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THE MASS AND RADIUS OF 40 ERIDANI B FROM *HIPPARCOS*: AN ACCURATE TEST OF STELLAR INTERIOR THEORY[1]

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ABSTRACT

The astrometric satellite *HIPPARCOS* has determined the distance to the important white dwarf 40 Eridani B with an accuracy surpassing 1%. We use this datum to redetermine the mass and radius of this star and find that $M = 0.501 \pm 0.011\ M_\odot$ and $R = 0.0136 \pm 0.00024\ R_\odot$. These values, the most accurately determined masses and radii for any white dwarf star, place 40 Eri B securely on the zero-temperature carbon core mass-radius relation, considerably improving observational confirmation of the theory of stellar degeneracy. The mass is now sufficiently high that binary star evolutionary channels are no longer required to account for it. The data exclude the possibility that the surface hydrogen layer is as thick as $10^{-4}\ M_\odot$.

Subject headings: stars: individual (40 Eridani B) — ultraviolet: stars — white dwarfs

1. INTRODUCTION

The white dwarf 40 Eridani B (=WD 0413−077) is of interest to astronomers in many fields of stellar astrophysics. Most important, it is one of the few white dwarfs that can be used to test directly the theory of stellar degeneracy, a Nobel Prize–winning idea (Chandrasekhar 1939) whose observational confirmation rests on a few somewhat uncertain data points (e.g., Shipman & Sass 1980; Schmidt 1996; Provencal et al. 1997). The white dwarf 40 Eri B is one of the most embarrassing data points, with a radius much too small for its assumed mass of 0.43 $M_\odot$ (Heintz 1974). The disagreement between astrometry and measurements of its gravitational redshift has been discussed in several papers over the years (Wegner 1980; Koester & Weidemann 1991).

Our understanding of 40 Eri B has wide implications. It is the smallest white dwarf star produced by single-star evolution. Indeed, its observed mass prior to *HIPPARCOS* was sufficiently small that binary star evolution (Iben 1987) was invoked to explain its existence, although the properties of the system virtually exclude mass exchange. The white dwarf 40 Eri B can serve as an interesting constraint on models of the evolution of low-mass stars and on other astrophysical determinations that depend on these models (such as the use of white dwarfs to determine the distances to globular clusters). It anchors the low end of the initial-final mass relation (Koester & Reimers 1996).

2. THE DISTANCE AND MASS OF 40 ERIDANI B

40 Eri B is the second brightest white dwarf and among the first to be discovered. Its parallax has been determined many times in the 20th century, mostly with long-focus refractors. Most researchers use parallaxes quite close to the Sproul parallax of 207 ± 2 mas (Heintz 1974). Many of us (e.g., Shipman 1979) questioned the rather small error bar. But until *HIPPARCOS*, we could only fret.

Therefore, one of us (H. L. S.) proposed this star, among others, as a target for the *HIPPARCOS* mission. *HIPPARCOS* is rather different from most astronomical facilities in that all targets were selected before launch, and the astrometric characteristics of all 118,322 stars in the *HIPPARCOS* Input Catalog (ESA 1997) were determined simultaneously. The *HIPPARCOS* parallax for 40 Eri A, which also applies to the physical companion 40 Eri B, is 198 ± 0.84 mas, somewhat smaller than the Sproul parallax. The quoted uncertainty is a standard product of the *HIPPARCOS* data reduction procedure and is robustly determined.

The difference between the two parallaxes appears small but is actually quite significant. The mass M of a star in a visual binary strongly depends on its parallax π: $M \propto \pi^{-3}$. We use the orbit published by Heintz (1974) combined with the *HIPPARCOS* parallax to determine a revised mass for 40 Eri B of 0.501 $M_\odot$. It is, however, critical to determine the uncertainties accurately.

The four quantities that enter into the calculation of the mass of 40 Eri B are the semimajor axis of the apparent orbit (in arcseconds), the orbital period, the parallax, and the fractional mass. Heintz's article gives no uncertainty for the first two, possibly because prior to *HIPPARCOS*, the parallax uncertainty dominated the overall error budget. We estimate an uncertainty for the semimajor axis by determining that the individual separations listed in Heintz (1974), as measured by the then most recent series of Sproul Observatory observations, differ from the calculated values with a mean $O - C = 2.5 \pm 6.4$ mas. The semimajor axis of the apparent orbit then becomes 6943 ± 6.4 mas. In a similar vein, we note that a 1 yr uncertainty in the period P results in a position angle error of 0°.2 in 50 yr, a value consistent with the values of $O - C$ listed by Heintz (1974). Thus, we adopt 1 yr for the uncertainty in P. The fractional mass of 40 Eri B is $f_B = 0.738 \pm 0.01$ (note that the value of $f = 0.26$ in Heintz's paper is the fractional mass of C), and we use the uncertainty quoted by Heintz, converting his probable error to a standard deviation.

