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## Modified-source gravity and cosmological structure formation

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**Abstract.** One way to account for the acceleration of the universe is to modify general relativity, rather than introducing dark energy. Typically, such modifications introduce new degrees of freedom. It is interesting to consider models with no new degrees of freedom, but with a modified dependence on the conventional energy-momentum tensor; the Palatini formulation of  $f(R)$  theories is one example. Such theories offer an interesting testing ground for investigations of cosmological modified gravity. In this paper we study the evolution of structure in these ‘modified-source gravity’ (MSG) theories. In the linear regime, density perturbations exhibit scale dependent runaway growth at late times and, in particular, a mode of a given wavenumber goes nonlinear at a higher redshift than in the standard  $\Lambda$ CDM model. We discuss the implications of this behaviour and why there are reasons to expect that the growth will be cut off in the nonlinear regime. Assuming that this holds in a full nonlinear analysis, we briefly describe how upcoming measurements may probe the differences between the modified theory and the standard  $\Lambda$ CDM model.

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**1. Introduction**

The concordance cosmological model describes a universe consisting of approximately 5% ordinary matter, 25% dark matter, and 70% dark energy. While it fits a wide variety of data, this model relies heavily on the existence of a ‘dark sector’ comprising 95% of the universe. Given the mysterious nature of dark matter and dark energy, and the fact that their existence is inferred exclusively through their gravitational effects, it is natural to wonder whether the apparent need for these components could be a sign that gravity is deviating from conventional general relativity (GR) on large scales.

Dynamical measurements that are taken to imply the existence of dark matter generally refer to length scales of kiloparsecs or greater, while evidence for dark energy comes from the acceleration of the universe, a phenomenon characteristic of the present Hubble radius of order one gigaparsec. The most precise experimental tests of GR, meanwhile, probe much smaller length scales [1]; Solar System measurements cover less than a milliparsec, while the binary pulsar PSR 1913 + 16 has an orbital semi-major axis less than a microparsec. We can therefore imagine that there is a large dynamical range over which gravity obeys Einstein’s equation to high precision, while behaving differently at sufficiently large scales.

Within the modified-gravity category there have been several different proposals: induced gravity in a class of extra dimensional models (Dvali–Gabadadze–Porrati (DGP) brane-worlds) [2]–[4]; phenomenological modifications of the Friedmann equation of cosmology [5, 6]; and direct, manifestly covariant modifications of the four-dimensional Einstein-Hilbert action [7]–[15]. (For some other ideas see [16]–[19].) In the last of these approaches, the simplest models involve an action obtained by integrating a function of the curvature scalar  $R$ ,

$$S = \int d^4x \sqrt{-g} f(R), \quad (1)$$

and varying with respect to the metric  $g_{\mu\nu}$ . These models were soon discovered to be dual to scalar-tensor theories, which lead to unacceptable deviations from GR in the solar system [20]. However, certain models based on inverse powers of more general curvature invariants [15] (particularly those involving the square of the Riemann tensor) have recently been shown to agree with Solar System tests [21] and to provide a good fit to the supernovae data [22], although tight constraints arise from the requirement that they be ghost-free [23]–[25].

On theoretical grounds, however, there are obstacles to constructing infrared modifications of GR that will escape detection on smaller scales. In a weak-field expansion around flat spacetime, GR describes the propagation of massless spin-2 gravitons coupled to the energy-momentum tensor  $T_{\mu\nu}$ , including that of the gravitons themselves. But such a theory is essentially unique; it has long been known that we can start with a field theory describing a transverse-traceless symmetric two-index tensor propagating in Minkowski space, couple it to the energy-momentum tensor of itself and other fields, and demonstrate iteratively that the background metric disappears, leaving instead a curved metric obeying Einstein's equation. It is therefore generally believed that infrared modifications of GR will necessarily involve the introduction of new degrees of freedom.

For the purpose of explaining the acceleration of the universe, however, there is a loophole in this argument. The properties of gravitons define both the propagation of linearized gravitational waves and the Coulomb form of the Newtonian gravitational potential in the weak-field limit. But there is more to gravity than gravitons, even in the infrared. In particular, the evolution of a Robertson-Walker cosmology is described by the Friedmann equation,

$$H^2 = \frac{8\pi G}{3}\rho - \frac{\kappa}{a^2}, \quad (2)$$

where  $\rho$  is the energy density,  $\kappa$  is the spatial curvature, and  $H$  is the Hubble parameter, related to the cosmological scale factor  $a(t)$  by  $H = \dot{a}/a$ . The Friedmann equation has nothing to do with gravitons; it is a constraint, relating the instantaneous expansion rate to the energy density and curvature. In fact, the Friedmann equation is a particular example of the Hamiltonian constraint of GR, relating the embedding of a partial Cauchy surface in spacetime to its intrinsic geometry and the energy-momentum tensor.

Because the Friedmann equation arises as a consequence of the constraint structure of GR rather than from the behaviour of gravitons, we are free to imagine modifying it without introducing new degrees of freedom. In particular, Einstein's equation relates the Einstein tensor to the energy-momentum tensor for matter,

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}. \quad (3)$$

In a theory describing a set of matter fields  $\chi_i$  with an action  $S_{(m)}[g_{\mu\nu}, \chi_i]$ , the energy-momentum tensor is defined by

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{(m)}}{\delta g^{\mu\nu}}. \quad (4)$$

We may ask, then, to what other sources could gravitons couple? In other words, we might define an alternative theory obeying

$$G_{\mu\nu} = 8\pi G \tau_{\mu\nu}(\chi_i), \quad (5)$$

where  $\tau_{\mu\nu}$  is some symmetric (0, 2) tensor that is conserved by virtue of the matter equations of motion, but differs from the conventional form (4). Since we have altered the right-hand side of Einstein's equation without introducing any new degrees of freedom, it may be possible to account for the acceleration of the universe without dark energy while remaining consistent with conventional tests of GR.

In fact this idea has been realized in the form of the Palatini formulation of  $f(R)$  gravity. As Flanagan has shown [10], the equations of motion obtained by separately varying the metric and connection in an action of the form (1) actually leads to a theory with *fewer* degrees of freedom; there is no propagating scalar, only the massless spin-two graviton of ordinary GR. In this paper we further consider models of this type, leaving behind the inspiration of the Palatini formulation of  $f(R)$  models and working directly with theories that have no propagating scalar, which we dub 'modified-source gravity' (MSG). The theories we consider are not precisely identical to the Palatini models, since we do not separately vary the metric and connection, so that the equations for the matter sector will be slightly different; nevertheless, there is an essential similarity between the two ideas.

