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Vortex loops cascade as a channel of quantum turbulence decay

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Abstract. We demonstrate that a single reconnection of two quantum vortices can lead to the creation of a cascade of vortex rings. Our analysis, motivated by the analytical results in localized induction approximation, involved high resolution Biot-Savart and Gross-Pitaevskii simulations. The latter showed that the rings cascade starts on the atomic scale, with rings diameters orders of magnitude smaller than the characteristic line spacing in the tangle, and thus capable of penetrating the tangle and annihilating on the boundaries. That way such process can be an efficient decay mechanism for sparse or moderately dense vortex tangle at very low temperatures.

1. Introduction

Turbulence in ^{4}He II is the state when the superfluid is penetrated by a tangle of quantum vortices of very small thickness. Such a tangle is created when the relative velocity of the superfluid and normal fluid component exceeds a certain threshold. The relative motion can be driven either mechanically, like in the spin-up experiment, or thermally as counterflow caused by non-uniform heating of the ^{4}He II mass. One of many issues related to superfluid turbulence is the question posed by Feynman (2) about the mechanism of quantum turbulence decay in the low temperature limit, $T \to 0$. All explanations of such free decay in a nearly frictionless situation invoke reconnections of quantum vortices as a starting point. The original Feynman's picture was reminiscent of the Richardson's cascade in classical turbulence. Quantum vortex loop would self-reconnect yielding two loops of smaller length. Such repeatedly occurring process would eventually transform the original vortex loop of macroscopic length into a sea of quantum excitations such as phonons and rotons. The currently prevailing view is that the energy contained in the tangle is transferred to smaller scales by a cascade of Kelvin waves triggered by reconnections and subsequently propagating along vortices (3; 4; 5; 6). The mechanism we propose here is also based on reconnections. We argue that vortices that are nearly antiparallel, when they reconnect, give rise to a sequence of very small vortex rings. The smallest rings in this cascade would be dissipated by friction and the somewhat larger rings could escape from the tangle disappearing into the container wall. The unusual characteristics of this sequence of rings is that the smallest rings are created first. Therefore, such type of reconnection, when it occurs, bypasses the cascade-type transfer from macroscopic scales to microscopic scales by generating microscopic scales first.

Our simulation on three different levels of physical description of ^{4}He II (LIA, BS and GP) clearly show that such sequences of vortex rings do indeed emerge from the reconnections of nearly antiparallel quantum vortices (1). Simple estimates of their importance in the overall dissipation of the tangle at $T \to 0$ suggest that they can play a role, at least in some parameter regimes (temperature and tangle density). The greatest remaining uncertainty is the frequency of such events in a real tangle and the statistics of the total line length in the created sequence. In our presentation we will give geometrical arguments and show the results of numerical simulations giving the statistics of the line-length loss.

2. Three levels of description of quantum vortices

The phenomenon of superfluidity is related to the Bose-Einstein condensation, and the flow in the superfluid in the 0K limit can be described with the Gross-Pitaevskii equation

$$i\hbar\partial_t\psi = \left(-\frac{\hbar^2}{2m}\nabla^2 + g\,|\psi|^2 - \mu\right)\psi, \quad \psi = \sqrt{\rho}\exp(i\phi), \tag{1}$$

where ψ is a 3D, complex scalar field of the order parameter. The flow itself is then given by the gradient of phase, $\boldsymbol{u} = \frac{\hbar}{m}\nabla\phi$. Although this fact implies potential flow, vorticity can still exist in connection with density singularities called quantum vortices. They are very thin (the vortex core radius a_s is of the order of 1 in superfluid ^{4}He) and have constant circulation $\kappa = h/m$.

This way, quantum vortex is almost perfect realisation of the vortex filament concept. Thus, in absence of the external driving agents, the flow can be fully understood as the motion of a 'gas' of vortex lines moving in the flow they induce:

$$\boldsymbol{u}(\boldsymbol{x}) = \frac{\kappa}{4\pi}\int \frac{\boldsymbol{s}' \times (\boldsymbol{x} - \boldsymbol{s})d\xi}{|\boldsymbol{x} - \boldsymbol{s}|^3}, \tag{2}$$

where $\boldsymbol{s}(\xi)$ denotes the curve representing vortex. This is called the Biot-Savart approximation (BS).

