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MULTISPACECRAFT MEASUREMENT OF ANISOTROPIC CORRELATION FUNCTIONS IN SOLAR WIND TURBULENCE

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ABSTRACT

Multispacecraft measurements in the solar wind are used to determine the field-aligned anisotropy of magnetohydrodynamic inertial range turbulence. The ratio of the parallel to perpendicular correlation lengths is measured by using time-lagged two-point correlations to construct a spatial autocorrelation function. The mean ratio obtained, 1.79 ± 0.36, is significantly greater than unity and therefore consistent with solar wind fluctuations being anisotropic with energy predominantly in wavevectors perpendicular to the large-scale mean magnetic field. In analyzing eight 40–60 minute intervals of multipoint magnetic field data from the four *Cluster* spacecraft, the degree of variation in the ratio of the parallel to perpendicular correlation lengths about the mean was larger than expected. This variation does not appear to be correlated with the solar wind velocity or the plasma beta. The ratio of parallel to perpendicular correlation lengths was also uncorrelated between different field components.

Subject headings: MHD — plasmas — solar wind — turbulence

1. INTRODUCTION

Turbulence is ubiquitous in high Reynolds number plasmas. A distinctive feature of solar wind turbulence (see Goldstein et al. 1995; Tu & Marsch 1995; Bruno & Carbone 2005; Horbury et al. 2005 and references therein) and plasma turbulence in general in the magnetohydrodynamic (MHD) regime is that the presence of a large-scale magnetic field induces anisotropy (Shebalin et al. 1983; Oughton et al. 1994). Of most interest is spectral anisotropy, which is anisotropy in the distribution of energy in wavevector space. This impacts on the propagation and acceleration of cosmic rays (Duffy & Blundell 2005), heating of the interplanetary plasma (Velli 2003), and other astrophysical phenomena. To measure this anisotropy requires that the fluctuations are measured from a variety of angles relative to the mean magnetic field direction. This is difficult to achieve using a single spacecraft, as fluctuations can only be measured in the solar wind flow direction. Here we adopt a multispacecraft approach by analyzing the anisotropy of the spatial autocorrelation function in solar wind turbulence using the four *Cluster* spacecraft.

The simplest model of anisotropic fluctuation symmetry is the "slab model." Excited wavevectors lie along the large-scale magnetic field direction producing a one-dimensional spectrum, and as a result, the correlation function decays with increasing scale parallel to the mean field but has no variation in the field-perpendicular direction. This model has a clear physical motivation in terms of Alfvén waves propagating along the mean field. The slab model has been widely used in past studies of solar wind turbulence to characterize the fluctuations and in cosmic-ray theory (Jokipii 1966), along with other applications. However, the slab model cannot explain the observed mean free paths of cosmic rays (see, e.g., Jaekel et al. 1994) and, at least in the case of incompressible MHD, does not permit wave-wave couplings and so cannot produce a turbulent Kolmogorov-like cascade (Oughton & Matthaeus 2005).

In contrast to slab, the "2D model" is characterized by excited wavevectors lying in the plane perpendicular to the mean magnetic field and therefore the correlation function decays only in the directions perpendicular, and not parallel, to the mean field. A simple and physically motivated interpretation of the 2D model was provided by Shebalin et al. (1983), who argued that the consequence of the resonant conditions for three-wave interactions was that energy is readily transferred perpendicular to the mean magnetic field in wavevector space but not parallel to it. This theoretical picture is supported by numerical simulations (Oughton et al. 1994; Milano et al. 2001), as well as experimental work in tokamaks (Zweben et al. 1979) and reversed-field pinch devices (Robinson & Rusbridge 1971).

Turbulent anisotropy has long been studied in the solar wind. Matthaeus et al. (1990) used single-spacecraft data to measure a highly anisotropic correlation function having a "Maltese cross" shape, which they interpreted as a superposition of both slab and 2D fluctuations. Bieber et al. (1996) used the ratio of the perpendicular to quasi-parallel power spectra, along with the dependence of the total power spectrum on the angle between the mean field and the solar wind flow direction, to measure the relative amplitudes of slab and 2D power, showing that ~85% by energy was in the 2D component.