Flanagan has argued that models of this type are experimentally excluded, as the effective matter action includes higher-order couplings not seen in particle-physics experiments. However, as we argue below, there are a number of reasons to believe that these constraints can be avoided; furthermore, the MSG field equations are an interesting toy model of cosmological modified gravity against which observations may be compared.

The possibility of distinguishing between modified gravity and dark energy by comparing the expansion history of the universe to the growth rate of cosmological perturbations has recently been emphasized from a variety of approaches [26]–[39]. Given the paucity of viable alternative theories of cosmological gravity, it is important to develop a model-independent perspective on the ways in which this distinction can manifest itself; we know of no better way to develop such intuition than to examine as many models as possible.

In this paper we therefore explore the cosmology of MSG by studying cosmological perturbation theory and extracting predictions for the growth of large-scale structure. We find that there is a substantial boost in the growth of the gravitational potential in comparison with ordinary lambda-cold dark matter ( $\Lambda$ CDM). We discuss the implications of this behaviour and why there are reasons to expect that the growth will cease in the nonlinear regime. Assuming that this holds in a full nonlinear analysis, we briefly describe how upcoming measurements may probe the differences between the modified theory and the standard  $\Lambda$ CDM model.

MSG is examined very much in the spirit of a toy model, as a laboratory to help understand the possible differences between theories of dark energy and theories that alter GR. It does not attempt to solve the cosmological constant problem (we set the vacuum energy to zero by hand), nor does it require less fine-tuning than any other typical model of dark energy (we choose the potential delicately so that acceleration happens at very late times), or is there any obvious way in which it would naturally appear as the low-energy limit of a more complete theory in the ultraviolet. Nevertheless, it is sufficiently difficult to find cosmological alternatives to GR that a simple explicit model can be useful in helping to focus efforts to distinguish between modified gravity and sources of dynamical dark energy.

## 2. MSG

### 2.1. Scalar-tensor and $f(R)$ theories

The Einstein-Hilbert action for GR is

$$S_{\text{EH}} = \int d^4x \sqrt{-g} \left( \frac{1}{2} M_{\text{P}}^2 R \right), \quad (6)$$

where  $R$  is the curvature scalar and  $M_{\text{P}} = 1/\sqrt{8\pi G}$  is the reduced Planck mass. Matter fields couple minimally to the metric  $g_{\mu\nu}$ . The propagating degrees of freedom of this theory are the two polarization states of a massless spin-two graviton.

One of the simplest ways to modify GR without introducing new fields is to consider actions that depend non-linearly on  $R$ ,

$$S_f = \int d^4x \sqrt{-g} f(R). \quad (7)$$

A particular example in which deviations from GR become important at small curvatures is  $f(R) = (M_{\text{P}}^2/2)(R - \mu^4/R)$  [7, 8]. Interestingly, however, the linear Einstein-Hilbert term (6) is the *only* function of  $R$  that propagates a spin-2 graviton by itself; any other function  $f(R)$  gives rise to a scalar-tensor theory, with both a spin-2 graviton and a spin-0 scalar [40]–[42]. This can be seen explicitly by considering an action with gravity coupled to a scalar  $\lambda$ ,

$$S = \int d^4x \sqrt{-g} [(R - \lambda) f'(\lambda) + f(\lambda)], \quad (8)$$

where  $f'(\lambda) = df/d\lambda$ . The field  $\lambda$  functions as a Lagrange multiplier, whose equation of motion sets  $\lambda = R$ . Plugging this back into (8) yields (7), so long as  $f'' \neq 0$  (that is, if  $f$  is anything other than linear). Defining a new scalar field  $\psi$  via

$$\psi = \frac{1}{2} \ln [f'(\lambda)], \quad (9)$$

we can perform a conformal transformation of the form

$$\tilde{g}_{\mu\nu} = e^{2\psi} g_{\mu\nu}. \quad (10)$$

The conformally transformed action becomes that of ordinary GR coupled to a propagating scalar field,

$$\tilde{S} = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{P}}^2}{2} \tilde{R} - 3\tilde{g}^{\mu\nu} (\nabla\psi)^2 - \tilde{U}(\psi) \right], \quad (11)$$

where  $(\nabla\psi)^2 \equiv g^{\mu\nu} \nabla_\mu \psi \nabla_\nu \psi$ . This is the Einstein-frame version of the model, in which the Ricci curvature appears by itself, but matter fields couple to the scalar  $\psi$  through the combination  $g_{\mu\nu} = e^{-2\psi} \tilde{g}_{\mu\nu}$ ; in the matter frame given by (7), there is no direct coupling between  $\psi$  and the matter fields, and free particles move along geodesics of  $g_{\mu\nu}$ . The potential is given by

$$\tilde{U}(\psi) = \frac{\lambda f'(\lambda) - f(\lambda)}{2f'(\lambda)^2} M_{\text{P}}^2. \quad (12)$$

In this expression,  $\Lambda$  is taken to be a function of  $\psi$  via inverting (9). (Note that our notation differs from that in [7].)

The problem with such a model is that, if we choose the function  $f(R)$  so that gravity is modified at small curvatures and the universe accelerates at late times, the scalar field will be very light, and the theory comes into conflict with Solar System tests of gravity. In terms of the Brans-Dicke parameter  $\omega$ , the model of [7] is nearly equivalent (apart from the small and presumably negligible potential) to a theory with  $\omega = 0$ ; meanwhile, the best current limits come from measurements of the Shapiro time delay from the Cassini mission, and give  $\omega > 40\,000$  [43]. However, recently some authors have claimed that the GR limit of the dynamical equivalence between  $f(R)$  and scalar-tensor theories is not well behaved, since  $f''(R) \rightarrow 0$ , and some  $f(R)$  theories behave in the Solar System in a manner perfectly consistent with current experimental limits [44]–[46].

