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# CHROMOSPHERIC LEAKAGE OF ALFVÉN WAVES IN CORONAL LOOPS

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## ABSTRACT

The recently observed rapid damping of coronal loop oscillations generated discussion regarding the physical mechanism responsible for the damping, with the leakage into the chromosphere suggested as one of the possible mechanisms. I study the leakage of Alfvén waves into the chromospheric footpoints of a coronal loop using the nonlinear viscoresistive 1.5-dimensional MHD model. The waves were launched by an initial transverse velocity profile in the loop. I find that the leakage time of long-wavelength transverse oscillations, such as the ones observed recently by the *Transition Region and Coronal Explorer* (*TRACE*), is 5 times longer than the observed damping time of the oscillations. For the loop recently observed by *TRACE*, I conclude that the observed damping of the long-wavelength oscillations is not due to leakage into the chromosphere. However, depending on particular chromospheric and coronal properties, chromospheric leakage may be significant in some events of coronal loop oscillations.

*Subject headings:* MHD — Sun: corona — Sun: magnetic fields — waves

## 1. INTRODUCTION

Recently, Aschwanden et al. (1999) and Nakariakov et al. (1999) observed coronal loop oscillations using the EUV telescope on board the *Transition Region and Coronal Explorer* (*TRACE*) satellite (Handy et al. 1998). The oscillation were triggered by a flare in several loops on 1998 July 14 at about 12:00 UT. Nakariakov et al. (1999) analyzed the decaying transversal oscillations of a long [$(130 \pm 6) \times 10^6$ m], thin [with a diameter of $(2.0 \pm 0.36) \times 10^6$ m], bright coronal loop in the Fe ix 171 Å emission line and found that the decay time of the oscillations was $14.5 \pm 2.7$ minutes for an oscillation with a frequency of $3.90 \pm 0.13$ mHz. Nakariakov et al. (1999) estimated that the footpoint leakage is insignificant based on the work of Berghmans & De Bruyne (1995) and concluded that the rapid oscillation damping is due to enhanced viscosity or resistivity in the coronal plasma. The enhanced dissipation may solve the existing difficulties with the leading coronal heating theories and address the main unresolved problem of solar coronal physics. Recently, Aschwanden et al. (2002) have documented transverse oscillations in 26 coronal loops in 17 events observed by *TRACE*.

De Pontieu, Martens, & Hudson (2001) studied analytically the leakage of Alfvén waves through the chromospheric footpoints of a coronal loop. They concluded that the damping time due to chromospheric leakage in a coronal loop observed by *TRACE* (Nakariakov et al. 1999) is 10–16 minutes. Thus, De Pontieu et al. (2001) claim that the damping of the observed coronal loop oscillations could be explained by the chromospheric leakage of Alfvén waves. Unfortunately, I find that their calculations contain a computational error, which resulted in an underestimated leakage time discussed in § 3. De Pontieu et al. (2001) included chromospheric damping due to ion-neutral collisions. However, they report no significant damping due to the ion-neutral collisions for long-period oscillations.

Previously, Leroy (1980) investigated the propagation of linear Alfvén waves by solving analytically the linear wave equation in an exponential atmosphere and has derived the reflectance coefficient of the Alfvén waves. Hollweg (1984) investigated the transmission and the reflection of Alfvén waves in a three-layer atmospheric model using the WKB approximation valid for short wavelengths and applied the reflection coefficient derived by Leer, Holzer, & Fla (1982) to Alfvén waves in the solar wind. Berghmans & De Bruyne (1995) studied the coronal loop oscillations driven by footpoint motions. They have estimated the energy confinement time of a typical coronal loop to be 100 Alfvén crossing times of the loop length (i.e., ∼30 times longer than the dissipation time of the *TRACE* loop).

To address the chromospheric leakage problem with fewer approximations than required by linear theory, and to resolve the confusion in the literature, I solve numerically for the first time the nonlinear 1.5-dimensional viscoresistive MHD model in order to investigate the leakage of the Alfvén waves through the chromospheric footpoints of a model coronal loop. The model loop represents the loop observed by *TRACE* (Nakariakov et al. 1999) and includes the coronal, chromospheric, and photospheric sections. The solution is one-dimensional in space. However, it includes all three components of $\boldsymbol{V}$ and $\boldsymbol{B}$; hence, it is termed 1.5-dimensional MHD.