While slab and 2D models represent idealized interpretations of real fluctuations, they do provide a useful parameterization of anisotropy in solar wind turbulence (Matthaeus et al. 1990; Bieber et al. 1996; Dasso et al. 2005). Most of the work has been done with single-spacecraft measurements (Matthaeus et al. 1990; Bieber et al. 1996; Dasso et al. 2005) because multipoint data have generally not been available. This situation has to some degree been alleviated in recent years due to a number of spacecraft in the near-Earth solar wind. Multispacecraft studies using correlation functions have made important advances using techniques such as simultaneous (zero time lag) two-point correlations (Zastenker et al. 2000; Richardson & Paularena 2001; Matthaeus et al. 2005). In this work we examine anisotropy in solar wind fluctuations using a novel multispacecraft technique intended to measure the anisotropic spatial autocorrelation function by using variable time lag cross-correlations, with data from the four *Cluster* spacecraft.

2. MULTISPACECRAFT TECHNIQUE

A single spacecraft in the solar wind, provided that the solar wind flow speed $\boldsymbol{V}_{\rm sw}$ is much greater than the local Alfvén speed $\boldsymbol{V}_{\rm A}$, can only measure the spatial autocorrelation function in the flow direction. Satisfying this condition means solar wind

fluctuations are convected past a spacecraft in a short time compared to the characteristic timescale on which the fluctuations vary, which is known as Taylor's hypothesis (Taylor 1938). For a time Δt between two observations, a spacecraft travels a distance $\Delta t \boldsymbol{V}_{\rm sw}$ in the plasma frame while solar wind fluctuations can propagate a distance $\Delta t \boldsymbol{V}_{\rm A}$ in the same time. Taylor's hypothesis is therefore equivalent to

$$\Delta t \boldsymbol{V}_{\rm sw} \gg \Delta t \boldsymbol{V}_{\rm A} \Rightarrow \boldsymbol{V}_{\rm sw} \gg \boldsymbol{V}_{\rm A}. \quad (1)$$

This condition is well satisfied in the near-Earth solar wind. In effect, the spacecraft magnetic field time series is a one-dimensional spatial sample through the plasma along the flow direction.

Linear sampling makes it impossible for a single spacecraft to measure the full three-dimensional structure of the fluctuations. A number of methods have been developed to estimate this structure using a single spacecraft (see, e.g., Matthaeus et al. 1990; Bieber et al. 1996; Dasso et al. 2005), but they all require a number of assumptions to be made. For example, Matthaeus et al. (1990) assumed that the statistical nature of the solar wind flow was independent of any particular flow regime, which is to say that the ensemble properties of flows in which the mean magnetic field is either nearly parallel or perpendicular to the flow are statistically similar. Matthaeus et al. also used long data sets that ranged from 62.5 up to 500 hr and therefore had to assume statistical stationarity over these long time intervals. Here we use a novel method of combining data from multiple spacecraft, first proposed by Horbury (2000) to measure the three-dimensional spatial autocorrelation function using only an hour of data, and can therefore eliminate some of these assumptions.

Consider a pair of spacecraft, 1 and 2, in a fast-moving plasma separated by a distance $\boldsymbol{r}_{12}$. Spacecraft 1 will sample a line through the plasma, producing a linear magnetic field time sample $\boldsymbol{b}^1(t)$, and similarly for spacecraft 2. Since $\boldsymbol{V}_{\rm sw} \gg \boldsymbol{V}_{\rm A}$, the time samples are spatial samples through the plasma, $\boldsymbol{b}^1(-\boldsymbol{V}_{\rm sw}t)$ and $\boldsymbol{b}^2(\boldsymbol{r}_{12} - \boldsymbol{V}_{\rm sw}t)$. The normalized time-lagged two-point cross-correlation of a component x of the magnetic field time series is defined as

$$R_x^{12}(\Delta t) = \frac{\langle \boldsymbol{b}_x^1(t)\boldsymbol{b}_x^2(t+\Delta t)\rangle}{\sqrt{\langle \boldsymbol{b}_x^1(t)\boldsymbol{b}_x^1(t)\rangle \langle \boldsymbol{b}_x^2(t)\boldsymbol{b}_x^2(t)\rangle}}, \quad (2)$$

where $\langle\cdots\rangle$ denotes a temporal average over t. Varying the time lag Δt corresponds to varying the vector separation $\boldsymbol{S}$ between the sampling points in the plasma frame:

$$\boldsymbol{S}(\Delta t) = \boldsymbol{r}_{12} - \boldsymbol{V}_{\rm sw}\Delta t. \quad (3)$$

Previous two-point correlations have only used zero time lags, which for any pair of spacecraft are only sensitive to a single vector separation in the plasma frame:

$$\boldsymbol{S} = \boldsymbol{r}_{12}. \quad (4)$$

Such two-point correlations have been the focus of recent multispacecraft studies (Richardson & Paularena 2001; Matthaeus et al. 2005). However, a major drawback of such a method is that comparison of the two time samples yields only a single correlation coefficient. Thus, in order to obtain any reasonable coverage of the spatial autocorrelation function, large quantities of data must be used. For example, Matthaeus et al. (2005) in producing a magnetic autocorrelation function used 264 data samples, each representing 24 hr of continuous magnetic field data.