Different strategies for avoiding this constraint have been explored. One approach is to consider models with inverse powers of other curvature invariants such as  $R_{\mu\nu}R^{\mu\nu}$  and  $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$  [15], [21]–[25]. As mentioned in the introduction, the degrees of freedom of such models differ from those of simple  $f(R)$  theories, and may be consistent with Solar System tests of gravity. Alternatively, one may consider the Palatini versions of  $f(R)$  theories, in which the metric and connection are treated as independent variables [9]–[12], [47]. Unlike in the case of the Einstein-Hilbert action,  $f(R)$  theories do not have identical equations of motion in the Palatini formulation as in the conventional scenario based on the metric alone. Indeed, Flanagan has shown that the scalar degree of freedom disappears entirely [10]. In the next section we explore a possibility of this form, not through the Palatini variation of an  $f(R)$  action, but simply by eliminating the scalar kinetic term by hand (although of course the formulations are related).

## 2.2. Eliminating the scalar

Our goal is to take the scalar-tensor action (11), equivalent to (7), and eliminate the propagating scalar degree of freedom. We choose the most brutally direct approach: simply erasing the kinetic term  $-3\tilde{g}^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi$  from (11). We are left with a new theory in which  $\psi$  is a Lagrange multiplier, without any dynamics of its own. Alternatively, we could imagine multiplying the kinetic term by a constant, and then taking the limit as the constant went to zero; in that limit, the scalar is still propagating, but decouples from any other fields. Generally, radiative corrections would tend to drive this constant to order unity. We will nevertheless set it to zero for purposes of this paper, accepting this as one of the fine-tunings that inevitably accompanies models of dynamical dark energy.<sup>4</sup>

The full Einstein-frame action for the theory, including additional matter fields  $\chi_i$ , is

$$\tilde{S} = \int d^4x \sqrt{-\tilde{g}} \left[ \frac{M_{\text{P}}^2}{2} \tilde{R} - \tilde{U}(\psi) \right] + \tilde{S}_{(\text{m})}[e^{-2\psi} \tilde{g}_{\mu\nu}, \chi_i]. \quad (13)$$

In the absence of the matter action, this model would be completely trivial. The scalar is non-propagating, and its equation of motion would fix  $\psi$  at any allowed value  $\psi_0$  at which  $d\tilde{U}/d\psi = 0$ . The model is then simply GR with a vacuum energy given by  $\tilde{U}(\psi_0)$ . The coupling to matter,

<sup>4</sup> Of course, the kinetic term may always be canonically normalized by a field redefinition. Setting the coefficient of the kinetic term to zero is therefore equivalent to taking the potential and the couplings all simultaneously to infinity.

however, leads to important consequences. We can undo the conformal transformation to return to the matter-frame metric  $g_{\mu\nu} = e^{-2\psi} \tilde{g}_{\mu\nu}$ , yielding

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{P}}^2}{2} e^{2\psi} R + 3e^{2\psi} (\nabla\psi)^2 - U(\psi) \right] + S_{(\text{m})}[g_{\mu\nu}, \chi_i], \quad (14)$$

where the new potential is

$$U(\psi) = e^{4\psi} \tilde{U}(\psi). \quad (15)$$

The actions (14) or (13) define MSG. We have left behind our original inspiration from  $f(R)$  gravity, so the potential  $U(\psi)$  is now simply a free function that defines the theory. In the form (14), the model resembles a conventional scalar-tensor theory, with a kinetic term for  $\psi$  and a direct coupling to the curvature scalar. However, there are implicitly derivatives of  $\psi$  in the  $(1/2)e^{2\psi} R$  term, which would be revealed after integration by parts, and would cancel the explicit kinetic term. The scalar field  $\psi$  is actually completely non-dynamical, as is evident from the Einstein-frame expression (13), in which no derivatives of  $\psi$  appear. In principle, this field could be integrated out exactly, and we will examine this approach in the next section; for the moment, it is more convenient to leave  $\psi$  explicitly in the action and field equations.

Let us examine the theory in the matter frame (14). The gravity equation of motion, obtained by varying with respect to  $g_{\mu\nu}$ , can be written

$$e^{2\psi} M_{\text{P}}^2 G_{\mu\nu} = T_{\mu\nu}^{(\text{m})} + T_{\mu\nu}^{(\psi)}. \quad (16)$$

Here,  $T_{\mu\nu}^{(\psi)}$  is the effective energy-momentum tensor for  $\psi$ ,

$$T_{\mu\nu}^{(\psi)} = -[U(\psi) + (\nabla\psi)^2 + 2\Box\psi]g_{\mu\nu} - 2\nabla_{\mu}\psi\nabla_{\nu}\psi + 2\nabla_{\mu}\nabla_{\nu}\psi, \quad (17)$$

where  $\Box \equiv g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}$ . The equation of motion for  $\psi$  is

$$\Box\psi + (\nabla\psi)^2 + \frac{1}{6M_{\text{P}}^2} e^{-2\psi} \frac{dU}{d\psi} - \frac{1}{6} R = 0. \quad (18)$$

Again, these equations bear a close resemblance to those of ordinary scalar-tensor gravity. However, we can take the trace of (16) to obtain

$$R = \frac{e^{-2\psi}}{M_{\text{P}}^2} [-T + 4U(\psi)] + 6(\nabla\psi)^2 + 6\Box\psi, \quad (19)$$

where  $T \equiv g^{\mu\nu}T_{\mu\nu}^{(\text{m})}$  is the trace of the matter energy-momentum tensor alone. Subtracting this from the scalar equation (18) leaves

$$\frac{dU}{d\psi} - 4U(\psi) = -T. \quad (20)$$

Thus, the trace of the gravity equation can be used to eliminate all spacetime derivatives from the scalar equation, leaving us with a *purely algebraic equation for  $\psi$  in terms of the matter fields*.

If we start with  $U(\psi)$ , we could use this equation to get  $T(\psi)$ , and invert that to get  $\psi(T)$ . If we start with  $\psi(T)$  [or equivalently  $T(\psi)$ ], we can then express the potential in terms of  $\psi$  as

$$U(\psi) = -e^{4\psi} \int_{\psi_0}^{\psi} e^{-4\psi'} T(\psi') d\psi'. \quad (21)$$

Lastly, if we know  $U(T)$  instead of  $\psi(T)$ , we could get  $\psi$  via

$$\psi(T) = - \int \frac{1}{T} \left( \frac{dU}{dT} + 4U \right) dT. \quad (22)$$

### 3. Comparison with experiment

#### 3.1. Solar System tests

Consider the vacuum equations, where  $T_{\mu\nu} = 0$ . Then from (20) the scalar is pinned at some value  $\psi_0$ , with

$$4U(\psi_0) - U'(\psi_0) = 0. \quad (23)$$

Looking back at (16), the vacuum gravitational field equation is

$$M_{\text{P}}^2 G_{\mu\nu} = -e^{-2\psi} U(\psi_0) g_{\mu\nu}. \quad (24)$$

Thus,  $e^{-2\psi_0} U(\psi_0)$  plays the role of an ordinary cosmological constant. Otherwise, the vacuum equation is precisely the vacuum Einstein equation. The Schwarzschild metric is therefore an exact solution of this theory (if the vacuum energy vanishes; otherwise it would be Schwarzschild-de Sitter). Gravitational waves in free space are completely identical to those in GR; the propagating degrees of freedom are simply those of a massless spin-2 graviton.

