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ORBITAL PERIOD CHANGES OF ALGOL-TYPE BINARIES: S EQUULEI AND AB CASSIOPEIAE

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ABSTRACT

The orbital periods of two Algol-type binaries, S Equ and AB Cas, are studied based on all available times of minima. It is found that the $O-C$ variations of both systems can be represented by a parabolic and a periodic term. S Equ has an accretion region due to mass transfer as shown by Richards and coworkers and Richards & Albright. On the other hand, Zola obtained a light-curve solution with a third light. Our study reveals evidence of mass transfer and third component stars in both systems. The parabolic terms in the $O-C$ variations are due to mass transfer from less massive to more massive components, which is consistent with the semidetached nature of both systems. The periodic terms are explained with the additional third stars in the systems. It is interesting that there is no sign of a magnetic braking effect from the cool secondary in the $O-C$ variation of either system.

Key words: binaries: individual (S Equulei, AB Cassiopeiae)

1. INTRODUCTION

S Equulei (BD +04°4584, HD 199454, HIP 103419) is a semidetached Algol-type eclipsing binary with an orbital period of $P = 3.4360969$ days (Catalano & Rodonò 1968; Piotrowski, Rucinski, & Semeniuk 1974; Cester et al. 1979; Zola 1992). The spectral types of the hotter and cooler components are B7 V and G8 III, respectively (Plavec 1983). Its spectra show single-peaked emission features with additional weak double-peaked emission at some epochs, indicating mass transfer activity and disk structure around the primary component (see, e.g., Richards & Albright 1999; Vesper, Honeycutt, & Hunt 2001). Although $O-C$ curves were published by several authors (Plavec 1966; Kreiner 1971), there was has been no detailed period study of the system. Just after our study was completed, we were informed of the paper by Qian & Zhu (2002) on the period variation of S Equ; they explain the period changes of S Equ in terms of a continuous period increase of $dP/dt = 1.27 \times 10^{-6}$ days yr^{-1} and a third body in the system.

AB Cassiopeiae (BD +70°193, HIP 12235) is also a classical Algol: $P = 1.3668755$ days, with A3 and K main-sequence stars (Tempesti 1971; Ando 1980; Rodríguez et al. 1998). Its primary component is a δ Scuti star with a pulsation period of $P = 0.0583$ days and amplitude $\Delta V \sim 0.05$ mag (Rodríguez et al. 1994). In the literature there is no spectroscopic evidence of mass transfer in the AB Cas system. Kaitchuck, Honeycutt, & Schlegel (1985) presented an investigation of many classic Algol-type stars as part of a search for emission lines from accretion disks around the primary star, but in the Hβ and Hγ spectral region no emission effects were seen for AB Cas. Although Kreiner (1971) and Ahnert (1973) reported an orbital period change in this system, a detailed orbital period study is absent so far.

2. ORBITAL PERIOD STUDY OF S EQUULEI

We have collected 123 visual and 14 photoelectric times of minimum light from the literature (see, e.g., the Eclipsing Binaries Minima Database).[1] We formed the $O-C$ (I) diagram, shown in Figure 1, by using Mallama's (1980) ephemeris,

$$\text{Min. I} = \text{HJD } 2{,}442{,}596.74348 + 3.4360969E \ . \quad (1)$$

As can be seen in Figure 1, the $O-C$ change is rather complex. However, the general trend of the $O-C$ (I) residuals can be represented by a parabola, which indicates a secular increase in the orbital period of the system. A parabolic least-squares fit to the $O-C$ (I) values leads to the following quadratic ephemeris, given with the mean error of each term:

$$\text{Min. I} = \text{HJD } 2{,}442{,}596.7510(3) + 3.4361009(2)E + 6.25(9) \times 10^{-9} E^2 \ . \quad (2)$$

As displayed in Figure 2, the quadratic ephemeris can provide a good fit to the mean trend of the $O-C$ (I) curve without describing any particular characteristics. Differences from the parabolic fit, $O-C$ (II), are also shown in Figure 2 (*bottom*), which indicate a sinusoidal change. Such periodic behavior may result from the light-time effect due to a third body that is dynamically bound to the eclipsing pair.