In this work we use a range of time lags and so are sensitive to a range of spatial scales within a single interval of data. The use of varying time lags means Taylor's hypothesis is satisfied by a more complicated condition, derived by Horbury (2000):

$$\frac{\boldsymbol{V}_{\rm A}\Delta t}{\boldsymbol{r}_{12} - \boldsymbol{V}_{\rm sw}\Delta t} \ll 1. \quad (5)$$

In practice this condition is well satisfied in the solar wind for most time lags. Any time lags that result in the left side of equation (5) being greater than 0.2 are considered to not satisfy Taylor's hypothesis and are removed.

As a result of using a range of time lags, we are sensitive to a range of vector separations and separation angles (the acute angle between the separation vector and the flow direction) in the plasma frame. By altering the time lag we are able to measure two-point correlations in the plasma frame at different length scales and at varying separation angles, all within a single data interval. This means that the angular dependence of the solar wind turbulence two-point correlations can be measured using a single interval of data by altering the time lags, in contrast to previous studies.

In our discussion of this novel technique, we considered a pair of spacecraft measuring correlation functions. However, it can be applied to any number of spacecraft: in this study we use the four *Cluster* spacecraft that provide six pairs of sampling points and therefore greater coverage in both angle and scale. It can also be used in conjunction with more sophisticated techniques such as cross-wavelets and structure functions, although only cross-correlations are used here.

3. RESULTS

We analyze 4 s resolution spin averaged magnetic field data from the magnetic field instrument on board the four *Cluster* spacecraft (Balogh et al. 2001). The data analyzed in this section were taken on 2006 March 5 from 5:05 to 5:45 UT. During this time the four *Cluster* spacecraft were in the solar wind at separations of ~10,000 km, and since the ion gyroradius was ~100 km, this is firmly in the inertial range. The average solar wind speed during this interval was 330 km s^{-1}, and the plasma beta was 1.1.

We define a right-handed orthogonal coordinate system in which the x-axis is aligned with the mean magnetic field, the y-axis is in the plane defined by the mean field and the solar wind velocity vector (which is nearly antisunward), and the z-axis completes the right-handed system. Using the multispacecraft technique described earlier, time-lagged two-point magnetic field cross-correlations are computed between pairs of spacecraft. Six separation vectors join the four *Cluster* spacecraft, which allows for six sets of correlation functions to be calculated for each component of the mean magnetic field aligned coordinate system. Figure 1 shows the computed cross-correlations as a function of time lag for the z-component of the magnetic field fluctuations. The shape of the correlation functions and the rate at which they fall off is a complex function of the separation vectors in the plasma frame, the variation of the separation vectors with time lag, and the structure of the fluctuations. Time lags that satisfy Taylor's hypothesis are then converted into spatial scales.

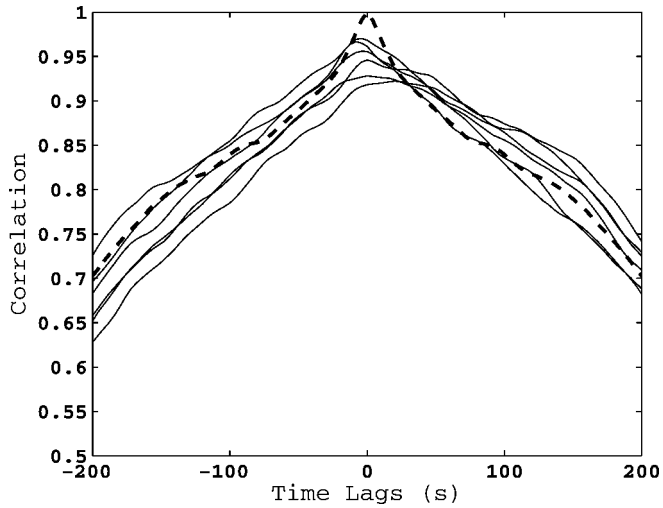

FIG. 1.—Cross-correlations between all six pairs of *Cluster* spacecraft as a function of time lag for the *z*-component of the magnetic field fluctuations on 2006 March 5 during the interval 5:05–5:45 UT. The autocorrelation from a single spacecraft is also plotted as a dashed line. The autocorrelation resembles the cross-correlations, but there are significant differences. The shape of the correlation functions and the rate at which they fall off is a complex function of the separation vectors in the plasma frame, the variation of the separation vectors with time lag, and the structure of the fluctuations.