In  $f(R)$  theories, Schwarzschild (for which  $R_{\mu\nu} = 0$ ) is generally also an exact solution to the vacuum field equations, even though those equations are not identical to Einstein's. However, it is not a unique solution, even with spherical symmetry; Birkhoff's theorem, which relies on Einstein's equation, fails to apply. This failure can ultimately be traced to the existence of new degrees of freedom; gravitating bodies are not only characterized by their mass, but also by a charge that couples to a new scalar field. MSG, in contrast, has no new degrees of freedom, and Einstein's equation holds in vacuum, so Birkhoff's theorem applies. The Solar System tests of gravity that rule out simple  $f(R)$  theories are completely compatible with MSG.

We are free to choose the 'potential'  $U(\psi)$  to satisfy phenomenological requirements. If our interest is in causing the universe to accelerate at late times without affecting local tests of gravity, these include the following:

- For energy densities  $\rho$  larger than some critical value  $\rho_*$  of order the present mass density of the universe, we want to recover conventional cosmological expansion ( $a \propto t^{2/n}$  when the total energy density is evolving as  $\rho \propto a^{-n}$ ). This is achieved if  $\psi \rightarrow \psi_* = \text{constant}$  at  $\rho \geq \rho_*$ . For constant  $\psi$ , the additional energy-momentum contributions from (17) are simply those of a constant effective vacuum energy  $U(\psi) < \rho_*$ . In particular, the effective gravitational constant will be constant, and we recover conventional Friedmann-Robertson-Walker cosmology.

- Since  $\rho_*$  is smaller than the density of any compact astrophysical object, the previous requirement guarantees that we will automatically get  $\psi = \psi_*$  inside planets and stars. The strength of gravity will not be significantly density-dependent, so the predictions of Solar System tests or the binary pulsar are unaffected without further restrictions.
- For  $\rho \leq \rho_*$  the universe should accelerate. In subsection 4.2 we consider one concrete choice of  $U(\psi)$  that accomplishes this goal. We might also ask that the value of  $U$  vanish when  $\rho = 0$ , so that Minkowski space is a solution to the theory; otherwise, late-time acceleration could simply be blamed on the new potential energy, which would not really be different from ordinary dark energy.

### 3.2. Matter interactions

Despite the characterization of MSG as a theory of ‘gravity,’ we can certainly imagine integrating out the scalar field in the Einstein-frame action (13) to obtain a modified matter Lagrangian coupled to GR. The result will be an action written purely in terms of the ordinary matter fields and gravity; if the matter action looks conventional in the matter frame, it will generally be an ungainly mess in the Einstein frame with  $\psi$  integrated out. As noted by Flanagan [10], this implies the existence of higher-order (nonrenormalizable) terms in the matter action, which will be proportional to inverse powers of a very small mass scale  $\mu$  characterizing the regime in which gravity is to be modified. Since  $\mu$  should be very low in models relevant to late-time acceleration, such new interactions would presumably be easily noticeable in experiments (and, needless to say, have not been seen).

In addition to the introduction of new nonrenormalizable interactions, another phenomenological problem identified by Flanagan [10] relates to the nonlinearity of the source for gravity in MSG. The effective energy density is a nonlinear function of the conventional matter-frame energy-momentum tensor. Thus, it is illegitimate to average the energy density of a particulate medium over some coarse-graining to obtain a fluid description. In the limit where a body is made of pointlike particles, the density is infinite at the locations of the particles and zero in between, and treating it as a smooth density profile would be a mistake.

Despite these reasonable concerns, for the rest of this paper we will imagine that MSG is experimentally viable and an averaged fluid description for the matter fields is sufficient. Our primary justification for sidestepping this important issue is the dramatic separation of scales between the regimes we are considering (evolution of cosmological perturbations) and those in which the above considerations become relevant (atomic and particle-physics scales). We therefore find it interesting to consider the MSG equations as a phenomenological description of gravity in the infrared, even if we do not have a viable model on small scales. In order to best understand how we can observationally distinguish dark energy from modified gravity, it is important to characterize different ways in which modified gravity could manifest itself cosmologically. Since the MSG equations provide a consistent dynamical description of gravity on very large scales, studying their observational consequences is a useful practical exercise.

Furthermore, it is certainly possible to imagine ways in which the problems of higher-order interactions and the nonlinear energy-momentum tensor could be overcome. Perhaps the most straightforward would be to imagine that the kinetic term for our auxiliary scalar  $\psi$  were not exactly zero, but rather some very small number; the fluctuations of this field could then serve to average over the density of particles on sufficiently small scales. Alternatively, if the dark matter is some smoothly-distributed bosonic condensate (such as axions), the fluid description used in

this paper could be completely accurate. In fact, it is important to note that experimental results relevant to the question of higher-order interactions do not actually take place in a true vacuum, but rather in the presence of whatever dark matter background exists in the Solar System. Since the local density of dark matter is likely to be substantially greater than that of the average density of the universe, terrestrial experiments take place in a regime where gravity is accurately described by GR and novel MSG effects are suppressed. Finally, Flanagan also notes [11] that the new interactions are strongly-coupled in the low-energy regime in which they are purportedly observable. All of these possibilities are interesting and subtle, and deserve greater attention than we will give them in this paper, where our interest is exclusively cosmological; further development of the theory to put it on more secure microphysical foundations would be very worthwhile.

## 4. Robertson-Walker cosmology

### 4.1. The modified Friedmann equation

In this subsection we examine the evolution of a homogeneous and isotropic universe, deriving the modified Friedmann equation for MSG. In the next subsection we focus on an explicit choice for the potential  $U(\psi)$  that leads to late-time acceleration.