## 2. THE 1.5-DIMENSIONAL MHD MODEL OF A CORONAL LOOP WITH CHROMOSPHERIC FOOTPOINTS

Taking the loop to be along the $x$-direction, I solve the normalized viscoresistive nonlinear 1.5-dimensional MHD equations in Cartesian geometry with the usual notations for the variables

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V_x) = 0, \quad (1)$$

$$\rho\left(\frac{\partial \boldsymbol{V}}{\partial t} + V_x \frac{\partial}{\partial x}\boldsymbol{V}\right) = (\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B} + \boldsymbol{F}_v, \quad (2)$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{V} \times \boldsymbol{B}) + \frac{1}{S}\frac{\partial^2}{\partial x^2}\boldsymbol{B}, \quad (3)$$

where the operator $\boldsymbol{\nabla} \times \boldsymbol{B} = -\hat{\boldsymbol{y}}\,(\partial B_z/\partial x) + \hat{\boldsymbol{z}}\,(\partial B_y/\partial x)$ in 1.5 dimensions. The viscous force due to the compressive viscosity is $\boldsymbol{F}_v = (4/3)\,\eta\,(\partial^2 V_x/\partial x^2)$. The compressive viscosity coefficient was set to $\eta = 10^{-6}$, and the Lundquist number $S =$

$10^6$. The values of the dissipation coefficients are small, and they do not affect the calculated leakage time significantly. In the above equations, the thermal pressure is neglected compared with the magnetic pressure (the zero-$\beta$ approximation). Due to this approximation, the energy equation is uncoupled from the above equations and is not needed to calculate the wave leakage. The zero-$\beta$ approximation is a good approximation in the coronal part of the loop and not as good in the chromospheric and photospheric parts, where $\beta \sim 1$. However, since we are dealing with nearly incompressible and nearly linear perturbations, the leakage time is not affected significantly by this approximation.

To represent the coronal and chromospheric parts of the loop, I use the following one-dimensional atmospheric model based on the model used by Leroy (1980), Hollweg (1984), and De Pontieu et al. (2001). In the model, the chromosphere and the photosphere are at both ends of the loop at $x_0 = 0, L$:

$$n(x) = \begin{cases} n_0, & 0 \le x \le x_{\rm ph}, \quad L \ge x \ge L - x_{\rm ph}, \\ n_0 e^{-|x-x_0|/h}, & x_{\rm ph} < x \le x_{\rm tr}, \quad L - x_{\rm ph} > x \ge L - x_{\rm tr}, \\ n_0 e^{-d/h}, & x_{\rm tr} < x < L - x_{\rm tr}, \end{cases} \quad (4)$$

where $n_0$ is the photospheric density, $x_{\rm ph}$, $L - x_{\rm ph}$ are the locations of the photospheric boundary, and $x_{\rm tr}$, $L - x_{\rm tr}$ are the boundaries of the transition region in the loop. Thus, the thickness of the transition region is $d = x_{\rm tr} - x_{\rm ph}$, and the chromospheric scale height is $h$. The corresponding Alfvén speed is given by

$$V_{\rm A}(x) = \begin{cases} V_{\rm A0}, & 0 \le x \le x_{\rm ph}, \quad L \ge x \ge L - x_{\rm ph}, \\ V_{\rm A0} e^{|x-x_0|/2h}, & x_{\rm ph} < x \le x_{\rm tr}, \quad L - x_{\rm ph} > x \ge L - x_{\rm tr}, \\ V_{\rm A0} e^{d/2h}, & x_{\rm tr} < x < L - x_{\rm tr}, \end{cases} \quad (5)$$