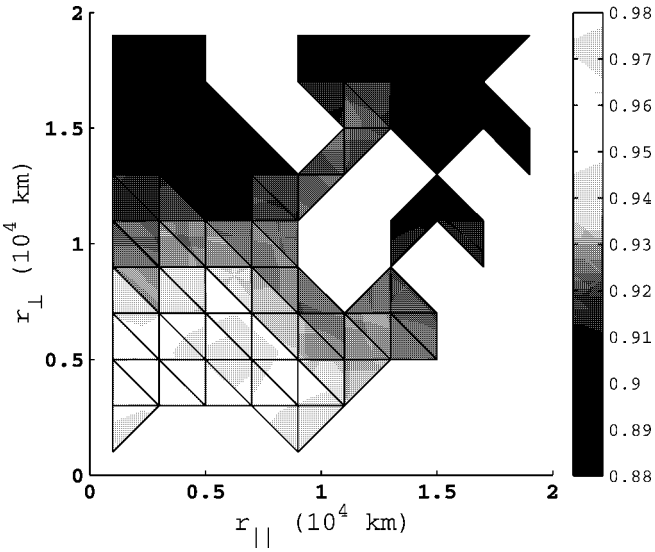

FIG. 2.—Spatial autocorrelation function of the *z*-component of the fluctuations for the same data as Fig. 1. The function is anisotropic: it does not decay equally in all directions, but rather the decay with increasing $r_\perp$ is more rapid than the corresponding decay with increasing $r_\parallel$, which is consistent with a dominant 2D component.

We assume axisymmetry about the mean magnetic field, which is motivated physically by variance anisotropy (Belcher & Davis 1971), as well as being a mathematical convenience (Bieber et al. 1996). This allows all the spatial autocorrelation values to be projected onto a two-dimensional plane spanned by two spatial separation coordinates: $r_\parallel$, which is parallel to the mean magnetic field, and $r_\perp$, the complementary perpendicular coordinate. Despite there being six vectors linking the four *Cluster* spacecraft, coverage of the spatial autocorrelation function is limited. For this reason, the data are binned and averaged. The binning process is carried out by superimposing a grid with squares of size $(2 \times 2) \times 10^3$ km over the spatial autocorrelation function, which extends from 0 to 2×10^4 km. The correlation function values in each grid square are averaged such that each data-containing grid square is represented by one number, the mean correlation in that grid square. Figure 2 shows the binned and averaged *z*-component of the spatial autocorrelation function. This correlation function decays more rapidly in the direction perpendicular to the mean magnetic field than in the field-parallel direction, consistent with a dominant 2D component. To estimate the anisotropy, we fit the spatial autocorrelation function to an "elliptical model." For each component of the magnetic field an autocorrelation function is constructed using one of the *Cluster* spacecraft. The elliptical model correlation function, $\boldsymbol{A}(r_\parallel, r_\perp)$, assumes the shape of the relevant autocorrelation function in all directions but uses an elliptical decay scaling:

$$\boldsymbol{A}(r_\parallel, r_\perp) = \boldsymbol{A}_0\left(\sqrt{(\alpha r_\parallel)^2 + (\beta r_\perp)^2}\right), \tag{6}$$

where $\boldsymbol{A}_0(-\boldsymbol{V}_{sw}\Delta t)$ is the single-spacecraft autocorrelation function that is only measured in the flow direction and α and β are free parameters that are computed by minimizing the variance between the spatial autocorrelation and the elliptical model. This model is intended to provide a first-order estimate of the anisotropy, and while it is not physically motivated, it does fit the data reasonably well.