Consider a flat Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{(1 - \kappa r^2)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (25)$$

where  $\kappa$  is zero, positive or negative depending on the curvature of the spatial hypersurface. It is not normalized, as we have chosen instead to set  $a = 1$  at the present time. The matter fields are taken to be a perfect fluid, with energy-momentum tensor  $T^\mu{}_\nu = \text{diag}(-\rho, p, p, p)$ . The 00 component of the gravitational field equation (16) becomes

$$3H^2 + 3\frac{\kappa}{a^2} = \frac{e^{-2\psi}}{M_{\text{p}}^2}[\rho + U(\psi)] - 3\dot{\psi}^2 - 6H\dot{\psi}, \quad (26)$$

where  $H = \dot{a}/a$ . We can convert the time derivatives of  $\psi$  into derivatives with respect to  $a$  by using

$$\dot{\psi} = \dot{a} \frac{d\psi}{da} = H \left( \frac{d\psi}{d \ln a} \right). \quad (27)$$

Collecting terms proportional to  $H^2$  and dividing, we obtain

$$H^2 = \left( 1 + \frac{d\psi}{d \ln a} \right)^{-2} \left[ \frac{e^{-2\psi}}{3M_{\text{p}}^2}[\rho + U(\psi)] - \frac{\kappa}{a^2} \right], \quad (28)$$

which serves as the cosmological evolution equation for MSG.

There are two obvious modifications from the conventional Friedmann equation: a potential-energy contribution  $U(\psi)$ , and a variable-strength effective Newton's constant,

$$8\pi G_{\text{eff}} = \frac{e^{-2\psi}}{M_{\text{p}}^2} \left( 1 + \frac{d\psi}{d \ln a} \right)^{-2}. \quad (29)$$

When  $\kappa = 0$  and  $\rho \gg U$ , this is the quantity that relates the energy density to the expansion rate. In fact, both the potential and the effective Newton's constant are simply functions of the density  $\rho$  and the pressure  $p$  through the  $\psi$  equation (20). The new equation is therefore reminiscent of the Cardassian model [5], in which the right-hand side of the Friedmann equation is a nonlinear function of  $\rho$ .

#### 4.2. A model

There is a great amount of freedom in the choice of MSG dynamics, represented by the arbitrary function  $U(\psi)$ . In this subsection we discuss one simple example that satisfies the criteria laid out in subsection 3.1, including late-time acceleration.

For simplicity we imagine that the matter is represented by a pressureless fluid, so that  $\psi$  can be thought of as a function of the matter energy density

$$\rho = -T = \rho_0 a^{-3}, \quad (30)$$

where  $\rho_0$  is the average cosmological energy density today. Given the relations (20), (21), and (22), we can choose to specify any of the three quantities  $\psi$ ,  $U$ , and  $\rho$  as a function of any one of the others, and their relationship to the third will be automatically determined.

We would like to choose behaviour for which  $\psi$  is approximately constant when the energy density is substantially higher than the present average cosmological density  $\rho_0$ , and decreases for  $\rho \ll \rho_0$ . A convenient form to choose is

$$e^{-4\psi} = \alpha \left( \frac{\rho_0}{\rho} \right) + 1, \quad (31)$$

with  $\alpha$  a dimensionless parameter defining the model. The limiting behaviour is given by

$$\psi(\rho \rightarrow \infty) = 0, \quad \psi(\rho \rightarrow 0) = \frac{1}{4} \ln(\rho) \rightarrow -\infty, \quad (32)$$

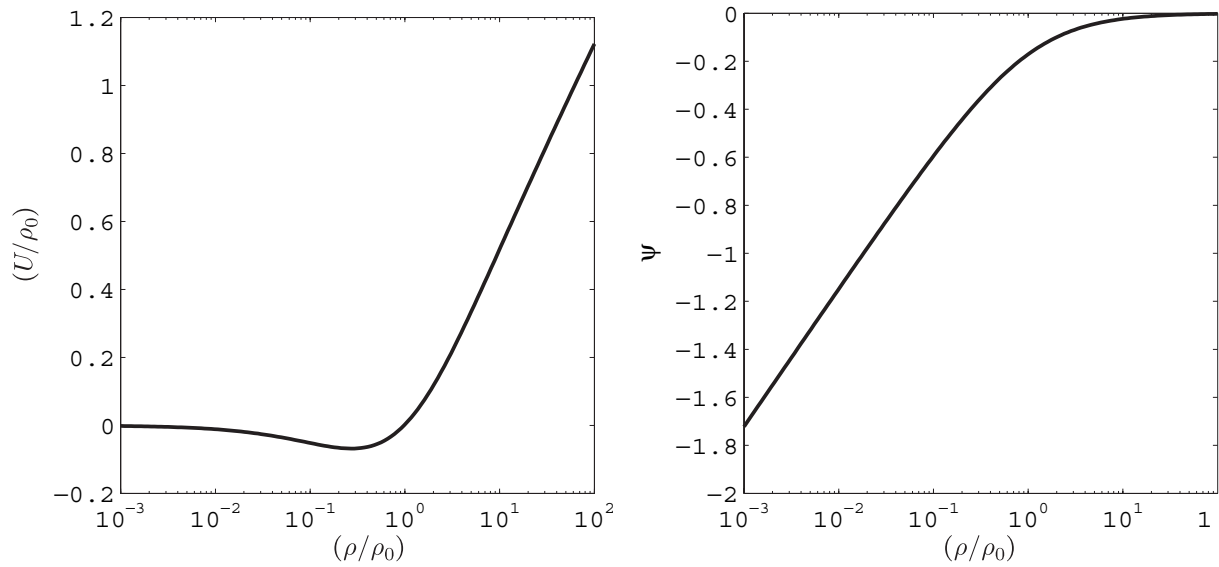
satisfying the aforementioned criteria.

The convenience of this model arises from our ability to analytically determine the potential as a function of the density,

$$U(\rho) = -\frac{\alpha\rho_0\rho}{4(\alpha\rho_0 + \rho)} \ln \left( \alpha \frac{\rho_0}{\rho} \right), \quad (33)$$

or equivalently as a function of  $\psi$ ,

$$U(\psi) = \alpha\rho_0 e^{4\psi} \left[ \psi - \frac{1}{4} \ln(1 - e^{4\psi}) \right]. \quad (34)$$



**Figure 1.** The potential  $U$  (in units of  $\rho_0$ ) and scalar field  $\psi$  (in units of  $M_P$ ) as functions of the cosmological energy density  $\rho$ .