where $V_{\rm A0} = B_0/(4\pi\rho)^{1/2}$ is the photospheric Alfvén speed and $B_0$ is the background magnetic field in the loop. In the model, I use $h = 200$ km, in agreement with the chromospheric model of Vernazza, Avrett, & Loeser (1981). Since in the photosphere $V_{\rm A0} = 10$ km s$^{-1}$ (De Pontieu et al. 2001) and in the corona $V_{\rm A} = V_{\rm A0} e^{d/2h} = 1000$ km s$^{-1}$ (Nakariakov et al. 1999), the chromospheric thickness is $d = 1842$ km, in agreement with recent helium line observations (Muglach & Schmidt 2001). Thus, I set the photospheric upper boundary at $x_{\rm ph} = 1000$ km and the transition-region upper boundary at $x_{\rm tr} = 2842$ km inside the loop. The loop length is $L = 130{,}000$ km (Nakariakov et al. 1999).

The Alfvénic oscillations in the loop are initialized at $t = 0$ with

$$V_y = V_{y0} \sin(kx), \quad (6)$$

where $k = 2\hat{k}\pi/L_0$ ($L_0 = L - 2x_{\rm tr}$ is the length of the coronal part of the loop). For the fundamental mode, I have used $\hat{k} = 0.5$, and for the high-frequency mode, I have used $\hat{k} = 5$. The typical amplitude of the oscillations in the corona is 20 km s$^{-1}$ $\ll V_{\rm A} = 1000$ km s$^{-1}$, and the oscillations are nearly linear. The initial density and the velocity profile are shown in Figure 1.

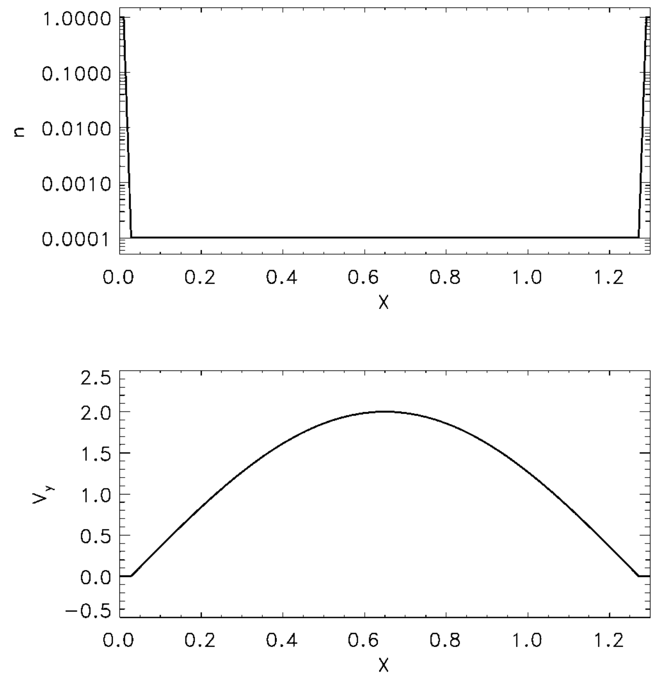


FIG. 1.—Initial state of the coronal loop. The normalized density (*upper panel*) drops exponentially in the transition region from $n = 1.0$ at the photospheric boundary to $n = 10^{-4}$ in the coronal part of the loop. The initial $V_y$ perturbation (*lower panel*) is a half-wave ($\hat{k} = 0.5$) in the coronal part of the loop. The velocity is in units of $V_{\rm A} = 10$ km s$^{-1}$, and the number density is in units of $10^{13}$ cm$^{-3}$.

## 3. MAIN RESULTS OF THE LINEAR THEORY

The linear theory predicts that the leakage time of the Alfvén waves is given by (e.g., Berghmans & De Bruyne 1995)

$$\tau_d = \frac{L}{|\ln R| V_{\rm A}}, \quad (7)$$

where $R$ is the wave reflection coefficient and $V_{\rm A}$ is the Alfvén speed in the coronal part of the loop ($V_{\rm A} = V_{\rm A0} e^{d/2h}$ in our loop model). When the Alfvén speed jumps discontinuously from the photospheric value to the coronal value, the reflection coefficient can be approximated by (e.g., Davila 1991)

$$R = \frac{V_{\rm A} - V_{\rm A0}}{V_{\rm A} + V_{\rm A0}}. \quad (8)$$

To model the *TRACE* loop, I take $V_{\rm A0} = 10$ km s$^{-1}$, $V_{\rm A} = 1000$ km s$^{-1}$, $L = 130{,}000$ km, and I get $R = 0.980$ and $\tau_d = 6500$ s, which is 7.5 times longer then the reported (Nakariakov et al. 1999) oscillation damping time of 870 s.