A representation of the $O-C$ (II) diagram by a periodic term based on Irwin's (1959) formulation can be given as

$$\text{Min. I} = \frac{K}{\sqrt{1 - e^2 \cos^2 \omega}} \left[\frac{1 - e^2}{1 + e \cos \nu} \sin(\nu + \omega) + e \sin \omega \right] , \quad (3)$$

where K is the semiamplitude of the light-time effect in units of days and is expressed as

$$K = \frac{a_{12} \sin i}{2.590 \times 10^{10}} \sqrt{1 - e^2 \cos^2 \omega} \ . \quad (4)$$

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[1] See http://www.oa.uj.edu.pl.

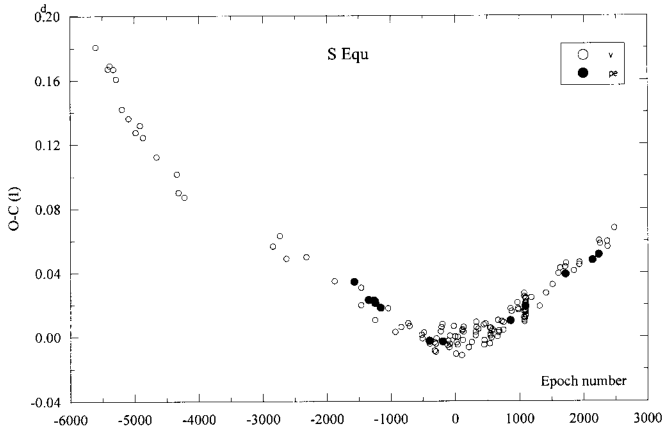

FIG. 1.—The $O-C$ (I) diagram for S Equ formed using Mallama's (1980) light elements.

In equations (3) and (4) a_{12}, e, i, and ω are the orbital parameters of the eclipsing pair around the center of mass of the triple system and ν is the true anomaly. The epoch of periastron passage and the period of this orbit, T and P, respectively, are hidden parameters in equation (3). The application of equation (3) to the $O-C$ (II) residuals yields the parameters of the third-body orbit. This fit is shown in Figure 2 (*bottom*). The representation seems to be satisfactory. Then we applied parabolic and periodic fits to the $O-C$ (I) residuals simultaneously. The least-squares solution yields the parameters of the third-body orbit and the coefficient of the quadratic term as listed in Table 1. The best fit including sinusoidal and parabolic variations and the remaining residuals are plotted against epoch numbers in Figure 3.

TABLE 1

ORBITAL PARAMETERS OF THE THIRD-BODY ORBIT, THE COEFFICIENT OF THE QUADRATIC TERM, AND THEIR STANDARD DEVIATIONS FOR S EQUULEI

Parameter	Value	Standard Deviation
T_0 (HJD)	42,596.7537	0.0008
P_0 (days)	3.4360982	0.0000002
$a_{12} \sin i$ (km)	3.54×10^8	8.34×10^6
e....................................	0.21	0.05
ω (deg)...........................	177	11
T (HJD)	38771	460
P (yr).............................	40.4	0.9
Q (days cycle^{-1})	5.55×10^{-9}	1.8×10^{-10}

The coefficient of the quadratic term is positive, which indicates an increasing period due to mass transfer from the less massive to the more massive component. This is to be expected, because S Equ is a semidetached classical Algol-type binary. From the quadratic term we have $dP/dt = 1.18 \times 10^{-6}$ days yr^{-1} and a conservational mass transfer rate $dM/dt = 3.97 \times 10^{-8}\ M_\odot$ yr^{-1}.

The 40.4 ± 0.9 yr periodic term (see Fig. 2) is not quite sinusoidal but exhibits a moderate eccentricity indicating a third body in an eccentric orbit as the cause of this term. The $O-C$ analysis allows us to determine the mass function of the unseen component and to estimate its mass by

$$f(m) = \frac{a_{12} \sin i}{P^2} = \frac{(m_3 \sin i)^3}{(m_{12} + m_3)^2}\ , \tag{5}$$

where m_3 and $f(m)$ are the mass and mass function of the third body and m_{12} is the total mass of the eclipsing pair.