As  $\rho \rightarrow 0$  we get

$$U(\rho \rightarrow 0) = \frac{\rho}{4} \ln \left( \frac{\rho}{\rho_0} \right) \rightarrow 0. \quad (35)$$

The fact that the potential vanishes at zero density guarantees that Minkowski space is a solution to the model. At large density we have

$$U(\rho \rightarrow \infty) = \frac{\alpha\rho_0}{4} \ln \left( \frac{\rho}{\rho_0} \right), \quad (36)$$

which rises more slowly than  $\rho$ , so that the potential does not dominate at early times. The behaviour of  $U$  and  $\psi$  as functions of  $\rho$  is shown in figure 1.

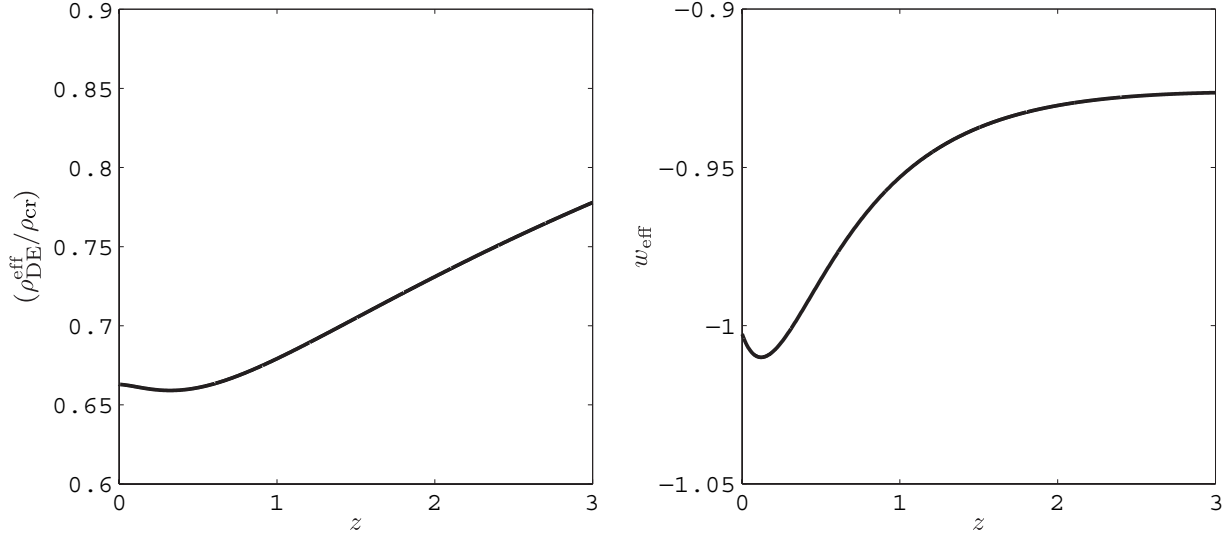
The other factor appearing in the Friedmann equation (28) involves  $d\psi/d \ln a$ , which can be written in this model as

$$\frac{d\psi}{d \ln a} = -3\rho \frac{d\psi}{d\rho} = -\frac{3\alpha\rho_0}{4(\alpha\rho_0 + \rho)}. \quad (37)$$

Putting everything together, the full Friedmann equation (28) for the specific model defined by (31) becomes

$$H^2 = \left( \frac{4\alpha\rho_0 + 4\rho}{\alpha\rho_0 + 4\rho} \right)^2 \left[ \frac{1}{3M_P^2} \left( \frac{\alpha\rho_0 + \rho - (\alpha\rho_0/4) \ln(\alpha\rho_0/\rho)}{\sqrt{\alpha\rho_0 + \rho}} \right) \sqrt{\rho} - \frac{\kappa}{a^2} \right]. \quad (38)$$

It seems unlikely that such an expression would have been guessed at as a phenomenological starting point for exploring cosmological dynamics in theories of modified gravity.



**Figure 2.** The plot on the left shows the effective dark-energy density, in units of the matter density today, that observers would reconstruct. (That is, the dark-energy density that would lead to equivalent behaviour for the scale factor if GR were correct.) The right plot shows the corresponding effective equation-of-state parameter as a function of redshift.

An observer, understandably assuming the validity of conventional GR, would interpret measurements of the expansion history  $a(t)$  in MSG in terms of the derived density and dynamics of an effective dark-energy component  $\rho_{\text{DE}}^{\text{eff}}$ . In terms of the (in principle) observable quantities  $H$  and  $\rho$ , the effective dark-energy density is

$$\rho_{\text{DE}}^{\text{eff}} = 3M_{\text{P}}^2 H^2 - \rho. \quad (39)$$

where the matter density of course evolves as  $\rho \propto a^{-3}$ . The effective equation-of-state parameter is

$$w_{\text{eff}} = -1 - \frac{1}{3} \frac{d \ln \rho_{\text{DE}}^{\text{eff}}}{d \ln a}. \quad (40)$$

The behaviour of these quantities as a function of redshift in the model defined by (31) is shown in figure 2.

We fix the parameters of our model by fitting the luminosity distance-redshift relation to the supernovae legacy survey (SNLS) data set of 115 type-Ia supernovae [48]; in addition we fit for the distance to the surface of last scattering: we use the third-year Wilkinson microwave anisotropy probe (WMAP) results [49] to fix its redshift,  $z_{\text{ls}} = 1088_{-2}^{+1}$ , the acoustic peak scale,  $\ell_{\text{A}} = 302_{-1.4}^{+0.9}$ , and its calibration through the matter density,  $\Omega_{\text{m}} h^2 = 0.1265 \pm 0.008$ . In doing the fit, we include both matter and radiation, with a total energy density  $\rho = \rho_{\text{m}}(1 + (R/a))$ , where  $R$  is the radiation to matter energy density ratio today. In fitting to the data, we allowed for the existence of spatial curvature, characterized by the parameter

$$C \equiv -\frac{3M_{\text{Pl}}^2 \kappa}{\rho_{\text{m}}^0}, \quad (41)$$

where  $\rho_m^0$  is the matter energy density today. The fit prefers a slightly open universe, but achieves approximately the same  $\chi^2$  as  $\Lambda$ CDM. The set of best fit parameters is

$$\alpha = 0.98 \quad C = 0.12 \quad h = 0.72. \quad (42)$$

Since we are fitting cosmological parameters in our model to the data, we need a consistent definition of the fractional densities  $\Omega_m$  and  $\Omega_\kappa$ . To do this, we begin by defining the critical density today in the usual way, via  $\rho_{\text{cr}}^0 = 3M_p^2 H_0^2$ , enabling us to write the Friedmann equation (38) as

$$H^2 = H_0^2 \frac{16(\alpha a^3 + 1)^2}{(\alpha a^3 + 4)^2} \left[ \frac{4 \left(1 + \frac{R}{a}\right) (\alpha a^3 + 1) - \alpha a^3 \ln(\alpha a^3)}{4a^3 \sqrt{\alpha a^3 + 1}} + \frac{C}{a^2} \right] \frac{\rho_m^0}{\rho_{\text{cr}}^0}. \quad (43)$$

