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Simulated Response of the MARIO SCHENBERG Detector to Gravitational Wave Signals with Noise

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Abstract. The Mario Schenberg gravitational wave detector has been constructed at its site in the Physics Institute of the University of Sao Paulo as programmed by the Brazilian Graviton Project under full financial support from FAPESP (the Sao Paulo State Foundation for Research Support). We are ready to do a first test run of the spherical antenna at 4.2 K with three parametric transducers and an initial target sensitivity of $h \sim 10^{-21} \text{ Hz}^{-1/2}$ in a 60 Hz bandwidth around 3.2 kHz.

We have built a computer code for determining the source direction and the wave polarization (solution of the inverse problem) in real time acquisition for strong signal-to-noise ratio cases. The digital filter used is a simple bandpass filter.

The “data” used for testing our code was simulated, it had both the source signal and detector noise. The detector noise includes the antenna thermal, back action, phase noise, series noise and thermal from transducer coupled masses. The simulated noise takes into account all these noise and the antenna-transducers coupling. The detector transfer function was calculated for a spherical antenna with six two-mode parametric transducers.

Finally, we were able to check at what distance Schenberg would detect some known sources. Here we present the results of these simulations.

1. Introduction

Gravitational Waves (GWs) are local space-time curvature perturbations caused by accelerated masses. These perturbations travel through spacetime with the speed of light and can excite quadrupolar normal-modes of elastic bodies. This principle is used for detecting GWs as it was proposed by Weber almost a half of a century ago [1].

In 1971 Forward suggested the use of a sphere as the antenna element of a resonant-mass detector [2]. He idealized a sphere with nine sets of electromechanical strain transducers placed along great circle routes. The tensor gravitational radiation components would be determined by five independent quantities from the nine transducer outputs.

Ashby and Dreitlein studied in detail the reception of GWs by an elastic self-gravitating spherical antenna [3] and Wagoner and Paik found the lowest eigenvalues for the monopole and quadrupole modes of a uniform elastic sphere [4].

In the 1990’s, Johnson and Merkowitz studied the antenna-transducer coupling problem and found an optimum configuration which minimizes the number of transducers while keeping them in a symmetric distribution on the antenna: the truncated icosahedron (TI) configuration [5, 6, 7]. Magalhaes and collaborators showed that distributions with more resonators also can

present some interesting symmetric properties [8, 9, 10]. Lobo also suggested a more elaborated symmetric distribution [11].

Nowadays some research groups have been constructing spherical GW detectors. One of them is the brazilian GW detector, the Mario SCHENBERG. SCHENBERG has a 65 cm-diameter and ~ 1150 kg copper alloy [Cu(94%)Al(6%)] spherical antenna that will operate at temperatures below 1 K [12]. These features are shared with the MiniGrail (The Netherlands). SCHENBERG is not operational yet so we use a model based in harmonic oscillators to simulate the detector behavior.

2. The Model

In order to develop a model for the response of the SCHENBERG detector to gravitational wave signals, we created a set of displacement equations for the antenna plus resonators system. That set of equations considers the dynamics of a spherical antenna coupled to 6 two-mode transducers.

The set of transducers follows the TI arrangement. That optimum distribution for the transducers allows us to monitor all those 5 quadrupolar normal modes of the sphere through the five “mode channels”, \mathbf{g} . By following this configuration we obtained an overall of 17 resonant modes (5 from the sphere and 12 from the set of transducers). We solved the equation of motion of such a system in order to find the eigenfrequencies which are between 3.17 – 3.24 kHz. We also obtained an expression for the mode channels which is given by

$$\tilde{\mathbf{g}}(\omega) = \tilde{\xi}(\omega)\tilde{\mathbf{F}}^S(\omega) + \tilde{\Omega}^{R_1}(\omega)\tilde{\mathbf{F}}^{R_1}(\omega) + \tilde{\Omega}^{R_2}(\omega)\tilde{\mathbf{F}}^{R_2}(\omega). \quad (1)$$

The quantities denoted by $\tilde{\mathbf{F}}^i(\omega)$ correspond to forces whose are acting on: the sphere (S), the intermediate mass (R_1), and the final mass (R_2). The transfer function matrix $\tilde{\xi}(\omega)$ is the proportionality ratio between the mode channels and the mode effective forces. We can easily refer the intermediate and final mass noises into the sphere by using the mode response function $\tilde{\Omega}^{R_i}(\omega)$ to the mode i of the transducers. In this way, we projected all noise sources to the antenna modes which is measured like

$$\tilde{\mathbf{g}}(\omega) = \tilde{\xi}(\omega)\tilde{\mathbf{F}}^S(\omega) \quad (2)$$

where $\tilde{\mathbf{F}}^S(\omega)$ denotes signal plus noise forces which are acting on the modes. Once the effective gravitational force at frequency domain is given by [13, 14, 15]

$$\tilde{\mathbf{f}}(\omega) = -\frac{1}{2}\omega^2\chi m_S R \tilde{\mathbf{h}}(\omega), \quad (3)$$

we can deconvolve the mode channel into the spherical amplitudes

$$\tilde{\mathbf{h}}(f) = -\frac{2}{\omega^2\chi m_S R}\tilde{\xi}^{-1}(\omega)\tilde{\mathbf{g}}(\omega). \quad (4)$$

Those spherical amplitudes correspond to the projected wave polarizations into sphere’s quadrupolar modes under incident angle. The strain amplitudes mimetizes both signal and noise and allows us to calculate the sensitivity curve for each quadrupolar mode of the antenna.

