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CLUSTER MERGER VARIANCE AND THE LUMINOSITY GAP STATISTIC

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ABSTRACT

The presence of multiple luminous galaxies in clusters can be explained by the finite time over which a galaxy sinks to the center of the cluster and merges with the central galaxy. The simplest measurable statistic to quantify the dynamical age of a system of galaxies is the luminosity (magnitude) gap, which is the difference in photometric magnitude between the two most luminous galaxies. We present a simple analytical estimate of the luminosity gap distribution in groups and clusters as a function of dark matter halo mass. The luminosity gap is used to define "fossil" groups; we expect the fraction of fossil systems to exhibit a strong and model-independent trend with mass: ∼1%–3% of massive clusters and ∼5%–40% of groups should be fossil systems. We compare our predictions to the luminosity gap distribution in a sample of 730 clusters in the Sloan Digital Sky Survey C4 Catalog and find good agreement. This suggests that theoretical excursion-set merger probabilities and the standard theory of dynamical segregation are valid on cluster scales.

Subject headings: cosmology: observations — dark matter — galaxies: clusters: general — galaxies: halos

1. INTRODUCTION

Virialized cold dark matter halos grow hierarchically by the merging of smaller virialized halos. Galaxies, which populate the halos, also grow hierarchically by the merging of preexisting galaxies. Halo merger rates can be estimated analytically using the excursion-set theory (Bond et al. 1991), which is commonly known as the extended Press-Schechter formalism (Press & Schechter 1974; Bower 1991; Lacey & Cole 1993). The merger rates can also be extracted from large-scale cosmological simulations. At the same time, direct measurement of merger rates in galaxy groups and clusters, however, is challenging. We here show that a simple statistic, the luminosity gap, can be applied to large galaxy surveys such as the Sloan Digital Sky Survey (SDSS) to test the predictions of the excursion-set merger probabilities.

Dark matter halos merge from the outside in. Mergers effectively begin when the components approach within a virial radius of each other; they conclude when the separate identities of the two halos have been erased, either by the merging of the halo centers or by the complete tidal disruption of the smaller halo. The duration of the merger defined this way depends on the rate at which dynamical friction induces orbital decay of the tidally truncated subhalos. If a galaxy is located at the center of each subhalo, the galaxies appear as separate objects until the conclusion of the merger. The timescale of orbital decay can exceed the age of the system; this is why the number of galaxies in a group or cluster (the "occupation number") exceeds unity.

A confirmation of this paradigm can be found in the existence of "fossil" groups of galaxies, which contain a single ultraluminous galaxy at the center and no other galaxy brighter than $L_\star$. The X-ray luminosity of fossil groups is comparable to that of rich clusters and indicates dynamical masses of about 10^{13}–10^{14} $M_\odot$. Jones et al. (2003) selected fossil groups with the criterion $\Delta\mathrm{mag}_{12} > 2$, where the luminosity gap $\Delta\mathrm{mag}_{12}$ is defined as the difference in photometric magnitude between the most and second most luminous galaxies in the group; we adopt this definition as well. Fossil groups have been identified as cluster-sized systems old enough that any previous merger with a halo hosting a $\gtrsim L_\star$ galaxy has completed so that all luminous galaxies have agglomerated onto the central galaxy. Their incidence rate is 8%–20% among observed systems in this mass range, at least if the mass is inferred from the X-ray luminosity (Jones et al. 2003). Numerical simulations by D'Onghia et al. (2005) predict a larger fraction of fossil systems: 33% ± 16% for a mass of 10^{14} $M_\odot$. These investigations suggest that fossil groups, poor clusters, and rich clusters can be distinguished by the time elapsed since the last major merger. We show here that the incidence rate of fossil groups can be estimated analytically from excursion-set theory.

In § 2 we compute the timescale on which dynamical friction drives orbital in-spiral of a tidally truncated subhalo inside the primary halo. In § 3 we estimate the subhalo mass distribution as a function of the subhalo mass and distance from the center of the primary halo. In § 4 we calculate the dependence of the luminosity gap on the halo mass. We also derive the fraction of fossil systems in groups and clusters as a function of mass. Finally, in § 5 we compare the predictions of our model to the luminosity gap distribution in 730 clusters from the SDSS C4 Catalog (Miller et al. 2005). Throughout the Letter we assume the standard cosmological model consistent with the results from the *Wilkinson Microwave Anisotropy Probe* (Spergel et al. 2003).