Now consider the asymptotic form of this equation as  $a \rightarrow 0$ . Ignoring the slowly-varying logarithm, we obtain

$$H^2 = H_0^2 \left[ \left(1 + \frac{R}{a}\right) \frac{1}{a^3} + \frac{C}{a^2} \right] \frac{\rho_m^0}{\rho_{\text{cr}}^0}, \quad (44)$$

which then allows us to consistently define the fractional densities today in precisely the same way as in  $\Lambda$ CDM:

$$\Omega_m = \frac{\rho_m^0}{\rho_{\text{cr}}^0} = 0.26 \quad \Omega_\kappa = \frac{C\rho_m^0}{\rho_{\text{cr}}^0} = +0.02, \quad (45)$$

where the numbers are the best-fit values to the supernova and cosmic microwave background data.

## 5. Linear perturbations

To study perturbations, we introduce small time- and space-dependent deviations from the background cosmological solutions for the metric and matter sources. Applying the equations of motion and linearizing, we then obtain a set of coupled first-order differential equations describing the coupled evolution of matter, radiation and metric perturbations as the universe expands.

The evolution equations for matter and radiation in the presence of a perturbed metric are given by the Boltzmann equations (for a particularly clear description of this see [50]). This formalism is independent of the equations of motion for the metric itself; moreover, as we work in the matter frame, the Boltzmann equations are formally the same as those derived in standard GR. However, these equations do contain new dynamics even if their structure is unchanged, since the background quantities on which they depend are solutions of modified background equations.

From (42), the best-fit background cosmology requires a small negative curvature. However, this will negligibly affect perturbations. Therefore, although we may include curvature in the background evolution, we neglect it in the treatment of perturbations. We consider scalar

perturbations to a flat FRW metric in conformal Newtonian gauge; the perturbed line element can be written as

$$ds^2 = -(1 + 2\Psi(\vec{x}, t)) dt^2 + a^2(1 + 2\Phi(\vec{x}, t)) d\mathbf{x}^2, \quad (46)$$

where  $\Phi$  and  $\Psi$  are spacetime-dependent gravitational potentials. We assume that the universe contains both radiation and dark matter, which allows us to parameterize the perturbations to the energy momentum tensor by

$$T^0_0 = -\bar{\rho}\delta \quad (47)$$

$$T^0_i = (1 + w)\bar{\rho}\partial_i q \quad (48)$$

$$T^i_j = w\bar{\rho}(\delta^i_j + \pi^i_j). \quad (49)$$

Here,  $\bar{\rho}$  is the background energy density,  $\delta \equiv \delta\rho/\bar{\rho}$ ,  $w$  is the equation-of-state parameter for the background fluid,  $q$  is the momentum density, and  $\pi^i_j$  is the anisotropic stress.

We can then linearize the modified Einstein equations (16) and obtain the Poisson and anisotropy equations for the perturbed cosmology in Fourier space,

$$\frac{k^2}{a^2}(\Phi + \delta\psi) + 3(H + \dot{\bar{\psi}})(\dot{\Phi} + \dot{\delta}\psi) - 3(H + \dot{\bar{\psi}})^2\Psi = \frac{e^{-2\bar{\psi}}}{2M_{\text{p}}}(\bar{\rho}\delta + (\bar{U}' - 2\bar{U} - 2\bar{\rho})\delta\psi) \quad (50)$$

$$\Phi + \Psi = -\frac{a^2 e^{-2\bar{\psi}}}{k^2 2M_{\text{p}}^2} w\bar{\rho}\pi - 2\delta\psi, \quad (51)$$

where  $\bar{\psi}$  and  $\bar{U}$  are the background quantities and  $\delta\psi$  is the perturbation to  $\psi$ . The scalar anisotropy  $\pi$  is related to the anisotropic stress by  $\pi^i_j = (\partial^i\partial_j - (1/3)\delta^i_j\partial^2)\pi$ , and is non-zero only for radiation. We therefore neglect  $\pi$  and  $\pi^i_j$  from now on.

Note that, in comparison with the unmodified ( $\Lambda$ CDM) version, the anisotropy equation (51) contains just one extra term, namely that involving  $\delta\psi$ . However, this term plays a crucial role in defining the difference between dark energy and modified gravity. Bertschinger [35] has recently emphasized that knowledge of both the modification to the Friedmann equation and the modification to the evolution of the anisotropy,  $\Phi + \Psi$ , is key for the understanding of the effects of the new gravity theory on cosmological structure formation. It is the latter which differentiates modified-gravity theories from models with dark energy, and therefore it is the existence of this new term which ultimately allows us to test the modified-gravity idea.

In contrast with the anisotropy equation, the evolution equation (50) has several new entries arising from the modification to the Einstein equation (16). Besides a rescaling of Newton's constant, there are terms that are functions solely of the scalar field, its perturbation and the potential. However, by using equation (20), we can express  $\bar{\psi}$  and  $\delta\psi$  in terms of the matter density and its perturbation. In particular, we have

$$\delta\psi = -\frac{1}{3} \frac{d\bar{\psi}}{d \ln a} \delta_{\text{m}}, \quad (52)$$

where  $\delta_m$  is just the matter density contrast, since radiation makes no contribution to  $T = -\rho + 3p$ .