3. The Sensitivity Curves

When usual noises (thermal, backaction, series and phase noises)[16] are added we find a ~ 60 Hz bandwidth. The detector presents $h \sim 10^{-21}$ Hz $^{-1/2}$ centered in 3206.3 Hz.

The Figure 1 presents the sensitivity curves for a quadrupole mode (all others are supposed the same) and the individual contributions from inputed noise sources. Those noise sources are referred into the sphere via transfer function (Equations from 1 to 4).

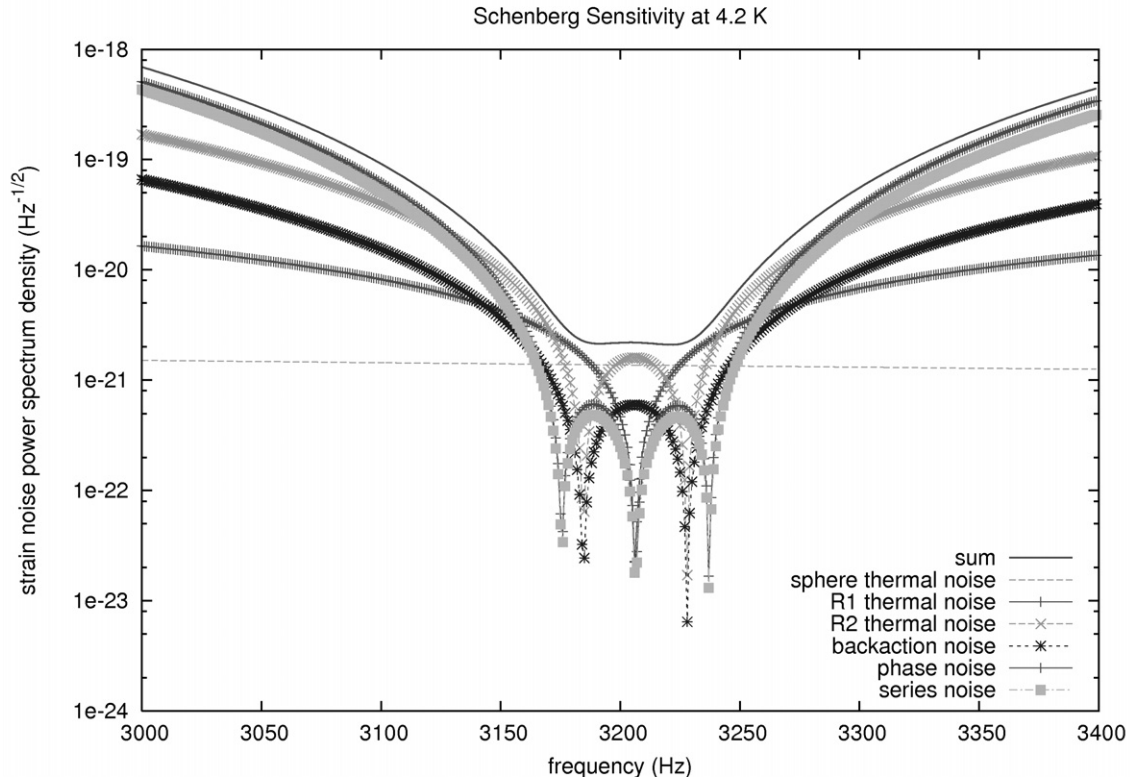


Figure 1. The SCHENBERG sensitivity curves at 4.2 K (upper line) for a single quadrupole mode and the individual contributions of the noise sources.

4. The Signal plus Noise

To simulate the SCHENBERG behavior when it is excited both by an incident gravitational wave and noise we needed to use some waveform. We adopted the AstroGravS Catalog (<http://astrogravs.gsfc.nasa.gov>) where a large number of gravitational waveforms are available. We used in this work, as an example, an adiabatic inspiral of a corotating binary [17], which was rescaled to $1.9 M_{\odot}$ neutron stars.

Signal and noise are expressed in distinct units (i.e. at frequency domain the signal amplitude is given in Hz^{-1} and the noise amplitude is given in $\text{Hz}^{-1/2}$), so we turn signal amplitude into another important quantity the characteristic strain which is defined as $h_c(f) = \sqrt{\Delta f |\tilde{h}^{GW}(f)|^2}$, where $|\tilde{h}^{GW}(f)|^2$ is the power spectral density of the signal at frequency f [18]. The characteristic strain is essentially the *rms* signal in a frequency interval of width Δf centered at frequency f and it has same units of noise sources.