The ratio α/β is related to the ratio of the parallel to perpendicular correlation lengths at the range of scales studied here and thus is a measure of anisotropy. A ratio smaller than unity implies the perpendicular correlation length is longer than the field-parallel correlation length, which is consistent with a dominant slab component, and vice versa for a dominant 2D component. The spatial autocorrelation shown in Figure 2 has $\alpha = 0.84$, $\beta = 0.49$, and $\alpha/\beta = 1.71$. Figure 3 shows a plot of the ratio α/β for all three magnetic field components using eight 40–60 minute intervals of *Cluster* data taken from the period 2006 February–March. In almost all cases the ratio α/β is larger than unity, consistent with solar wind turbulence being anisotropic with energy mostly in wavevectors perpendicular to the mean magnetic field. The mean values of the α/β ratio for all the magnetic field components are 1.91 ±

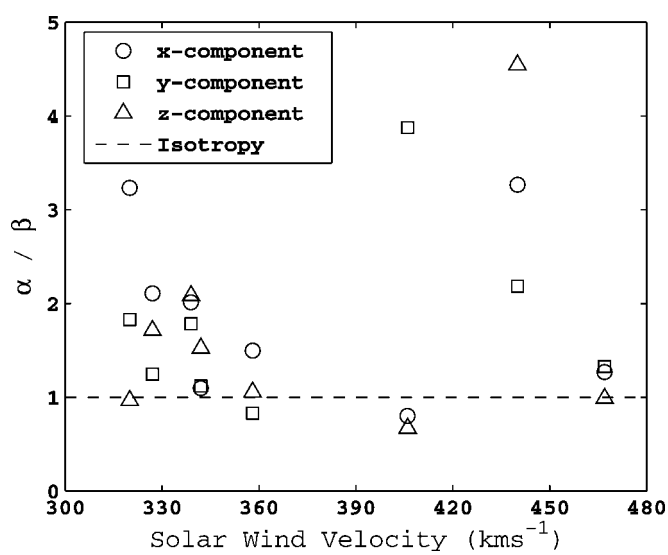

FIG. 3.—Anisotropy ratio α/β for all three components of the magnetic field fluctuations using eight 40-60 minute intervals of *Cluster* FGM data. Most of the data points are above unity, which is consistent with solar wind turbulence being anisotropic with a dominant 2D component.

0.33 for the x-component (parallel to the mean field), 1.77 ± 0.34 for the y-component, and 1.69 ± 0.44 for the z-component. The mean ratio is, within errors, equal for all the magnetic field components, although there is an indication that anisotropy is more pronounced in the component parallel to the mean field. A notable feature of Figure 3 is the large degree of variation of the ratio α/β about the mean. This variation does not appear to be correlated with the solar wind velocity or the plasma beta. The magnetic field components of the α/β ratio are also uncorrelated between each another. It is unclear whether this variability results from the inability of the simplified elliptical model to accurately represent the data or whether it is a real effect indicating that the individual intervals of *Cluster* data exhibit varying degrees of anisotropy.

4. DISCUSSION

A wide range of work has suggested that solar wind turbulence may be described as a composite of slab and 2D components (Matthaeus et al. 1990; Bieber et al. 1996; Dasso et al. 2005). Applying time-lagged two-point correlations using the four *Cluster* spacecraft in the solar wind, we obtain a mean value of 1.79 ± 0.36 for the ratio of parallel to perpendicular correlation lengths. Within the context of the slab and 2D models, our results support the view that the slab model alone is an inadequate description of solar wind turbulence and that a mixture of both slab and 2D models with a dominant 2D component represents a more realistic description. The adoption of such a description of solar wind turbulence has assisted in the modeling of MHD turbulence (Galtier et al. 2005), as well as improving our understanding of phenomena closely related to MHD turbulence such as cosmic-ray propagation (Shalchi et al. 2006).

The multispacecraft technique that we have described enables solar wind turbulence to be studied without many of the assumptions and restrictions associated with single-spacecraft observations. Among other benefits, this technique allows measurements to be made at various angles to the mean field direction simultaneously and to compare between these angles within a single data interval, removing one of the key assumptions of previous analyses. This method also represents an improvement on simultaneous two-point correlations because it extracts more information out of the available data. The multispacecraft method also has the advantage that it can be used in conjunction with other data analysis techniques such as structure functions and cross-wavelets, and is not limited to cross-correlations.

The present analysis applies to turbulence in a range of solar wind streams, and so extension to disturbed regions such as corotating interaction regions and coronal mass ejections would be worthwhile to determine whether anisotropy is enhanced by the compressions and rarefactions in such regions. In addition, work has already begun on using both numerical and modeling techniques to determine a more physically motivated correlation function model with which to fit the data. Finally, the separations between the *Cluster* spacecraft during this analysis were ~10,000 km, which is within the inertial range. A possible future project would be to repeat this analysis at separations close to, if not within, the dissipation scale (Leamon et al. 1998, 1999, 2000) to examine how the balance of slab and 2D components is different in the kinetic regime.

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