2. DYNAMICAL EVOLUTION

Consider a subhalo of mass M_s merging into a primary halo of mass $M_h \geq M_s$ at redshift z_m. We define a "merger" as the time when the center of subhalo crosses the virial radius of the new composite halo of mass $M = M_h + M_s$. After the halos coalesce, the subhalo spirals toward the center of the composite halo. The effective dynamical mass of the subhalo decreases after the merger because bound mass is tidally stripped as the orbit of the subhalo decays. We denote the bound mass by $M_s(R_s)$, where R_s is the tidal truncation radius. This is related to the separation r of the subhalo from the center of the composite halo via $M_s(R_s)/R_s^3 = M(r)/r^3$, where $M(r)$ is the mass

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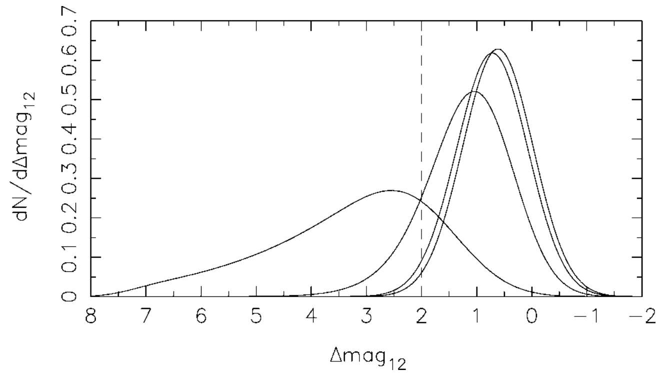

FIG. 1.—Probability distribution for the luminosity gap $\Delta\mathrm{mag}_{12}$ at $z = 0$ calculated using eq. (6). From left to right, the curves correspond to halos with mass $M = 10^{12.5}$, $10^{13.5}$, $10^{14.5}$, and $10^{15.5}$ $M_{\odot}$. We assume $\ln \Lambda = 2$, $M_{\min} = \frac{1}{2}M$, and $R = R_{\mathrm{vir}}(M)$ throughout. While the initial mass of the satellite is always smaller than the mass of the primary, the luminosity of the satellite galaxy may exceed that of the primary because of the scatter in the L_c-M relation, leading to a negative luminosity gap; the measured quantity is $|\Delta\mathrm{mag}_{12}|$.

of the composite halo contained within radius r. This relation yields R_s and $M_s(R_s)$ in terms of r.

The subhalo experiences a torque $T = |\boldsymbol{r} \times \boldsymbol{F}|$, where $\boldsymbol{r}$ is its position relative to the center of the composite halo and $\boldsymbol{F}$ is force of dynamical friction (Chandrasekhar 1943), $|\boldsymbol{F}| = 4\pi G^2 M_s(R_s)^2 \rho(r) \ln(\Lambda)/v^2$. Here $\rho(r)$ is the density of the composite halo at distance radius r, $\ln(\Lambda)$ is the Coulomb logarithm (which in principle depends on the orbit of the subhalo and on the orbital phase space distribution of dark matter), and v is the velocity of the subhalo. To simplify the calculations we assume circular orbits. Then the velocity of the subhalo is the circular velocity in the composite halo $v = [GM(r)/r]^{1/2}$. In numerical simulations of satellites in halos, $\ln(\Lambda) \sim 2$ (Velázquez & White 1999; Fellhauer et al. 2000).

The galaxies spiral toward the center of the composite halo at the rate $dr/dt = (dJ/dr)^{-1} T/M_s(R_s)$, where $J = [GM(r)r]^{1/2}$ is the specific angular momentum associated with the orbit of the subhalo. We assume that the density profile of the halo follows the form of Navarro et al. (1997), with the concentration parameter from the Bullock et al. (2001) fit. The orbital decay rate can be integrated to find the distance of the subhalo from the center of the composite halo as a function of time.

3. SUBHALO DISTRIBUTION

We would like the probability that a subhalo of mass M_s is located at distance r from the center of the composite halo of mass M. The extended Press-Schechter formalism does not directly yield such an expression because halos grow through an entire hierarchy of mergers. We present a variation of one of the established models for the subhalo mass function (e.g., Nusser & Sheth 1999; Fujita et al. 2002; Sheth 2003; Lee 2004; Oguri & Lee 2004). These models ignore a number of issues, such as halo triaxiality and the evolution of substructure within substructure. Our confidence in their validity stems from their success in reproducing subhalo statistics in large-scale numerical simulations (e.g., Zentner et al. 2005), gravitational lensing observations (e.g., Natarajan & Springel 2004), and the cluster luminosity function (Cooray & Cen 2005).[5]

Lacey & Cole (1993) give an expression for the fraction of

[5] Note that the known problems with the extended Press-Schechter merger rates are not severe for the mass ratios of interest (Benson et al. 2005).