Obviously this theory fails to describe the event of vortex collision. Then, guided by the results of the GP considerations, one assumes that vortices reconnect. Because of this fact, results of the numerical simulations in BS regime might be dependent on the details of the phenomenological procedure used to handle reconnections. While on the other hand GP simulations can't handle structures larger than few hundred a_s due to computational complexity, one must blend the results from both regimes to obtain reasonable numerical view on any reconnection-dependent phenomenon.

The Biot-Savart regime gives an occasion to easily generalise the theory up to the lambda point by introducing modifications to the vortex reaction on the flow depending on the temperature-dependent parameter α. This takes into account 'friction' appearing at non-zero temperatures which causes the lines to shrink and eventually decay if not pinned to the boundaries.

The BS approximation can be further simplified by assuming that the motion of a fragment of the vortex line is mainly driven by the velocity induced by its local neighbourhood, giving purely geometrical theory, called Local Induction Approximation (LIA). In dimensionless units and Frenet–Serret frame, the equation of motion for vortex reads

$$\dot{\boldsymbol{s}} = c\hat{\boldsymbol{b}} + c\alpha\hat{\boldsymbol{n}}, \tag{3}$$

where c is the local curvature and $\hat{\boldsymbol{b}}$ and $\hat{\boldsymbol{n}}$ are the binormal and normal versors, respectively.

Superfluid turbulence is the state when the fluid is penetrated by a random tangle of numerous quantum vortices. It can be generated in the laboratory and is known to decay effectively even at very small temperatures, which implies that there exists a non-friction mechanism of decay.

3. Vortex loops cascade

In our analysis, we are interested in the angle- or corner-like structures which can be considered as an approximation of the configuration occurring immediately after the reconnection of two straight lines. When the initial vortex configuration consists of two half-lines with a common origin, the line motion is equivalent to a homothety $\boldsymbol{s}(\xi,t) = \boldsymbol{S}(l)\sqrt{\beta t}$, with $l := \xi/\sqrt{\beta t}$. Analytic solutions of such evolution in the LIA approximation have been found by Buttke (7), for $T = 0$, and Lipniacki (8; 9; 10), for $T > 0$. As showed by Svistunov (11) for $T = 0$ and later extended by us for the general case, such solutions may contain self-crossings of the vortex filament for certain values of temperature and sufficiently small (8.5°) asymptotic angle between the lines far away, see Figure 1A.

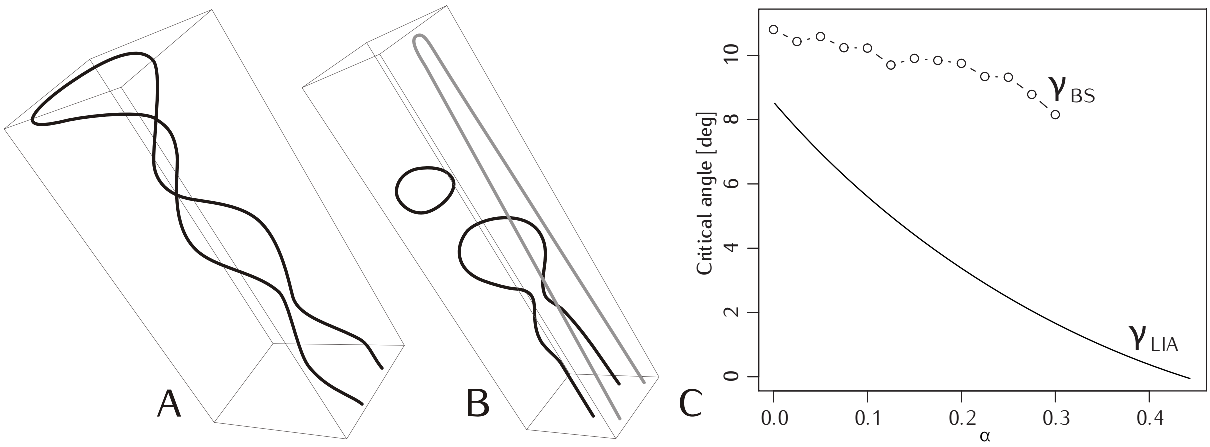

Figure 1. A) Self-similar solution in the LIA approximation for $\gamma = 5°$ and $\alpha = 0.01$. B) Corresponding solution in the BS regime. Grey line shopws the initial condition. C) The critical angle $\gamma_{LIA}(\alpha)$ below which self-similar solutions contain self-crossings and $\gamma_{BS}(\alpha)$ below which the Biot-Savart solutions generate vortex rings.