The resulting mass error budget is shown in Table 1. The entries in Table 1 are multiplied by the relevant constant to reflect the contributions of each quantity to the uncertainty in

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TABLE 1
MASS ERROR BUDGET

Observed Quantity	Contribution to Mass Error Budget
Apparent semimajor axis	0.0028
Period	0.008
Parallax	0.015
Mass ratio	0.013
Net error $\Delta M/M$	0.022
Mass of 40 Eri B	$0.501 \pm 0.011\ M_\odot$

TABLE 2
RADIUS ERROR BUDGET

Observed Quantity	Contribution to Radius Error Budget
Parallax	0.0042
Flux calibration	0.01
Monochromatic flux (and temperature)	0.009
V magnitude	0.01
Net error $\Delta R/R$	0.017
Radius of 40 Eri B	$0.0136 \pm 0.00024\ R_\odot$

the mass of 40 Eri B. The following relation, derived by standard error propagation techniques applied to Kepler's third law, makes these contributions more explicit:

$$(\Delta M/M)^2 = (3\Delta a/a)^2 + (2\Delta P/P)^2 + (3\Delta\pi/\pi)^2 + (\Delta f/f)^2. \tag{1}$$

The entry in Table 1 corresponding to the *HIPPARCOS* parallax is, then, $3\Delta\pi/\pi$.

Equation (1) presumes that the uncertainties are statistically independent. Our results indicate that the dominant contributions to the mass error budget are the parallax and fractional mass. Consequently, our rather simple treatment of the uncertainties in a and P is roughly correct. To our knowledge, no measurements of either separation or position angle have been made for 30 years. A current measurement of these quantities would be invaluable in confirming both Heintz's orbit and our error estimates.

3. THE RADIUS OF 40 ERIDANI B AND ITS UNCERTAINTY

The radius of 40 Eri B has been determined by several investigators using model atmosphere calculations. The two most recent determinations are from Koester & Weidemann (1991) and Bergeron, Saffer, & Liebert (1992). We adjusted both radii to the *HIPPARCOS* parallax, using the parallaxes quoted in those papers, to obtain values of 0.0134 $R_\odot$ and 0.0139 $R_\odot$, respectively. One way to determine the radius of 40 Eri B is simply to average the two values and use the average difference between each individual determination as the uncertainty. While this procedure comes up with what we believe to be the correct answer, a more extensive discussion of the uncertainty is needed to illuminate the reasons behind the difference and the prospects for improvement.

Shipman (1979) describes the physical basis of any radius determination. The fundamental relation between the monochromatic Eddington flux H, the stellar distance D, and the stellar radius R is

$$f = 4\pi H(R/D)^2. \tag{2}$$

The stellar flux f is generally determined by a star's visual magnitude. Its uncertainty is a combination of random errors in the magnitude of an individual star and systematic errors in transforming this magnitude into a flux, either through the zero point of the bolometric correction or through the use of equation (2) more explicitly. We take the random uncertainty in m_v (40 Eri B) to be 0.02 mag, following Koester & Weidemann (1991). A more subtle problem is the accuracy of the transformation of stellar magnitude to flux. A careful review of recent literature on this issue (Hayes & Pasinetti 1985; Oke 1990) shows no need to revise a conclusion one of us reached some time ago (Shipman 1979): no matter how accurate any ground-based measurement of m_v is on its own internal system, there is a residual uncertainty of 2% relating to the conversion of visual magnitude to absolute flux. *Hubble Space Telescope* (*HST*) measurements may produce a modest improvement to a precision of 1.1% (Colina & Bohlin 1994; Bohlin, Colina, & Finley 1995). Since there is no reason to believe that the flux of 40 Eri B is known any more accurately than that of a random ninth-magnitude star, we take 2% as an estimate of the systematic uncertainty in transforming m_v to a flux.

Since the parallax uncertainty is well known, the remaining source of uncertainty is our knowledge of the monochromatic stellar flux H, in this case in the visual band. This quantity is normally produced by a standard model-atmospheres program and is a function of effective temperature. Recent temperature determinations include 16,400 K and 16,730 K (Bergeron et al. 1992) and $17,000 \pm 200$ K (Koester & Weidemann 1991). These determinations, based on different model atmosphere codes, suggest that $\Delta T \approx 300$ K. In this temperature range, our model atmospheres show that $H \propto T^{1.6}$, and so $\Delta H/H \approx 1.6\Delta T/T$, or 0.018. Experience in using various model atmosphere programs suggests that uncertainties arising from the numerical methods employed and from the input physics are at least a factor of 2 smaller than that produced by the temperature uncertainty.

