of our daily life. This theory, though, still constitutes a veiled mystery at a deeper level of understanding, for there is a lack of a clear interpretation of the physics underlying quantum systems. This is somehow connected to its widely accepted interpretation, namely the Copenhagen interpretation [1], which not only does not allow us to think of quantum systems as we do of classical ones, but it just forbids such a thing.

A feasible way to surmount this drawback (although surely not the final one) comes through Bohmian mechanics [2–5]. According to this approach to the quantum world, systems are described in terms of trajectories evolving in configuration space. Rather than constituting a step backwards towards the classical paradigm, this quantum-mechanical formulation puts at the same descriptive level both classical and quantum systems, though the latter's behaviour differs from the former's one through the dynamical particularities induced by the quantum laws of motion. For instance, these laws give rise to motions constrained to the configuration space, uncertainty principles, interference phenomena, discreteness, and other features which are absent in classical systems. Of course, in the statistical limit, i.e., when an appropriate sampling over many of such Bohmian trajectories is carried out, the results of the standard quantum mechanics are recovered.

On the other hand, we find that in principle there are no restrictions to export the core idea in Bohmian mechanics, namely the possibility to reformulate quantum mechanics as a hydrodynamic-like theory, as well as the ideas emerging from it to other physical contexts where waves or distributions are the primary descriptor. This is somehow the other way around that

Madelung [6] considered in 1926, when he tried to provide a hydrodynamical interpretation of quantum processes based on the analogy between the latter and classical hydrodynamical flows. Within this approach, Bohmian trajectories were just considered to be the "tracks" displayed by some hypothetical *tracer particles* left on top of the quantum fluid [7], something very similar to what can be found in modern studies of flows, where the trajectories displayed by a number of tracer particles is recorded and subsequently analysed in order to extract information about the flow. By means of this method, for example, studies revealed anomalous diffusion and Lévy flights in two-dimensional rotating flows [8, 9]. However, closer in spirit to the present discussion, we find the recent observations by Couder and coworkers [10–14] as well as by Bush [15], where a seemingly classical analogue of Bohmian motion is found for droplets bouncing on a vertically vibrating bath of the same fluid.

The purpose of the present work is to pose an analysis and discussion of some interesting properties of quantum systems and their consequences, which arise when they are investigated from a Bohmian perspective. Furthermore, the link that can be established between the Bohmian viewpoint and its application or extension to other non-quantum wave theories will also be discussed. In order to make clear the arguments exposed, the work has been organized as follows. In Section 2, the essential elements/concepts of Bohmian mechanics are introduced as well as some important consequences implied by this theory. The transfer of knowledge from Bohmian mechanics to other classical wave theories, such as electromagnetism, is discussed in Section 3 in the context of light waveguiding. To conclude, a series of final remarks are highlighted in Section 4.

2. What can be learnt from Bohmian mechanics?

2.1. Some elementary background

In the standard Bohmian approach [2], one usually starts considering the wave function in polar form,

$$\Psi(\mathbf{r},t) = R(\mathbf{r},t)e^{iS(\mathbf{r},t)/\hbar}, \tag{1}$$

which is substituted into the (single-particle) non-relativistic, time-dependent Schrödinger equation,

$$i\hbar\,\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\,\nabla^2 + V\right)\Psi, \tag{2}$$

to yield the system of real coupled equations

$$\frac{\partial R^2}{\partial t} + \nabla\cdot\left(R^2\,\frac{\nabla S}{m}\right) = 0, \tag{3}$$

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2}{2m}\frac{\nabla^2 R}{R} = 0. \tag{4}$$

In brief, the continuity (or conservation) equation (3) rules the ensemble dynamics, i.e., the number of particles described by the probability density has to remain constant; the quantum Hamilton-Jacobi equation (4), on the other hand, describes the time-evolution of the phase field, which governs the motion of quantum particles through the guidance equation

$$\mathbf{v}_B = \dot{\mathbf{r}} = \frac{\nabla S}{m}. \tag{5}$$

In this way, the simple transformation described by (1) allows us to go a step beyond standard quantum mechanics. In the latter, Ψ is all what is needed to describe and interpret the experimental outcomes; in Bohmian mechanics, on the other hand, the evolution of a *tracer*

particle [7] according to the information supplied by Ψ (the guiding field) is also considered to further understand the quantum system dynamics.