Such solutions are not consistent with the assumptions behind the LIA approximation from which they are derived, but their existence suggests that the reconnection of two straight vortex lines at a sufficiently small angle may lead to a series of vortex self-reconnections and the creation of a cascade of vortex rings of increasing diameter. We have confirmed the creation of such a scenario by performing a series of numerical Biot-Savart simulations starting form a configuration of slightly rounded angle, see Figure 1B. Due to non-local interactions, the critical angle for that process (γ_{BS}) predicted from the BS simulations is slightly higher and less dependent on temperature than that computed under the LIA and reaches 10.4° in 0K, see Figure 1C. In the $T = 0$ limit, the radii of consecutive rings are following geometric sequence, with quotient inversely proportional to the reconnection angle – starting from one for almost antiparallel lines and reaching infinity when the angle approaches critical angle.

We have checked whether this mechanism persists when one starts the simulation from the configuration of two straight vortex lines inclined at small angle and separated by a small distance instead of rounded angle (which is an idealised effect of their reconnection). Sample propagation of such setting is showed on Figure 2A. One can see that the vortex lines are first approaching each other forming a pyramid-like structure; then they collide and reconnect in the tip point, reconnect and form two rounded-angle-like structures which then undergo vortex loop separation analogous to this previously observed. Our analysis show that the angle at the vertex for both of them is equal to the asymptotic angle, for angles in the range analysed in this study (0-15°). Thus, the critical angle in this setting is the same as for idealised case.

Finally, we have performed simulations in Gross-Pitaevskii regime to verify that this phenomenon is not artefact caused by phenomenological resolving of reconnections in Biot-Savart regime. We have been simulating the the behaviour of two straight vortices inclined at small angle (1-7°), with initial separation of $4a_s$. Example evolution for 4° inclination is shown on Figure 2B.

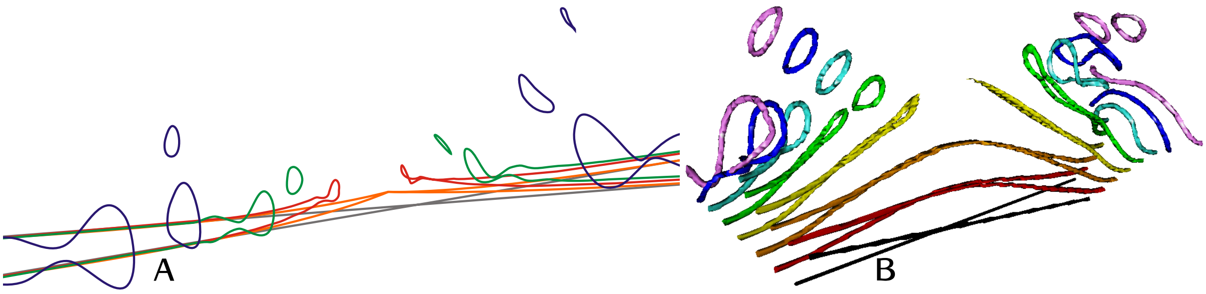

Figure 2. A) Biot-Savart and B) Gross-Pitaevskii simulations of the reconnection of two initially straight vortex lines in $T = 0$K, inclined at 5° angle. The initial separation is $2 \times 10^3 a_s$ in the BS and $4a_s$ in the GP simulation.

These simulations have confirmed the course of events observed in the BS regime (Figure 2A) — the creation of a pyramid-like structure, the reconnection in its tip and the creation of a series of vortex loops. Moreover, it showed that the first produced rings are of an atomic scale, having radii of about $3a_s$.

4. Conclusion

In this work we analyse the faith of a two reconnecting vortex filaments in the case of small reconnection angles. Our analysis was motivated by analytic results obtained in Localised Induction Approximation (LIA), in which self-similar solutions corresponding to a shape of vortex lines after reconnection proved to have self-crossings. Based on both fine-scale Biot-Savart regime simulations, we have shown that at more exact level this effect corresponds to a production of a series of vortex loops of an increasing diameter, provided that the asymptotic angle between reconnecting lines is smaller than 10.4°. Because the Biot-Savart simulations of the processes heavily dependent on reconnection may be biased by the implementation of the simulator, we have also analysed the problem in the Gross-Pitaevskii regime. The results of those simulations have confirmed the previous findings, moreover showed that the first vortex rings produced in the process have diameters of an order of vortex core thickness. This way this phenomenon leads to a direct transfer of energy to even smallest scales; furthermore, small rings have a greater chance of penetrating the tangle and anihilate on container boundary.

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