Leroy (1980) investigated the propagation of the Alfvén waves in an isothermal atmosphere by solving analytically the linear-wave equation in the exponential atmosphere and found that in the long-wavelength limit, the reflection coefficient is given by

$$R \sim \tanh(d/4h). \quad (9)$$

It is interesting to note that for a given $V_{\rm A0}$ and $V_{\rm A}$ and an exponential chromospheric model given by equation (5), equation (9) reduces to equation (8). This is due to the fact that from equation (5), we have $d/4h = 0.5 \ln(V_{\rm A}/V_{\rm A0})$, which upon substitution into equation (9) produces equation (8). Thus, in the long-wavelength limit, and for a given $V_{\rm A0}$ and $V_{\rm A}$, the Alfvén

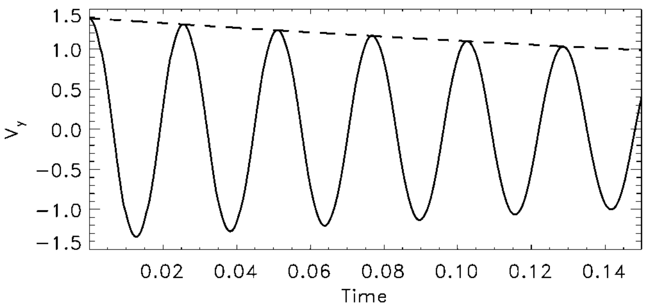


FIG. 2.—Temporal evolution of $V_y$ at $x = 0.33$ in the coronal loop. When a finite-thickness chromosphere ($h = 200$ km, $d = 1842$ km) is included, the leakage time is 4406 s. The exponential decay fit is shown with the dashed line. The time axis is in units of $10^4$ s, and the velocity is in units of 10 km $s^{-1}$.

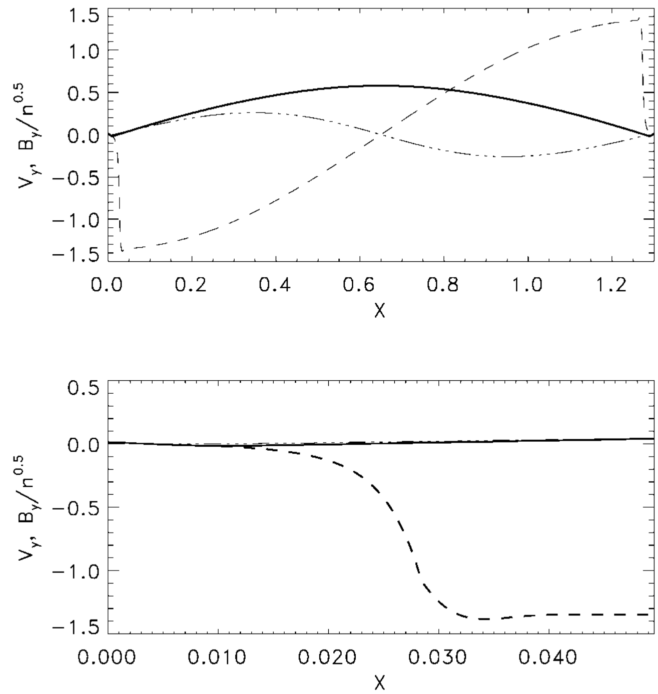


FIG. 3.—Spatial dependence of $V_y$ (*solid line*), $V_x$ (*dash-dotted line*), and $B_y/\sqrt{n}$ (*dashed line*) at $t = 0.0939 = 939$ s with $\hat{k} = 0.5$. The upper panel shows the full length of the loop. The lower panel shows the details of the variables in the transition region. The *x*-axis is in units of $10^5$ km, and the *y*-axis is in units of 10 km $s^{-1}$.

wave leakage time is independent of the thickness and the scale height of the chromosphere. The long-wavelength limit is a good approximation since the observed coronal loop length is much larger than the chromospheric thickness and since the wavelength of the observed oscillations is of the order of the loop length. Obviously, for short-wavelength oscillations ($\hat{k} \gg 1$), this limit is not applicable.