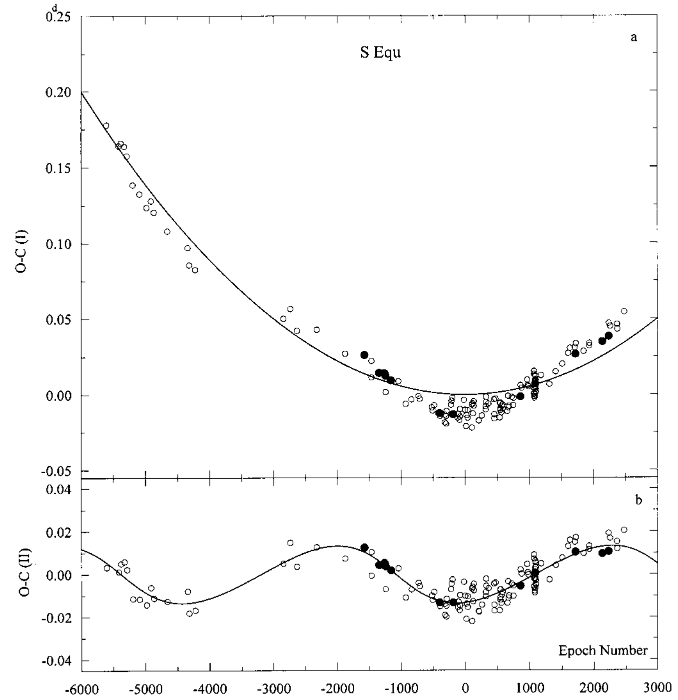

FIG. 2.—(*a*) $O-C$ (I) diagram for S Equ and its parabolic representation (*solid line*). (*b*) The residuals, $O-C$ (II), from the parabolic best fit and its representation of a third-body orbit.

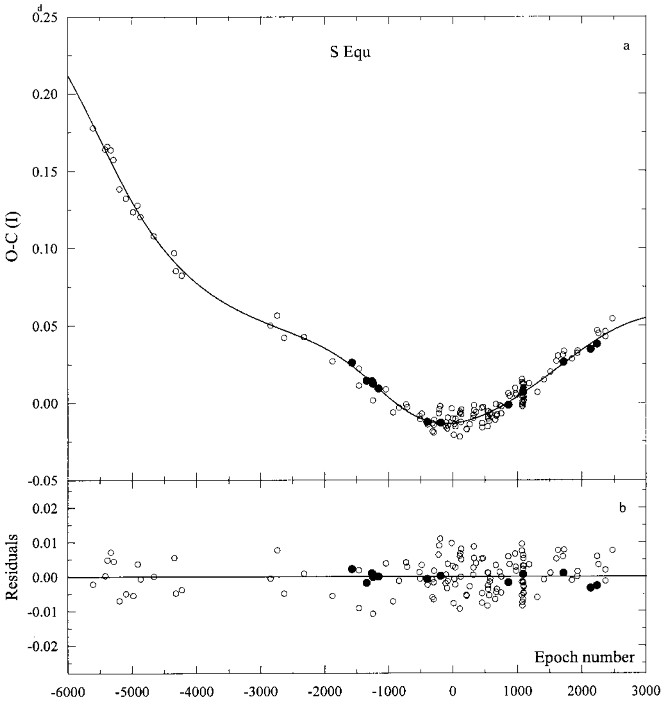

FIG. 3.—(*a*) $O-C$ (I) diagram for S Equ and its representation by a best-fit parabolic plus a periodic term (*solid line*). (*b*) The remaining residuals from the representation.

The mass function for the third star was calculated as $0.0081 \pm 0.0002\ M_\odot$. The mass of the third body cannot be obtained exactly if we do not know the inclination of its orbit. We estimated the mass of the third body as 0.41, 0.44, 0.55, and 0.91 $M_\odot$ for inclinations of 90°, 70°, 50°, and 30°, respectively.