Now, the same description can be formally inferred by simply assuming that the evolution of any quantum system can be identified with the diffusion of a (quantum) fluid throughout configuration space, as formerly proposed by Madelung [6]. Therefore, as in the case of classical fluids, the elementary quantities of this approach would be the (probability) density of the quantum fluid and its associated probability current density, $\mathbf{J}$, both related through the conservation (or continuity) equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0, \tag{6}$$

where $\mathbf{J} = \rho \mathbf{v}_H$ and $\mathbf{v}_H$ is a local hydrodynamic velocity field. The probability density and the probability current density are related to the wave function (1) by the transformation relations

$$\rho = R^2, \qquad \mathbf{J} = R^2 \, \frac{\nabla S}{m}, \tag{7}$$

with the local velocity field being

$$\mathbf{v}_H = \frac{\mathbf{J}}{\rho} = \frac{\nabla S}{m}, \tag{8}$$

which is equivalent to the Bohmian one, $\mathbf{v}_B$, given by (5) (hence, from now on, both velocity fields will be denoted by $\mathbf{v}$). The integration in time of (8) generates a family of streamlines or paths (for each wave function considered) along which the quantum fluid propagates, just as in the case of a classical fluid. As it can be inferred, the first equality goes beyond Bohmian mechanics and allows to define streamlines in any system characterized by a certain density and a vector that transports it through the corresponding configuration space, regardless whether such a density describes a quantum system or not.

2.2. The non-crossing rule

One of the most relevant properties revealed by Bohmian mechanics is that quantum fluxes cannot cross in configuration space. That is, at a certain time t, each point on configuration space has a uniquely-defined value of the local velocity field, except for regions where the wave function displays a node (i.e., $\Psi = 0$). Therefore, two or more Bohmian trajectories cannot cross through such a point at the same time t or, equivalently, *quantum systems cannot reach the same configuration at a given time following different quantum streamlines*. In terms of the standard quantum mechanics, this just means that paths can be labelled, although they cannot be experimentally *observed* because this implies changing the process and therefore generating new, alternative paths. Obviously, this changes dramatically our physical view of the superposition principle, which states that any linear combination of solutions of Schrödinger's equation is also a formally valid solution to the same equation. Accordingly, given a coherent superposition of two counter-propagating wave packets, they can be evolved in time separately and then recombined at any time to determine when and where an interference pattern. Formally, there is no problem in proceeding in this way. However, from a (hydro) dynamical viewpoint the evolution of both wave packets has to be accounted for together for the interference pattern to be observable (which is what actually happens in an experiment), this implying that interpretations of physical phenomena cannot directly rely upon the superposition principle. Bohmian mechanics provides us an answer for this based precisely on the non-crossing rule.

In order to make clear such a statement, consider the wave packets above are represented by two free expanding Gaussian wave packets,

$$\Psi(x,t) = \left(\frac{1}{2\pi \tilde{\sigma}_t^2} \right)^{1/4} e^{-(x-x_{\rm cl})^2/4\tilde{\sigma}_t \sigma_0 + i p_0 (x - x_{\rm cl})/\hbar + iEt/\hbar}, \tag{9}$$

where the time-dependence of their spreading is given by

$$\sigma_t = |\tilde{\sigma}_t| = \sigma_0 \sqrt{1 + \left(\frac{\hbar t}{2m\sigma_0^2}\right)^2}. \tag{10}$$

The wave packet (9) propagates along the classical trajectory $x_{\rm cl} = x_0 + v_0 t$ (with $v_0 = p_0/m$), while the associated Bohmian trajectories are given by

$$x(t) = x_{\rm cl} + \frac{\sigma_t}{\sigma_0}\ [x(0) - x_0]\,, \tag{11}$$

with $x(0)$ denoting the initial position of the corresponding Bohmian trajectory. As can be noticed, there are two contributions to the evolution of the tracer particles, one coming from the classical drift and another one from a purely quantum origin [16, 17]. The former makes the particle to evolve according to only the laws of classical mechanics; the latter makes it to deviate in such a way that its motion accommodates to the evolution of the quantum fluid. The time-scale that separates both types of motion (to some extent) is given in this particular case by the characteristic time $\tau \equiv 2m\sigma_0^2/\hbar$ [16, 17]. Depending on the relative value of t when compared with τ, three regimes can then be found (in analogy with the regimes of optical physics [7]):