De Pontieu et al. (2001) use the following expression for the timescale of the leakage through the ends of the loop:

$$\tau_d = \frac{2L}{(2 - R_1 - R_2)V_A}, \tag{10}$$

where $R_1$ and $R_2$ are the reflection coefficients at the loop's footpoints. Unfortunately, the factor of 2 in the above equation was omitted in equation (15) of De Pontieu et al. (2001). Also, De Pontieu et al. (2001) have used the energy reflection coefficient $R_e \equiv R^2$ instead of the wave amplitude reflection coefficient $R$ to estimate the leakage time of the *TRACE* loop, which resulted in the erroneously short leakage time in their paper. Below, I provide the correct values. Using the energy reflection coefficient in a sunspot of $R_{e,\,\mathrm{spot}} = 0.90$ and in a plage of $R_{e,\,\mathrm{plage}} = 0.94$, calculated by De Pontieu et al. (2001), and the correct equation (10), I get $\tau_d = 3179$ s, which is 4.4 times longer than the observed damping time of the loop oscillations.

The energy reflection coefficient of Alfvén waves was obtained by Leer et al. (1982) for Alfvén waves in the solar wind (their eqs. [44] and [46]) and used by Hollweg (1984) and De Pontieu et al. (2001) in coronal loops. For solar parameters, $R_e$ can be approximated by

$$R_e \approx 1 - 2\pi\alpha, \tag{11}$$

where the value of $\alpha = 2h\omega/V_A$ is small. In the model, $h = 200$ km, $\omega = 2\pi/256 = 0.0245$ rad $s^{-1}$, and $V_A = 1000$ km $s^{-1}$. Thus, $\alpha = 0.0098$, $R_e = 0.938$, and $R_1 = R_2 = R \equiv R_e^{0.5} = 0.9685$, and using equation (10), I get $\tau_d = 4127$ s. For short-wavelength Alfvén waves, $\omega = 0.245$ rad $s^{-1}$ and $R = 0.62$, and I get $\tau_d = 342$ s. Equation (7), valid for $R \approx 1$, produces similar leakage times. For arbitrary wavelength, $h$, and $d$, equation (33) of De Pontieu et al. (2001) or equation (65) of Leroy (1980) can be used to calculate $R$.

## 4. NUMERICAL RESULTS

The linear analytical calculation of the leakage time is sensitive to the estimated theoretical value of the reflection coefficient. In the numerical model, the leakage time is calculated from the decrease of the wave amplitude in the loop as a function of time, without the need to estimate the reflection coefficient (which could be calculated from the numerically determined $\tau_d$). For this purpose, I have solved equations (1)–(3), with the initial density profile (eq. [4]) and velocity perturbation (eq. [6]), using the fourth-order Runge-Kutta method for the temporal solution and fourth-order derivatives in space. I have used 4096 grid points to resolve the loop's structure and to minimize the effects of numerical dissipation. The boundary conditions were open at both ends of the loop, allowing flows through the footpoints. In the figures, the velocities are in units of 10 km $s^{-1}$, the time is in units of $10^4$ s, and the distance is in units of $10^5$ km.