The change of the systemic velocity of the binary should be about 3.6 km s^{-1} while orbiting around the center of mass of the triple system. We estimate the maximum angular separation between the third body and the binary system to be about $0\rlap{.}''051$ for a distance of 400 pc. If we assume that the third body is a main-sequence star and its orbit is coplanar with the binary orbit, then its mass would be about 0.4 $M_\odot$. Using the mass-luminosity relation for main-sequence stars given by Demircan & Kahraman (1991), we can estimate the bolometric absolute magnitude of the third body to be about 8.4 mag, whereas the bolometric absolute magnitude for the primary component of the system is about 1.3 mag. Thus, the third star, if it exists, should be about 7 mag fainter than the S Equ system, and it will be difficult to detect spectroscopically.

3. ORBITAL PERIOD STUDY OF AB CASSIOPEIAE

All available times of primary and secondary minima (altogether 620 visual, six photographic, nine photoelectric, and one CCD) were collected from the literature. In addition, four primary times of minimum light were obtained at Ege University Observatory (EUO) and Çanakkale Onsekiz Mart University Observatory (ÇOMUO) for this study and are listed in Table 2.

The $O-C$ residuals were computed with the linear ephemeris given by Rodríguez (1998):

$$\text{Min. I} = \text{HJD } 2{,}447{,}483.4980 + 1.3668783E\ . \qquad (6)$$

All of the $O-C$ residuals are plotted versus epoch number in Figure 4.

As displayed in Figure 4, the $O-C$ variation of the system is very similar to that of S Equ. The following quadratic ephemeris for the system is obtained from parabolic representation of the $O-C$ variation:

$$\text{Min I} = \text{HJD } 2{,}447{,}483.5075(2) + 1.36688317(6)E + 6.40(9) \times 10^{-10}E^2\ . \qquad (7)$$

The residuals from the parabolic fit show a periodic variation (see Fig. 5, *bottom*). We used the same procedure for the analysis of the $O-C$ diagram of AB Cas as for S Equ.

The coefficient of the quadratic term indicates a period increase with a rate of $\Delta P/P = 0.03$ s yr^{-1}, which may be due to mass transfer from the less massive to the more massive component. A conservational mass transfer rate was estimated from the quadratic term as $dM/dt = 5.6 \times 10^{-8}$ $M_\odot\ \mathrm{yr}^{-1}$.

TABLE 2

Observed Times of Minimum Light of AB Cassiopeiae

T (HJD)	Error	Filter	Method	Observatory
52,162.3596.......	0.0002	*B*, *V*	Photoelectric	EUO
52,166.4584.......	0.0003	*B*, *V*	Photoelectric	EUO
52,322.2834.......	0.0007	*B*, *V*	Photoelectric	EUO
52,490.4170.......	0.0005	. . .	CCD	ÇOMUO

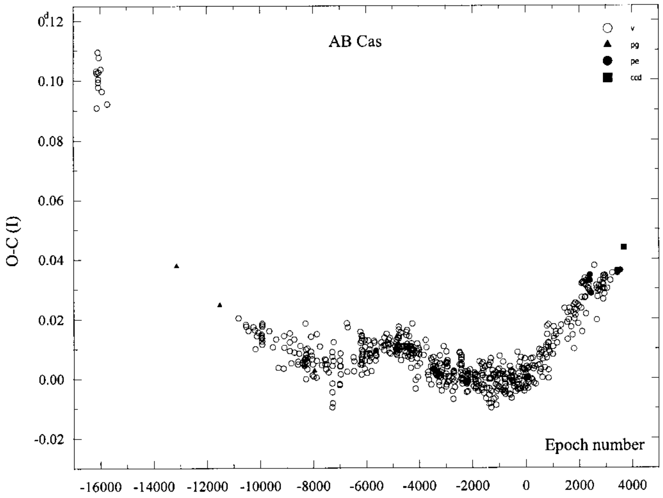

Fig. 4.—The $O-C$ (I) diagram for AB Cas

The tilted sinusoidal variation in Figure 5 suggests a light-time effect caused by the presence of a third body. The orbital parameters of the three-body system and the coefficient of the quadratic term were determined by using equation (3) plus parabolic change simultaneously, and the results are listed in Table 3. Figure 6 shows the best-fit parabolic plus periodic representation of the $O-C$ diagram and the residuals from the best fit.