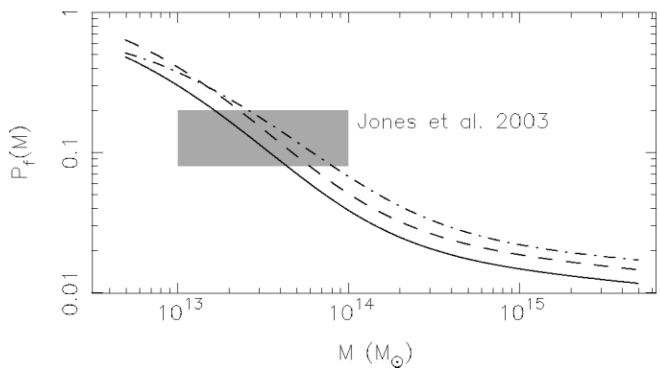

FIG. 2.—Probability $P_f(M)$ that a halo of mass M contains a fossil system of galaxies. The curves assume $\ln \Lambda = 1$ and $M_{\min} = \frac{1}{2}M$ (*solid line*), $\ln \Lambda = 2$ and $M_{\min} = \frac{1}{2}M$ (*dashed line*), and $\ln \Lambda = 1$ and $M_{\min} = \frac{3}{4}M$ (*dot-dashed line*). The shaded rectangle is the measurement of Jones et al. (2003).

mass of a halo with mass M at redshift z that lies in progenitors with masses between M_s and $M_s + dM_s$ at redshift z_m:

$$f(M_s, z_m|M, z) = \frac{\delta_c(z_m) - \delta_c(z)}{(2\pi)^{1/2}[\sigma^2(M_s) - \sigma^2(M)]^{3/2}} \left|\frac{d\sigma^2(M_s)}{dM_s}\right| \times \exp\left\{\frac{[\delta_c(z_m) - \delta_c(z)]^2}{2[\sigma^2(M_s) - \sigma^2(M)]}\right\}, \quad (1)$$

where $\sigma(M)$ is the mass variance on scale M and $\delta_c(z)$ is the critical overdensity for collapse at redshift z. The progenitor mass function is obtained by multiplying $f(M_s, z_m|M, z)$ by the halo multiplicity factor, $dN/dM_s = (M/M_s)f$.

Lacey & Cole (1993) define the formation redshift as the

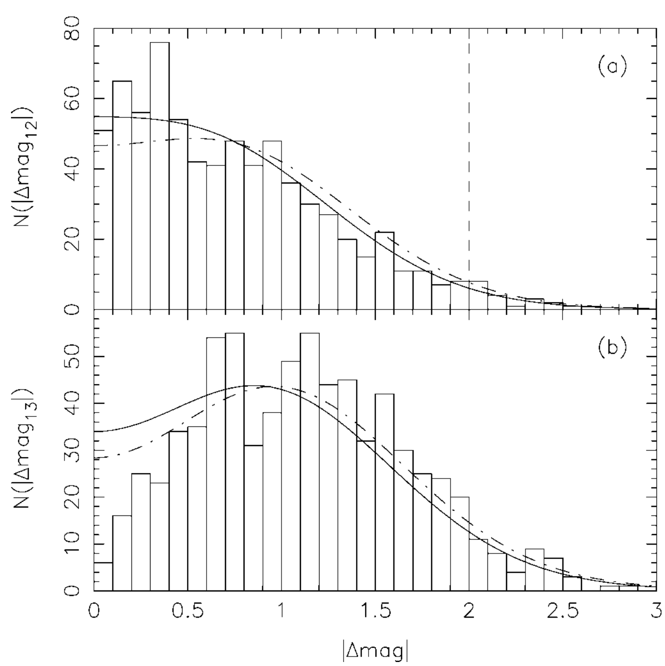

FIG. 3.—The r-band luminosity gap distribution from 730 clusters in the SDSS C4 Catalog (Miller et al. 2005). We evaluate the luminosity gap relative to (*a*) the first and second most luminous galaxies and (*b*) the first and the third most luminous galaxies. We also show predictions of our model with $\ln \Lambda = 1$ and $M_{\min} = \frac{1}{2}M$ (*thick line*) and with $\ln \Lambda = 1$ and $M_{\min} = \frac{1}{2}M$ (*dot-dashed line*). Fossil systems are located to the right of the dashed line; $P_f \approx 2.9\%$ in the data and 2.7% in the model.

time at which the most massive progenitor halo contains half of the mass of the final halo. Equation (1) is integrated with respect to M_s and differentiated with respect to z_m to obtain the growth rate of the fraction of the halo mass in large objects; we interpret this as the probability that a halo at redshift z formed at a redshift between z_m and $z_m + dz_m$,

$$\frac{dP}{dz_m}(z_m, M, z) = \frac{d}{dz_m}\int_{M_{\min}}^{M} f(M_s, z_m|M, z)\, dM_s, \qquad (2)$$

where $M_{\min} \geq \frac{1}{2}M$ is a minimum mass cutoff.