- *Ehrenfest or Huygens* ($\tau \equiv 2m\sigma_0^2/\hbar \ggg t$): Bohmian trajectories follow classical ones (in agreement with Ehrenfest's theorem).
- *Near-field or Fresnel* ($\tau \equiv 2m\sigma_0^2/\hbar > t$): Bohmian trajectories start to diverge from their classical counterparts.
- *Far-field or Fraunhofer* ($\tau \equiv 2m\sigma_0^2/\hbar \ll t$): Bohmian trajectories display a sort of rectilinear, uniform evolution due to the stationarity of the wave function far from any interaction potential.

Therefore, by inspecting the topology displayed by the quantum trajectories, one readily notices that the quantum flow evolves from an initially confined fluid to a linearly expanding one, undergoing at times of the order of τ a sort of internal boosting which bursts it open. However, this does not provide any clue on the true dynamics.

In order to understand the importance of not disentangling the joint evolution of both wave packets, as indicated by the superposition principle, consider the full wave function,

$$\Psi(\mathbf{r}, t) = \psi_1(\mathbf{r}, t) + \psi_2(\mathbf{r}, t), \tag{12}$$

where the $\psi_i = \rho_i^{1/2} e^{iS_i/\hbar}$ are properly normalized and have the shape indicated by (9), with $x_{2,0} = -x_{1,0} = x_0 > 0$ and $v_{1,0} = -v_{2,0} = v_0 > 0$. It is straightforward to notice that

$$\rho = \rho_1 + \rho_2 + 2\sqrt{\rho_1\rho_2}\cos\varphi, \tag{13}$$

$$\mathbf{J} = \frac{1}{m}\left[\rho_1\nabla S_1 + \rho_2\nabla S_2 + \sqrt{\rho_1\rho_2}\nabla(S_1 + S_2)\cos\varphi + \hbar\left(\rho_1^{1/2}\nabla\rho_2^{1/2} - \rho_2^{1/2}\nabla\rho_1^{1/2}\right)\sin\varphi\right], \tag{14}$$

with $\varphi = (S_2 - S_1)/\hbar$. Substituting these expressions into (5) leads to the equation of motion

$$\dot{\mathbf{r}} = \frac{1}{m}\frac{\rho_1\nabla S_1 + \rho_2\nabla S_2 + \sqrt{\rho_1\rho_2}\nabla(S_1 + S_2)\cos\varphi}{\rho_1 + \rho_2 + 2\sqrt{\rho_1\rho_2}\sin\varphi} + \frac{\hbar}{m}\frac{\left(\rho_1^{1/2}\nabla\rho_2^{1/2} - \rho_2^{1/2}\nabla\rho_1^{1/2}\right)\sin\varphi}{\rho_1 + \rho_2 + 2\sqrt{\rho_1\rho_2}\cos\varphi}. \tag{15}$$

The crossed term in (13) and (14) arises from the interference of the probability densities and current densities associated with ψ_1 and ψ_2 after assuming the coherent superposition (12). However, as seen from (15), the local velocity field keeps a more complicated form, which cannot

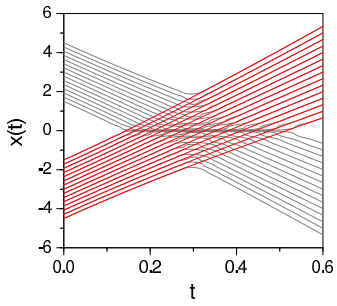

Figure 1. Bohmian trajectories associated with a coherent superposition of two Gaussian wave packets (gray lines) and with a single Gaussian wave packet (red lines).