The resulting radius error budget is shown in Table 2. As is the case in Table 1, the terms reflect standard error propagation through equation (2). The table lists $0.5\Delta f/f$, since that is how flux error contributes to radius error. For the sake of argument, we assume statistical independence of all sources of uncertainty.

Both recent determinations of the radius of 40 Eri B, adjusted for the *HIPPARCOS* parallax, fall within our error bars. Koester & Weidemann's radius is smaller, as we would expect, since their temperature is slightly higher. The agreement between the error estimate in Table 2 and the difference between the two findings provides some confidence that the uncertainties in this process are reasonably well understood.

The most important conclusion from Table 2 is that the parallax error is only a small contributor to the net radius uncertainty. Such a circumstance is rare in stellar astronomy. If the monochromatic flux from 40 Eri B could be measured on the *HST* standard system to the precision quoted by Colina & Bohlin, and if the temperature could be determined more definitively, the uncertainty in the radius could be reduced by a factor of about 2.

The improvements discussed above are well within reach and force us to face the question of how accurate model at-

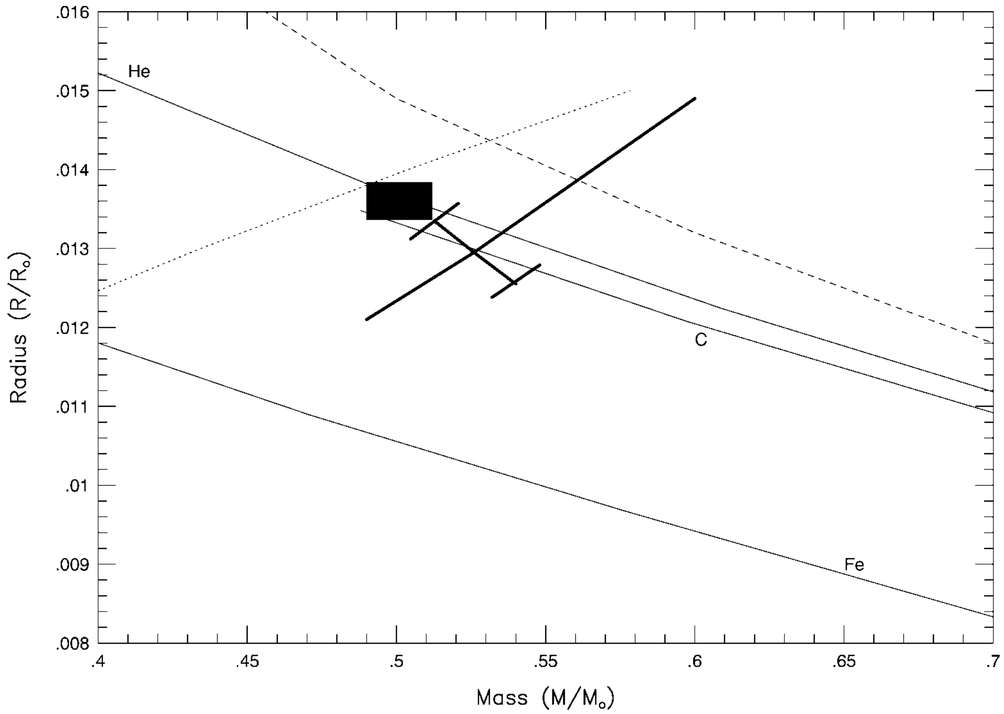

FIG. 1.—The mass-radius relation for 40 Eri B. The solid lines labeled He, C, and Fe denote the zero-temperature mass-radius relation of Hamada & Salpeter (1961). The dashed line is a Wood (1994) relation for an 15,000 K hydrogen surface white dwarf, the dotted line is the relation determined from surface gravity, and the heavy lines show the results determined from gravitational redshift measurements. The filled box shows our astrometrically determined mass and its error bound.

mosphere programs really are. We note that the two temperature determinations we use are produced from two computer codes independently developed over the years by Koester and Bergeron. To the best of our knowledge, these two codes do not share one common line of software. Our present findings agree with those of Kruk et al. (1997), who find that the relative fluxes from model atmosphere programs agree with each other, and with Hopkins Ultraviolet Telescope observations, at the same 1% level at which the *HST* team claims to have determined a flux scale for visual magnitudes.

4. ASTROPHYSICAL IMPLICATIONS

The astrometrically determined mass and radius given above can be independently verified using measurements of surface gravity and gravitational redshift. We present the results of the comparison in graphical form in Figure 1 and in tabular form in Table 3.

Stellar surface gravity is determined by fitting model atmospheres to observations, paying particular attention to the breadth of the hydrogen lines. The model predictions are more sensitive to the input physics than is the case with monochromatic fluxes, since the breadth of the hydrogen lines is directly determined by the hydrogen line broadening theory. Consequently, uncertainties in the model fits are difficult to estimate even if the data are good. However, our prediction agrees well with the observed value (Table 3).