Following Oguri & Lee (2004), to approximate the probability that a halo of mass M acquired a subhalo of mass M_s at redshift z_m, we multiply the subhalo mass function by the formation redshift distribution $d^2N/dM_s\, dz_m \sim (dN/dM_s)(dP/dz_m)$. This approximation is heuristic; doing better requires integrating over merger trees. Such a treatment would improve upon the imprecise definition of "formation time" (Cohn & White 2005) and its likely correlation with subhalo properties, but we defer that to future work. The present calculation is similar to sampling over merger tree halo formation histories, except that we ignore all but the couple of most-massive surviving subhalos and do not keep track of the order in which the mergers take place.

The probability that a subhalo of mass M_s lies a distance $r < R_{\rm vir}(M, z)$ from the halo center is given by $d^2N/dM_s\, dr = (d^2N/dM_s\, dz_m)(dz_m/dr)$, where the average density of the halo is 200 times the mean density of the universe within the virial radius, and $dz_m/dr = (dr/dt)^{-1}(1 + z_m)H(z_m)$ is the inverse of the derivative of the separation at redshift z with respect to the merger redshift z_m, where $H(z_m)$ is the Hubble parameter at this redshift.

The luminosity gap is independent of the subhalo position, so we integrate $d^2N/dM_s\, dr$ with respect to radius to obtain[6]

$$\frac{dN_R}{dM_s}(M_s) = \int_{z_m(M_s,R)}^{z_m(M_s,0)} \frac{d^2N}{dM_s\, dz_m}(M_s, z_m)\, dz_m, \qquad (3)$$

where $R \leq R_{\rm vir}$ is a constant radius and $z_m(M_s, r)$ is implicitly defined by the relation

$$R_{\rm vir}[z_m(M_s, r)] - r = \int_{z}^{z_m(M_s,r)} \frac{dr}{dz'_m}\, dz'_m. \qquad (4)$$

We solve equation (4) approximately by evaluating the virial radius at z rather than z_m and by evaluating dr/dz_m at $R_{\rm vir}(z)$. The latter approximation is justified because dr/dz_m depends on radius only weakly, especially when $r \sim R_{\rm vir}$.

The integration of equation (3) over the interval (M_s, M) yields the number of subhalos $N_R(M_s)$ with mass greater than M_s. This defines a one-to-one relation between M_s of a particular subhalo and the expected number of subhalos above this threshold. It is straightforward to show that the distribution of the kth most massive surviving subhalo mass is

$$\frac{dN}{dM_k} = -\frac{d}{dM_k}[F_k(M_k)e^{-N_R(M_k)}], \qquad (5)$$

where $F_2(M_2) = 1$ and $F_3(M_3) = 1 + N_R(M_3)$.

[6] We suppress the explicit dependence on M and z everywhere.

4. THE LUMINOSITY GAP DISTRIBUTION

We next calculate the distribution of luminosities of the first and second most luminous satellites in a galaxy cluster. Thus we need to relate the initial mass of a subhalo to the luminosity L_c of its central galaxy. At $z \approx 0$, L_c in any halo is tightly correlated with its mass with a functional form $L_c(M) = L_0(M/M_0)^a[b + (M/M_0)^{cd}]^{-1/d}$ (Vale & Ostriker 2004; Cooray & Milosavljević 2005a, 2005b). Our fit to the r-band luminosities in Seljak et al. (2005) yields $L_0 = 5.7 \times 10^9\, L_\odot$, $M_0 = 2 \times 10^{11}\, M_\odot$, $a = 4$, $b = 0.57$, $c = 3.78$, and $d = 0.23$. The relation possesses lognormal intrinsic scatter $dN_{\rm scat}/dL(L|M) = [(2\pi)^{1/2} \ln(10)\Sigma L]^{-1} \exp\{-\frac{1}{2}\log[L/L_c(M)]^2/\Sigma^2\}$ (Cooray & Milosavljević 2005b), where in the r-band $\Sigma \approx 0.17^{+0.02}_{-0.01}$ in clusters (Cooray 2006).