The orbital period of the eclipsing binary around the center of mass of the triple system was determined to be 40.3 ± 0.9 yr. We found that the semiamplitude of the

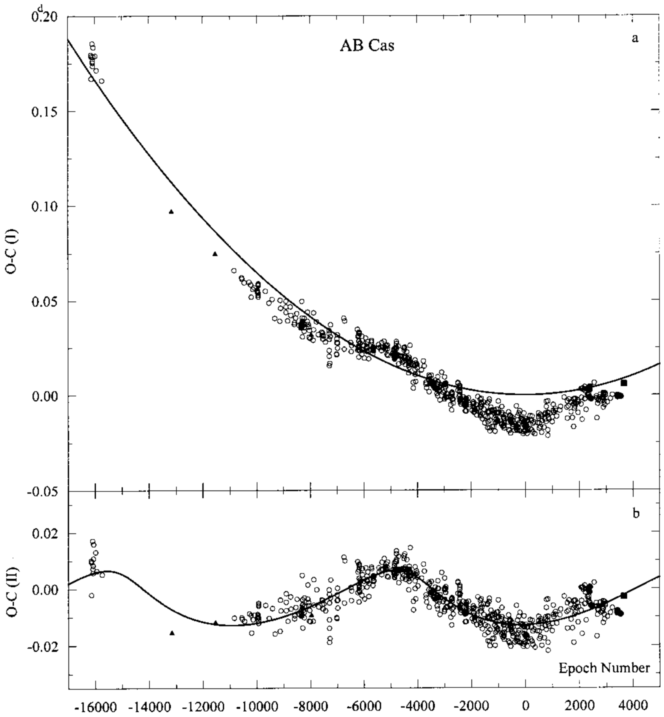

Fig. 5.—(*a*) $O-C$ (I) diagram for AB Cas and its parabolic representation (*solid line*). (*b*) The residuals, $O-C$ (II), from parabolic best-fit and its representation of a third-body orbit.

TABLE 3

Orbital Parameters of the Third-Body Orbit, the Coefficient of the Quadratic Term, and Their Standard Deviations for AB Cassiopeiae

Parameter	Value	Standard Deviation
T_0 (HJD)	47,483.5148	0.0007
P_0 (days)	1.3668841	0.0000001
$a_{12} \sin i$ (km)	2.57×10^8	8.45×10^6
e	0.39	0.04
ω (deg)	125	6
T (HJD)	41,770	196
P (yr)	40.3	0.9
Q (days cycle^{-1})	6.5×10^{-10}	2.0×10^{-11}

systemic velocity accompanying the light-time effect is 2.8 km s^{-1} and the semimajor axis of the eclipsing pair's orbit around the common center of gravity of the triple system is $a_{12} \sin i = 1.72$ AU. By using these values in equation (5), we estimated the mass function of the third body as $f(m_\odot) = 0.003\ M_\odot$. Then we computed the mass of the hypothetical third body as 0.31, 0.33, 0.42, and 0.67 $M_\odot$ for inclinations of 90°, 70°, 50°, and 30°, respectively. The angular separation between the eclipsing pair and the third star at the periastron and apastron points of the orbit were estimated to be 0''.020 and 0''.046. In this calculation, the distance of 300 pc for AB Cas was taken from the *Hipparcos* Catalogue. If the third star is a main-sequence star of 0.31 $M_\odot$, then by using Demircan & Kahraman's (1991) formulation we can estimate that it will be about 8 mag fainter than the AB Cas system.