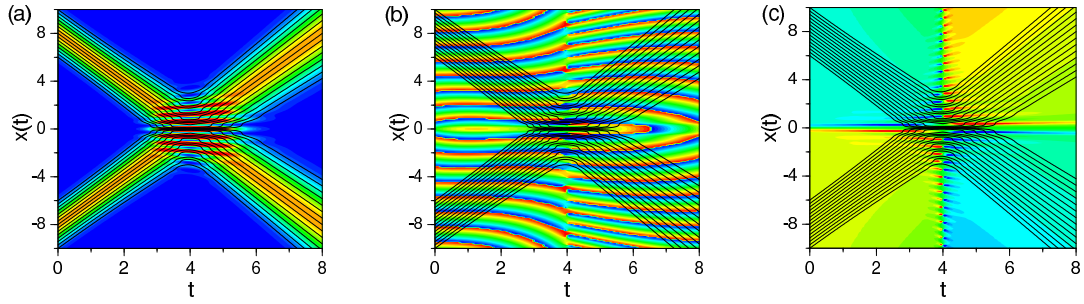

Figure 2. Contour-plots representing the evolution in time of the probability density (a), the phase field (b) and the local velocity field (c) of an initial coherent superposition of two Gaussian wave packets. The Bohmian trajectories (black lines) associated with this problem are also superimposed to each graph.

be expressed as a simple superposition, thus indicating that the dynamics is going to be far more complex. This is illustrated in Fig. 1, where the single wave-packet Bohmian trajectories (red) diverge from the behaviour displayed by the two wave-packet ones as they approach interference region, ending up eventually in the wrong place. This can be readily understood by looking at the three graphs displayed in Fig. 2: although the probability density (part (a)) does not provide any clue, both the phase field (part (b)) and the local velocity field (part (c)) immediately make apparent the dynamical consequences of the superposition.

It is worth stressing that the non-crossing takes place regardless of the presence of the quantum potential. Rather it is an intrinsic or inherent property associated with the dynamics generated through a guidance condition. In classical mechanics we can also observe something similar, but in phase space, where phase-space orbits cannot cross through the same point at the same time. In such a case, the "guiding field" is the classical action (this was actually the idea that led Schrödinger in the derivation of his wave theory of quanta [18]). Nevertheless, in the two wave-packet problem presented above, the non-crossing can still be well represented by the action of an effective potential acting on a single wave packet, with the form of a repulsive wall and an attractive region, both depending on time [17].

2.3. Quantum probability tubes

The possibility to study quantum processes in terms of Bohmian trajectories brings in another interesting property, namely the possibility to define *quantum probability tubes* [19], i.e., tubes in configuration space along which the integral of the probability density at a time t remains constant regardless of changes in their profile or in the shape of the enclosed sections of ρ. In order to show this, consider the so-called *restricted probability*, defined as

$$\mathcal{P}(t) \equiv \int_{\Omega} \rho(\mathbf{r}, t) d\mathbf{r}, \tag{16}$$

which gives the probability to find the system inside a certain region of interest Ω of the corresponding configuration space at a time t [20–22]. From standard quantum mechanics (and making use of the divergence or Gauss-Ostrogradsky theorem), the variation with time of $\mathcal{P}(t)$ can be expressed as

$$\frac{d\mathcal{P}(t)}{dt} = \int_{\Omega} \frac{\partial \rho}{\partial t}\, d\mathbf{r} = -\int_{\Omega} (\nabla \cdot \mathbf{J})\, d\mathbf{r} = -\int_{\Sigma} \mathbf{J} \cdot d\mathbf{S}, \tag{17}$$

where Σ is the boundary of Ω. That is, the losses/gains of $\mathcal{P}(t)$ inside Ω are described by the outgoing/ingoing probability flow through Σ, which is accounted for by the quantum current density $\mathbf{J}$. Defining an arbitrary region Ω in standard quantum mechanics is rather simple. However, how can this region be propagated in time so that $\mathcal{P}(t)$ keeps the same value along time?