When the Alfvén speed jumps discontinuously from 10 km $s^{-1}$ in the photosphere to 1000 km $s^{-1}$ in the corona, the calculated leakage time of the Alfvén waves with $\hat{k} = 0.5$ is 6453 s, in excellent agreement with the theoretical value predicted by equations (7) and (8). With a chromosphere ($h = 200$ km and $d = 1842$ km), the decay of the oscillations proceeded at a higher rate than in the previous case, and the leakage time is $\tau_d = 4406$ s. The temporal evolution of $V_y$ in the coronal part of the loop ($x = 0.33$) for the initial velocity perturbation (eq. [6]) is shown in Figure 2. The corresponding oscillation frequency is $\omega = 0.0245$ rad $s^{-1}$. The exponential decay fit is shown with the dashed line. This leakage time is in good agreement with the value predicted by equation (10) using $R = R_e^{1/2}$ given by equation (11), and it is about 7% longer.

In Figure 3, the spatial dependence of $V_y$, $V_x$, and $B_y/\sqrt{n}$ at $t = 0.0939 = 939$ s is shown. The $V_x$ component of the velocity is produced by the nonlinear coupling of the Alfvén wave to the fast wave. Note that the wavelength of $V_x$ is half the wavelength of $V_y$ and $B_y$. However, since the amplitude of the wave is small (2%) compared with the coronal Alfvén speed, the nonlinearity does not affect the dynamics or the leakage time significantly. The upper panel shows the full length of the loop. The lower panel shows the details of the variables in the transition region.

To investigate the temporal evolution of short-wavelength Alfvén waves, I have initialized $V_y$ with $\hat{k} = 5$. In this case, I find that the leakage time is 397 s, in good agreement with the value predicted by equation (10) using $R = R_e^{1/2}$ given by equation (11). The numerical leakage time is about 16% longer than the value based on linear theory. I find that the nonlinearly produced $V_x$ is more significant in the short-wavelength oscillations because of larger spatial gradients than in the long-wavelength oscillations. However, the effect of the nonlinear $V_x$ on the leakage time is small.

## 5. DISCUSSION AND CONCLUSIONS

The recent *TRACE* observations of damped coronal loop oscillations generated discussion regarding the possible damping mechanism. Nakariakov et al. (1999) estimated the leakage to be nearly 2 orders of magnitude longer than the damping time and concluded that the damping is due to enhanced viscosity or resistivity in the solar corona. This conclusion has an important impact on coronal heating theories since the enhanced dissipation solves the difficulties with existing wave heating or reconnection theories. Based on an erroneous calculation, De Pontieu et al. (2001) concluded that the damping is due to the leakage of Alfvén waves into the chromospheric footpoints of the loop and that dissipation is due to ion-neutral collisions in the chromosphere.

Using nonlinear viscoresistive 1.5-dimensional MHD equations, I model the leakage in the *TRACE* loop observed by Nakariakov et al. (1999). I find that the leakage time in this loop is 4406 s, which is 5 times longer than the observed damping time. Based on this model, I conclude that the chromospheric footpoint leakage cannot explain the observed rapid oscillation damping, in agreement with the conclusion of Nakariakov et al. (1999). I find that short-wavelength Alfvén waves will leak on a much shorter timescale and that the nonlinear effects do not affect the leakage time in the small-amplitude (compared with $V_A$) coronal loop oscillations.

Although I conclude that the footpoint leakage in not important in the damping of oscillations in the particular loop observed by *TRACE,* I note that the leakage time depends on several observed and assumed parameters. An increased or decreased chromospheric scale height will leaded to a faster or slower wave leakage, with fixed photospheric and coronal Alfvén speeds, and loop length. Similarly, the values of photospheric and coronal Alfvén speeds, the loop length, and the wavelength can vary from one location to another in an active region and may be different in various active regions and in various loops. These factors will affect the leakage time, and the footpoint leakage may become more or less important in some events of coronal loop oscillation.

The dynamic variations and shocks in the chromosphere were not included in the model. These effects will result in small-scale variations and sharp gradients in $V_A$ that will lead to an even higher wave reflection and a lower wave leakage rate than the presently calculated rate. In the present 1.5-dimensional MHD coronal loop model, the geometric divergence of the magnetic field near the footpoints is not included. This divergence can contribute further to the reflection of Alfvén waves and increase the leakage time, even without a change in the value of the Alfvén speed. To account for the magnetic divergence requires multidimensional modeling of the coronal loop.

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