5. The Problem of Incompleteness

The fact that only the quadrupolar modes have been considered instead of all spherical harmonic functions yields to a problem of incompleteness on the sphere's reconstruction. The sum of the five quadrupolar modes (Figure 2) with equal degrees of excitation implies on amplified regions which point to a preferential direction (Figure 3) when only noise is considered. From this result we produce a “gain mask” which normalizes the output position map.

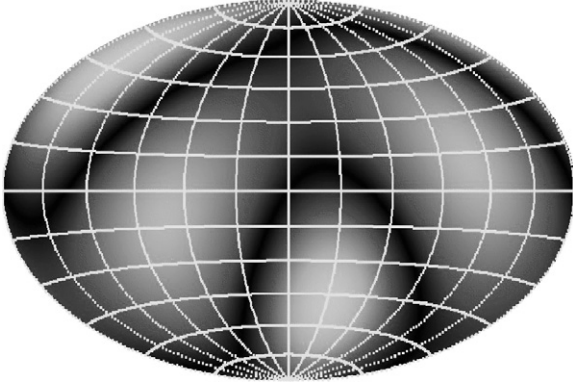


Figure 2. The sum of the five quadrupolar modes over the sphere in a Hammer-Aitoff projection (the brightest areas represent the highest absolute amplitudes).

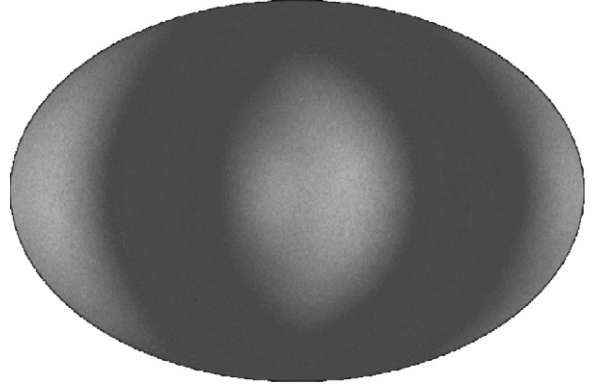


Figure 3. When the direction is calculated that pattern points to some preferential direction. So, the noise isn't uniformly distributed over the sphere.

6. The Amplitude Signal-to-Noise Ratio

The amplitude signal-to-noise ratio [19] was estimated as

$$ASN R = \sqrt{\int_{-\infty}^{\infty} \tilde{W}(f) \frac{|\tilde{h}^{GW}(f)|^2}{S_N(f)} df} \quad (5)$$

where $\tilde{W}(f)$ represents the applied filter function. $S_N(f)$ is the equivalent noise profile [20]. It denotes the sum of the noises which acts on each normal mode. It is given by

$$S_N(f) = \sum_{m=1}^5 h_m(f)^2, \quad (6)$$

where $h_m(f)$ is the sensitivity of mode m as presented in Figure 1. It is easy to show that sum represents the total noise energy on the sphere.

7. The Results

We numerically solved the inverse problem to recover the informations about an input source as its incident direction, polarizations and spectrum.

The Figure 4 shows the incident direction of a source with $ASN R \sim 3$, using just a simple band-pass filter. That low signal-to-noise ratio makes the determination of source location to be spreaded and gives an error in that position. Such error is determined by the solid angle element $\delta\Omega = \delta(\cos\theta)\delta\phi$.

The Mario SCHENBERG detector will be able to detect nearby events (up to Magellanic clouds) at its first goal sensitivity (4.2K) just by using a simple band-pass filter.

For $ASN R < 3$ (simple band-pass filter) another approach must be used. We tested the efficiency of a matched filter and checked that it works well when the waveform is known. However it has a poor performance when the input signal doesn't match perfectly with the template.

The presented method can be applied in a real time data analysis software which will be able to emit an on-line warning to be checked. This method is very flexible and can be adapted for the real case.

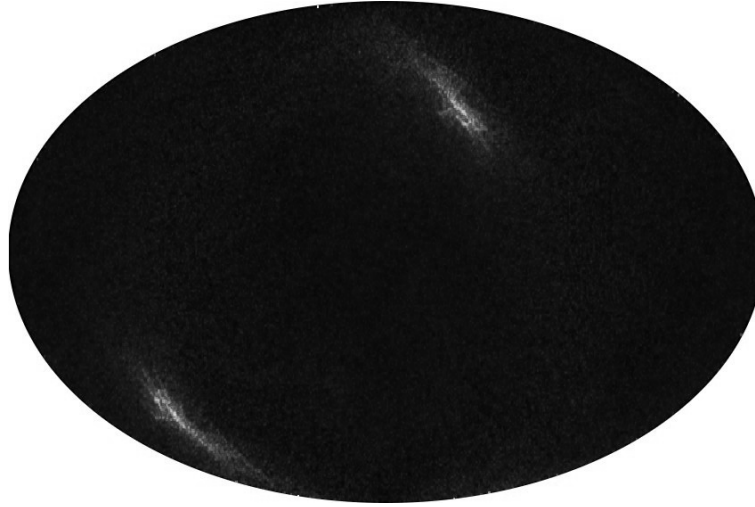


Figure 4. Position of a source with $ASNR \sim 3$.

Acknowledgments

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