Although the answer to the question posed above can be quickly answered in classical mechanics by using arguments based on the Liouvillian structure of this theory [23], the same cannot be found in standard quantum mechanics, but in Bohmian mechanics and, more specifically, in its non-crossing property. As shown elsewhere [19], if the surface Σ is the geometric place formed by a (discrete or continuous) set of initial conditions, because of the non-crossing property, any probability enclosed inside such a set will remain constant at any subsequent time. This is based on the fact that at any given time t no trajectory inside or outside the boundary defined by the set of *separatrix* trajectories will be able to penetrate into or exit from Ω respectively. Therefore, if the probability $P(t)$ is measured in terms of the accumulation of trajectories embraced by Σ, i.e.,

$$\mathcal{P}(t) \approx \sum_{i=1}^{N} \delta(\mathbf{r}_i(t) \in \Sigma(t)), \tag{18}$$

this number must remain constant at any time provided $\Sigma(t)$ is a causal map of $\Sigma(0)$ at time t. An important consequence of this result is that *any restricted probability can be determined directly from the initial state if the end points of the associated separatrix trajectories are known.* This means that, in principle, one could determine (or, at least, estimate) final probabilities without carrying out the full calculation [24], but directly from the particular region covered by the initial wave function causally connected with the feature of interest (e.g., a transmitted amplitude or a diffraction peak), since

$$\mathcal{P}_{\infty} = \int_{\Sigma_{\infty}} \rho(\mathbf{r}_{\infty}) d\mathbf{r}_{\infty} = \lim_{t\to\infty} \int_{\mathbf{r}(t)} \rho[\mathbf{r}(t)] d\mathbf{r}(t) = \lim_{t\to\infty} \mathcal{P}(t) = \int_{\mathbf{r}_0} \rho[\mathbf{r}_0] d\mathbf{r}_0 = \mathcal{P}_0. \tag{19}$$

Here, $\rho(\mathbf{r}_{\infty})d\mathbf{r}_{\infty}$ is the probability to find the system confined within the layer of configuration space defined by the separatrix trajectories ending at $\mathbf{r}_{\infty}$ and those ending at $\mathbf{r}'_{\infty} = [\mathbf{r} + d\mathbf{r}]_{\infty}$. Analogously, $\rho[\mathbf{r}(t)]d\mathbf{r}(t)$ is the probability to find the system confined within the layer defined by the sets of separatrix trajectories $\mathbf{r}(t)$ and $\mathbf{r}'(t) = [\mathbf{r} + d\mathbf{r}](t)$ at a time t. Note that, since the probability density is being evaluated along quantum trajectories, it is preferably denoted as $\rho[\mathbf{r}(t)]$ instead of $\rho(\mathbf{r}, t)$, with $\mathbf{r}_{\infty} = \mathbf{r}(t \to \infty)$.

In order to illustrate this result, consider the scattering of a wave packet with a rather arbitrary shape by a barrier, as displayed in Fig. 3(a) (details on this calculation can be found in [19]). The initial and final probability densities are denoted by the black and red solid lines, respectively, while the barrier is indicated by the blue shadowed region. The propagation goes on until the probability in the region covered by the barrier (intra-barrier resonance) becomes negligible, which can be better seen in Fig. 3(b), where the transmission (green dashed line), reflection (blue dash-dotted line) and intra-barrier resonance (red dotted line) probabilities are