The gravitational redshift of 40 Eri B has been measured repeatedly since Munch (1944). Again, the agreement between our astrometric prediction and the observed values is satisfactory.

A most interesting astrophysical application of the current results is to the theory of stellar degeneracy. Since its publication by Chandrasekhar (1939), several generations of astronomers have sought to test the predictions of the white dwarf mass-radius relation. The situation in the early 1980s, summarized by Shipman & Sass (1980), was not particularly sat-

TABLE 3

AGREEMENT WITH PREDICTION

Parameter	Astrometric Prediction (this Letter)	Observed Value	Reference
Surface gravity (log g)	7.87 ± 0.02	7.85	...
Gravitational redshift	23.3 ± 0.6	25.8 ± 1.4	1
		26.5 ± 1.5	2
		23.9 ± 1.3	3
		28	4
		23 ± 5	5
		21 ± 2	6
		22 ± 9	7

REFERENCES.—(1) Reid 1996; (2) Koester & Weidemann 1991; (3) Wegner 1980; (4) Greenstein et al. 1977; (5) Greenstein & Trimble 1972; (6) Popper 1954; (7) Munch 1944.

isfactory; only three stars could be used to test this important theory, and two lay 1.5 σ from the predicted relation. Since then, others have added data points to this test, but agreement between theory and experiment is still poor (e.g., Schmidt 1996; Provencal et al. 1997). As shown in Figure 1, our results place 40 Eri B precisely on the zero-temperature mass-radius relation for carbon cores predicted by Hamada & Salpeter (1961) over 25 years ago.

The primary contribution of modelers since Hamada & Salpeter (1961) is the recognition that a white dwarf with an envelope of reasonable thickness and nonzero temperature will not fall on the Hamada-Salpeter relation. We use the Wood (1994) models to determine that the radius of 40 Eri B is too small to fit a Wood model with a canonical $10^{-4}\,M_{\odot}$ hydrogen layer (DA). This finding is consistent with other results (e.g., Shipman 1979, 1995), arguing that the ratio of DA to non-DA white dwarfs at cool temperatures suggests that DA white dwarfs do not all have the same hydrogen layer mass but have a variety of H layer thicknesses ranging from 10^{-4} to $10^{-7}\,M_{\odot}$.

For years, 40 Eri B has presented a puzzle to theoreticians who struggled with its origin. A star of 0.43 $M_{\odot}$ could not be produced by single-star evolution. Our current mass of 0.50 $M_{\odot}$ turns around the arguments of Iben (1987), showing that the mass is entirely consistent with single-star evolution and can set some constraints on models of single-star evolution. Indeed, the controversy regarding the appropriateness of standard mass-loss formulas such as the often-used Reimers scheme (Willson, Bowen, & Struck 1995) may benefit from such a firm anchor as 40 Eri B.

A related consideration is the use of white dwarf masses in very old stellar systems to determine such things as globular cluster distances and the age of the universe (Renzini et al. 1996). These methods depend critically on knowing the masses of white dwarfs in globular clusters. The lowest mass Galactic white dwarf can provide important constraints and confirmation of Renzini's method. Our mass of 40 Eri B is entirely consistent with the assumptions of Renzini et al. (1996).

5. CONCLUSIONS AND PROSPECTS FOR THE FUTURE

We determine the mass and radius of 40 Eri B to be $M = 0.501 \pm 0.011\,M_{\odot}$ and $R = 0.0136 \pm 0.00024\,R_{\odot}$. These values are consistent with the mass-radius relation. They are also consistent with our picture of single-star evolution and confirm that no special evolutionary models are needed to account for 40 Eri B. Our Letter includes an extensive discussion of the uncertainties in the mass and radius and their origins.

HIPPARCOS parallaxes are sufficiently accurate that we anticipate considerable improvement in our knowledge of the mass-radius relation with more extensive investigation of the white dwarfs in the *HIPPARCOS* catalog. Such an investigation is currently under way and should permit us to produce a picture more extensive and precise than that currently in the literature.

40 Eri B itself offers prospects for improvement. A most useful contribution to the radius determination would be a precise temperature. Perhaps 40 Eri C contaminates the spectrophotometry and produces inconsistencies in the result; good *IUE* data do exist on this star. Another advance would be a determination of the absolute flux of 40 Eri B to an accuracy of 1% in any wave band, not just the visual.

The radius of 40 Eri B is now better determined than the mass. It has been 30 years since the last measurement of separation and position angle of this pair. An additional precise measurement could confirm the orbit used in this Letter. The prospects for significant reduction in the mass uncertainty are not large, since the parallax provides a large contribution to the mass error budget.

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