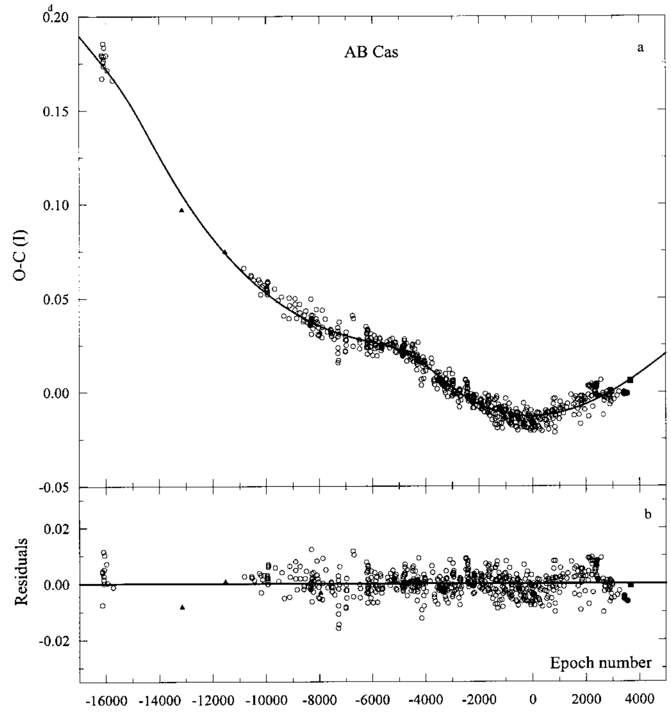

Fig. 6.—(*a*) $O-C$ (I) diagram of AB Cas and its representation by a summation of a parabolic and a periodic term (*solid line*). (*b*) The residuals from the representation.

4. SUMMARY AND DISCUSSION

A detailed analysis of the $O-C$ diagrams of two classical Algol-type binaries, S Equ and AB Cas, was performed. The $O-C$ variations of the systems suggest that the orbital periods of both show secular increases with a periodic oscillation. The secular period increase rates were obtained as $\Delta P/P = 0.102$ s yr^{-1} and $\Delta P/P = 0.03$ s yr^{-1} for S Equ and AB Cas, respectively, requiring mass transfer rates of about $dM/dt = 3.97 \times 10^{-8}\ M_\odot$ yr^{-1} and $dM/dt = 5.6 \times 10^{-8}\ M_\odot$ yr^{-1} from Roche lobe–filling low-mass secondaries to the primaries. Richards & Albright (1999) reported that S Equ shows emission lines in its spectra due to an accretion disk structure resulting from mass transfer. In addition, Vesper et al. (2001) showed, using spectral data, that the system has an accretion structure. A light-curve analysis by Zola (1992) of S Equ needed a third light ($L_3 = 0.06$ in B, 0.07 in V), which should be provided by a third component star in the system, and the periodic oscillation in the $O-C$ variation should be another sign of the existence of a third body.

Although the quasi-periodic $O-C$ variations are expected because of the stellar activity variations of the Algol binaries' cool secondaries, it is difficult for this effect to produce perfectly smooth and tilted periodic oscillation components (indicating eccentricity) in the systems' $O-C$ variations. We applied the third-body hypothesis (in eccentric orbits) to the analysis of these periodic $O-C$ variations.

The minimum masses of the hypothetical third stars are estimated to be 0.41 and 0.31 $M_\odot$ for S Equ and AB Cas, respectively, which are probable masses under the assumption of coplanarity for the binary and third-body orbits. As noted in § 2, a third body of mass 0.41 $M_\odot$ for S Equ would be about 7 mag fainter than the binary itself. From Pogson's formula, this quantity implies a fractional third light of about 0.001, which is at least an order of magnitude smaller than the fractional third light derived by Zoła (1992). If Zoła's estimate is reliable and if this light is that of a third body, then its main-sequence mass should be about 1.05 $M_\odot$. This quantity, on the other hand, requires through equation (5) that the inclination i of the third body be about 28° and that it definitely not be coplanar with the binary orbit. Such a prediction can be checked by a new light-curve analysis of S Equ. In fact, the existence of a third body in S Equ and its properties may be revealed by combining the more reliable multicolor light-curve analysis with the $O-C$ analysis. A similar, further study is also required to demonstrate the existence of a third body in the AB Cas system.

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