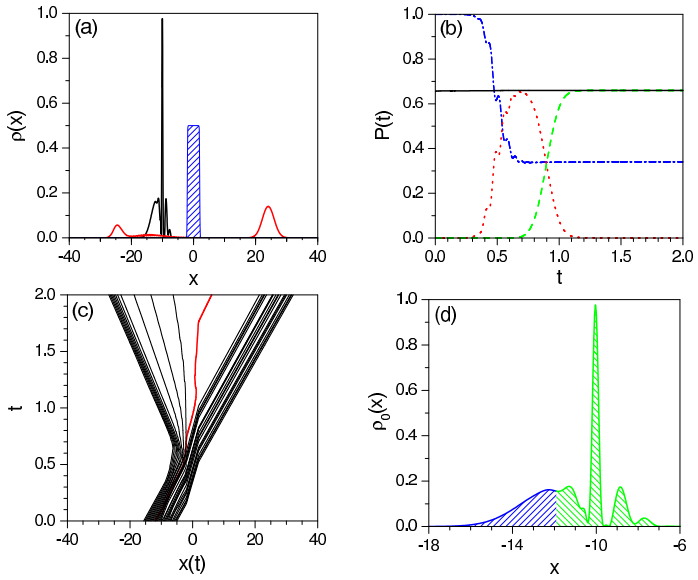

Figure 3. Scattering of a general wave packet by an almost square potential barrier. (a) Initial (black line) and final (red line) probability densities; the barrier is denoted with the blue shadowed region. (b) Time-dependence of different restricted probabilities: transmission (green dashed line), reflection (blue dash-dotted line) and intra-barrier resonance (red dotted line) probabilities. The black solid line denotes the transmission probability directly computed from the initial state (see text for details). (c) Bohmian trajectories illustrating the tunnelling dynamics. The separatrix trajectory is displayed with the red thicker line. (d) Initial probability density split according to the separatrix initial condition: initial conditions in the green shadowed region contribute to transmission and those from the blue shadowed one to reflection.

recorded along time. These are the three restricted probabilities considered in this example, with $\Omega_{\rm trans}$ being the region beyond the right edge of the barrier, $\Omega_{\rm res}$ the region bound between the two edges of the barrier, and $\Omega_{\rm refl}$ the region to the left edge of the barrier. After $t \approx 1.15$, $\mathcal{P}_{\rm res} \approx 0$ and $\mathcal{P}_{\rm trans}$ reaches its maximum, asymptotic value; $\mathcal{P}_{\rm refl}$, on the contrary, reaches its minimum earlier, at $t \approx 0.75$. This means than $\mathcal{P}_{\infty,\rm trans}$ can be computed by simply choosing as the separatrix trajectory a Bohmian trajectory that reaches the left tail of the transmitted wave packet in part (a) and propagating it backwards in time, as seen in part (c). Integrating the probability density between the position of this trajectory at each time and infinity (for practical purposes), it is then found that $\mathcal{P}_{\infty,\rm trans}$ remains constant all the way, as the black straight line shows in part (b). Regarding the initial set (probability density), it is found (see part (d)) that any initial condition started to the right of the separatrix initial condition will get into the transmitted region, while if it starts to its left, the trajectory becomes reflected.

3. Where can this knowledge be transferred to?

It is very common to separate quantum physics from other wave theories based on the difference in the treatment of probability amplitudes with respect to other types of waves (which are considered as perturbations propagating through a given medium or vacuum). However, the way how these wave theories operate is in essence the same, for both include the same type of concepts (e.g., coherence, interference, diffraction, tunnelling, discreteness, etc.) and principles (e.g., uncertainty, superposition). Taking this into account, it is possible to establish a sort of feedback regarding the understanding of both wave phenomena, in general, and quantum phenomena, in particular, through the hydrodynamic (Bohmian) approach considered above.

To briefly show how this transfer of knowledge can be done, consider the case of electromagnetic fields, for example. Bohmian mechanics was formerly formulated to describe massive particles, resulting very appealing to explain event-to-event experiments like those carried by Merli *et al* [25], Tonomura *et al.* [26], or Shimizu *et al* [27]. Now, there are also event-to-event experiments with light, such as those performed by Dimitrova and Weis [28], which claim for a treatment on equal footing. As shown by Prosser [29], this is actually possible by directly considering Maxwell's equations (which put electromagnetism at the same level of Schrödinger's wave mechanics [30]). In this case, given an electromagnetic field[1] defined by an electric field $\mathbf{E}(\mathbf{r})$ and a magnetic field $\mathbf{H}(\mathbf{r})$, the two key elements necessary to define the corresponding streamlines are the electromagnetic energy density and the Poynting vector (electromagnetic current density), i.e.,

$$U(\mathbf{r}) = \frac{1}{4}\left[\epsilon_0 \mathbf{E}(\mathbf{r})\cdot\mathbf{E}^*(\mathbf{r}) + \mu_0 \mathbf{H}(\mathbf{r})\cdot\mathbf{H}^*(\mathbf{r})\right]. \tag{20}$$