We assume that the most luminous galaxy is located at the center of the composite halo. Then the remaining subhalos cannot contain the most luminous galaxy in the cluster; the most luminous galaxy in a subhalo is the second most luminous member of the cluster. The distribution of galaxy luminosities associated with the kth most massive subhalo dN/dL_k is obtained by integrating dN/dM_k multiplied by the $dN_{\rm scat}/dM_k(L_k|M_k)$ over the subhalo mass M_k.

The luminosity gap $\Delta{\rm mag}_{1k}$ is the (observed) magnitude difference between the first and kth most luminous galaxies in a cluster. The most luminous galaxy is the central galaxy, and the second one lies in the largest surviving subhalo. In systems of mass M the gap is distributed as

$$\frac{dN}{d\Delta{\rm mag}_{1k}}(\Delta{\rm mag}_{1k}|M) = \int_0^\infty\int_0^\infty \frac{dN}{dL_k}(L_k, M)\frac{dN_{\rm scat}}{dL}(L|M) \times \delta\left[\Delta{\rm mag}_{1k} + \frac{5}{2}\log\left(\frac{L_k}{L}\right)\right] dL_k\, dL, \qquad (6)$$

where $\delta(x)$ is the Dirac delta function. In Figure 1 we plot $dN/d\Delta{\rm mag}_{12}$ for halos of various masses; the median luminosity gap is larger in smaller halos.

The incidence rate $P_f(M)$ of "fossil" systems ($\Delta{\rm mag}_{12} > 2$) can then be calculated by integrating equation (6). In Figure 2 we plot $P_f(M)$ for several combinations of input parameters. On scales $M \sim 10^{14}\, M_\odot$, the probability that the system is fossil is about 3%–6%, fairly close to the observed value (Jones et al. 2003). It is, however, smaller than the same probability estimated from the simulations of D'Onghia et al. (2005) (see § 1). The three curves illustrate the errors we expect from our simplified treatment; they do not affect our qualitative results but may be distinguishable with more extensive observations.

5. THE LUMINOSITY GAP IN SDSS C4 CLUSTERS

We measured the luminosity gap distribution on 730 clusters in the SDSS C4 Cluster Catalog (Miller et al. 2005) at mean redshift $\langle z\rangle = 0.087$. The three brightest cluster members were identified from the SDSS photometry as being within a projected 500 h^{-1} kpc radius of the center of the cluster. In addition, these galaxies must have $m_r - m_i$ colors that lie within 2 σ of the E/S0 cluster ridgeline as determined by spectroscopically confirmed members (Visvanathan & Sandage 1977). We utilize extinction-corrected model-fit magnitudes and apply $z = 0$ K-corrections (ver. 3.2; Blanton et al. 2003).

The masses of these clusters are estimated from the total r-band luminosities via $\log(h^{-1}M) \approx -2.46 + 1.45 \log(h^{-2}L_r)$; 95% lie in the range (0.5–10) $\times 10^{14}(h/0.7)^{-1} M_\odot$. The total luminosity is a better mass estimator than the line-of-sight velocity dispersion or the richness of the cluster (Miller et al. 2005; see also Lin et al. 2004; Popesso et al. 2005 and references therein). A histogram of the luminosity gap distribution is shown in Figure 3. The mean luminosity gap is $\langle \Delta \mathrm{mag}_{12} \rangle \approx 0.75$.

To predict the luminosity gap distribution of the C4 sample, we multiply $dN/d\Delta\mathrm{mag}_{12}$ of equation (6) at $z = 0$ by the mass function of C4 clusters and integrate over mass. The resulting composite model luminosity gap distribution is shown by the thick solid and dot-dashed lines in Figure 3*a*. The agreement of the model and data is remarkable for the choice of the dynamical friction parameter $\ln\Lambda = 1$. For $\ln\Lambda \gtrsim 2$, the model distribution develops a local minimum at $\Delta\mathrm{mag}_{12} = 0$ and its maximum shifts to $\Delta\mathrm{mag}_{12} \sim 0.6$ (Fig. 3, *dot-dashed line*). Such behavior is not apparent in C4 clusters, but it has been detected in luminous red galaxies (LRGs) in SDSS imaging data by Loh & Strauss (2006), who measured the luminosity gap within 1.0 h^{-1} Mpc of LRGs and found that the peak moved to positive values in underdense fields. These curves illustrate the sensitivity of the luminosity gap to the detailed properties of mergers; clearly the qualitative fit is excellent, but more detailed model-fitting may in the future enable tests of particular aspects of merger dynamics.

In Figure 3*b* we present the luminosity gap distribution relative to the third most luminous galaxy in the cluster. The model overpredicts the frequency of small $\Delta\mathrm{mag}_{13}$ because it treats the luminosities of the second and third most luminous galaxies as independent, whereas the former must exceed the latter by definition.

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