$$\mathbf{S}(\mathbf{r}) = \frac{1}{2}\,\mathrm{Re}\left[\mathbf{E}(\mathbf{r})\times\mathbf{H}^*(\mathbf{r})\right], \tag{21}$$

respectively. Since the electromagnetic energy density is transported through space in the form of the Poynting vector, a local velocity field can be defined [31] in analogy to (8), which reads as

$$\mathbf{S}(\mathbf{r}) = U(\mathbf{r})\mathbf{v}, \tag{22}$$

from which electromagnetic energy flow lines or photon paths are obtained by integrating the equation

$$\frac{d\mathbf{r}}{ds} = \frac{1}{c}\frac{\mathbf{S}(\mathbf{r})}{U(\mathbf{r})}, \tag{23}$$

along the arc-length coordinate s (which can be referred to a proper time $\tau = s/c$, with c being the speed of light). By means of this procedure, the interference of diffracted polarized beams was studied [32, 33], showing a good agreement with experiments carried out later on to infer the average paths displayed by photons [34].

The previous discussion focussed on Maxwell's equations. However, it is not necessary to stay at such a level in order to define hydrodynamic streamlines. As mentioned above, this can be done at any level where the elementary descriptive tool is a wave equation. Thus, consider, for example, the transmission of a light pulse through a waveguide within the small-angle or paraxial approximation [31]. Assuming that the optical axis is oriented along the z-axis and the electromagnetic field passing through the waveguide is time-harmonic, the field can be approximated by a plane wave along the z-direction modulated by a certain complex-valued amplitude,

$$\Phi(\mathbf{r}) = \phi(\mathbf{r})e^{ik_z z}, \tag{24}$$

where $k_z = n_0 k$, $k = 2\pi/\lambda$, λ is the light wavelength in vacuum and n_0 is the bulk refractive index. Substituting this expression into Helmholtz's equation,

$$\nabla^2\Phi + n^2k^2\Phi = 0, \tag{25}$$

with n being the refractive index inside the waveguide, one readily finds

$$2ik_z\,\frac{\partial\phi}{\partial z} + \frac{\partial^2\phi}{\partial z^2} = -\nabla_\perp^2\phi + (k_z^2 - n^2k^2)\phi, \tag{26}$$

[1] Time-independent electromagnetic fields will be considered here for simplicity, although time-dependent ones should be used, in general.

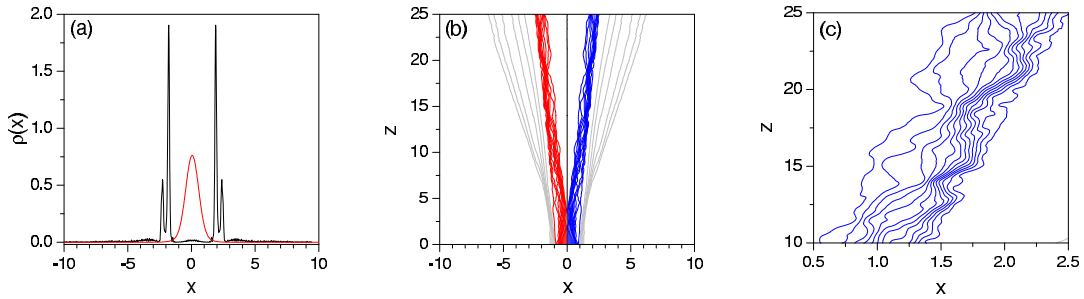

Figure 4. Light transport through a waveguide with the shape of a Y-junction. (a) Initial (black) and final (red) probability densities. (b) Optical streamlines illustrating the dynamics inside the waveguide. (c) Enlargement of part (b).

where $\nabla_\perp^2$ is the transverse part of the Laplacian (for example, in Cartesian coordinates, it reads as $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$). If, apart from the paraxial approximation, it is also assumed that the slowly varying envelope approximation holds (i.e., the amplitude ϕ varies slowly in space compared to $2\pi/k_z$), the highest-order derivative in z can be neglected (i.e., $\partial^2\phi/\partial z^2 \approx 0$) and Eq. (26) can be recast as

$$i\,\frac{\partial\phi}{\partial z} = -\frac{\nabla_\perp^2\phi}{2k_z} + \frac{k}{2n_0}\left(n_0^2 - n^2\right)\phi. \tag{27}$$

This equation is, indeed, isomorphic to Schrödinger's one, with z playing the role of the evolution parameter (rather than the time t) and $k_z = n_0 k$ the role of an "optical mass". Equation (27) has been used, for example, to study the design of waveguides with optimal conditions of light transmission [35–37]. However, regarding our discussion here, it also brings the possibility to define a family of *optical streamlines* if the pulse $\phi(\mathbf{r})$ is expressed in polar from, as $\phi(\mathbf{R}, z) = \rho^{1/2}(\mathbf{R}, z)e^{iS(\mathbf{R},z)}$, where $\mathbf{R}$ denotes the vector perpendicular to the propagation direction (z). In this case, the local velocity field reads as

$$\mathbf{v} = \frac{d\mathbf{R}}{dz} = \frac{\nabla_\perp S}{k_z}, \tag{28}$$

which gives rise to such streamlines when integration over z is carried out.

In Fig. 4 we have an example of how these trajectories look like in the particular case of a Y-junction [38], which splits the initial pulse (red line) into two emerging pulses (black line), as seen in part (a). As it can be noticed in part (b), also here the electromagnetic flux follows the non-crossing rule, which allows to distinguish between different paths (and therefore to also define density tubes) without disturbing the experiment, also in agreement with the experiment performed by Kocsis *et al* [34]. Actually, in the enlargement of part (c), a behavior mimicking that of the frustrated internal reflection (inside the waveguide) can be observed, although contrary to geometric rays, the optical streamlines follow the wavy behaviour dictated by the pulse inside the wave, thus being affected by diffractions and interferences.

4. Final remarks

We are used to consider that quantum mechanics is about small objects. However, this is just a misunderstanding, for quantum mechanical behaviours arise whenever objects are simple enough, i.e., whenever the action of any internal or external surrounding degrees of freedom does not play any relevant role in their dynamics. As an example, just consider the interference patterns that can be obtained from large organic molecules [39] or Bose-Einstein condensates [40, 41]. Such a misunderstanding comes from technological lacks in the past, when quantum

mechanics was developing, as well as *black-box* interpretations, like the Copenhagen one. The possibility to carry out interference diffraction of large objects, particle by particle, requires new viewpoints and interpretations that may go beyond the blind ones available at present.

In this sense, the role of Bohmian mechanics as a theory of quantum motion that describes the evolution of tracer particles when travelling along quantum fluids (i.e., the fluid associated with the diffusion of the wave function in a configuration space) is highly needed. Even if at a statistical level predictions are the same as those already provided by standard quantum mechanics, the fact that hydrodynamic-like analyses can be carried out, elucidating properties such as the flux non-crossing, results as very appealing, for it allows us to understand the (quantum) physical world in a different manner, without conceptual restrictions.

Furthermore, such approaches should also be extended to other fields of physics, for they could help us to get different insights of already well-known and established processes and phenomena. This is the case, as it has been seen, of the so-called classical electromagnetism. In this case, not only theoretical descriptions have been developed to better understand the evolution of the electromagnetic energy through space, but also experiments have been motivated that have corroborated the models proposed [34]. Of course, there is also no doubt about the important role these approaches present from a pedagogical viewpoint [42] by allowing us to connect the distribution of energy (probability) through space with its diffusion (current densities).

Finally, I would like to close this work with a simple, but striking reflection. Usually, one is always tempted to think that only those physical problems that are directly connected with a particular practical application are of interest and, therefore, worth spending time and efforts to investigate them. Of course, this is a legitimate point of view regarding where efforts and resources should be targeted to. However, this strategy cannot be used to get deeper into the physical theories and approaches behind such problems in order to get a better and deeper understanding of them. This is precisely what happens with quantum mechanics and, by extension, with wave theories. Nowadays nobody questions their use, but there are less and less researchers facing the problem of the understanding of the quantum world and the paradoxes it brings in to our intuition, which in spite of all is *shaped* by a classical world.

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