The time-space  $(0, j)$  component of the linearized Einstein equation is

$$\dot{\Phi} - H\Psi = \frac{e^{-2\bar{\psi}}}{2M_{\text{Pl}}^2} \bar{\rho}(1+w)q + \dot{\bar{\psi}}\Psi - \delta\dot{\bar{\psi}} + H\delta\bar{\psi} + \dot{\bar{\psi}}\delta\bar{\psi}. \quad (53)$$

Combining this with (50) and making use of (52), we obtain an algebraic equation for the potential  $\Psi$ . At late times, we are interested in modes well within the Hubble radius, and we can also neglect contributions from radiation. We then have

$$\frac{k^2}{a^2}\Psi = - \left[ \frac{e^{-2\bar{\psi}}}{2M_{\text{Pl}}^2} \left( 1 + \frac{d\bar{\psi}}{d \ln a} \right) \bar{\rho}_m - \frac{k^2}{3a^2} \frac{d\bar{\psi}}{d \ln a} \right] \delta_m. \quad (54)$$

This constraint relates the potential  $\Psi$  directly to the matter variables.

## 6. The growth of linear structure

In the previous section we have derived the basic linearized equations for the evolution of perturbations in the context of MSG. In this section we study the growth of linear structure.

At late times, when all the modes of interest have entered the horizon, and radiation and momentum flow are negligible, we can combine (54) with the Boltzmann equations for dark matter to obtain a second-order differential equation governing dark matter perturbations

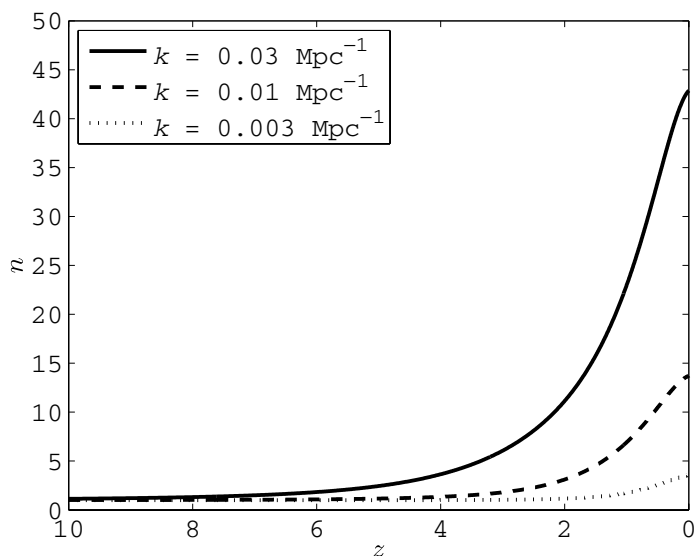
$$\ddot{\delta}_m + 2H\dot{\delta}_m - \left[ \frac{e^{-2\bar{\psi}}}{2M_{\text{Pl}}^2} \left( 1 + \frac{d\bar{\psi}}{d \ln a} \right) \bar{\rho}_m - \frac{k^2}{3a^2} \frac{d\bar{\psi}}{d \ln a} \right] \delta_m = 0. \quad (55)$$

The first term in the bracket multiplying  $\delta_m$  is the same as the term in the Poisson equation in GR, modified by the fact that the value of Newton's constant is evolving as the average density in the universe decreases beneath the critical value at which the modifications to GR become important.

The second term in the coefficient of  $\delta_m$  in (55) is very different in nature: it introduces a scale dependence in the growth of structure. For negative  $d\bar{\psi}/d \ln a$ , small scales will begin to grow more quickly than large scales once the universe approaches the accelerating phase. In figure 3, we give an example of this enhanced growth for the choice of potential corresponding to the model defined by (31) with the best fit parameters (42). We have plotted the exponent  $n$  of

$$\delta_m \equiv \delta_0 \exp \left[ \int d \ln a n(a) \right], \quad (56)$$

with  $\delta_0$  a constant. The rate of growth is strongly scale-dependent once the universe enters the epoch of acceleration, with the exponent departing from  $n = 1$  (holding for all modes during matter domination) and tending to  $n \propto k$  at late times. The  $k$ -dependence of the growth rate will



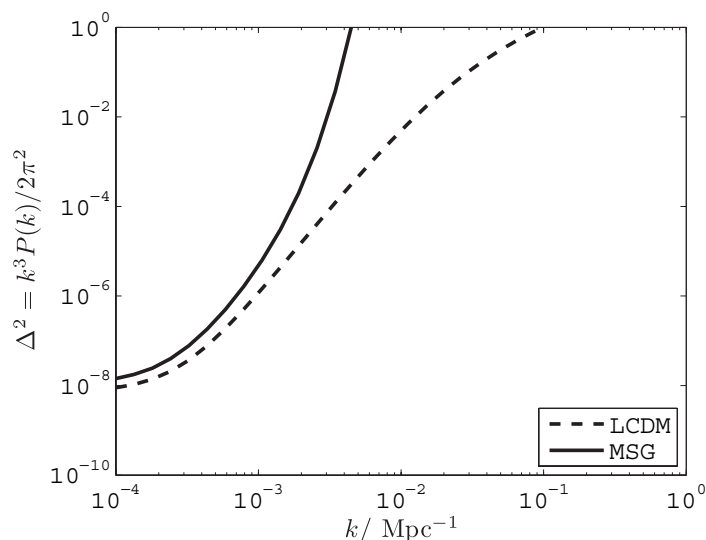
**Figure 3.** The exponent of the growth of matter density perturbations, as defined in (56) for a selection of modes, solved for the model defined by (31) with the best fit parameters (42) (near-horizon effects are ignored). The rate of growth is very strongly scale-dependent once the cosmology enters the epoch of acceleration, with the exponent departing sharply from  $n = 1$  and tending to  $n \propto k$  at late times.

result in smaller scales reaching nonlinearity extremely quickly after the onset of acceleration. Thus, we generically expect that the matter power spectrum will be nonlinear at larger scales than in  $\Lambda$ CDM.

In fact, this enhanced growth of scales which are small but still in the linear regime seems to be a generic feature of MSG. The effect can be traced to the existence of the new  $k$ -dependent term in equation (56) for  $\delta_m$ : small scales will witness enhanced growth so long as the potential  $U(\psi)$  is chosen so that  $d\bar{\psi}/d \ln a < 0$ . But, as inspection of (28) shows, it is precisely this behaviour that makes the universe accelerate at late times, by increasing the effective value of Newton's constant in the modified Friedmann equation. It therefore seems difficult to avoid this phenomenon simply by a clever choice of the potential  $U(\psi)$ .

A possible loophole in this argument would be to consider models in which  $\psi$  were nearly constant in the present era, but the contribution of the potential  $U(\psi)$  itself to the right-hand side of (28) were to induce cosmological acceleration. One might reasonably complain that this case is simply a dark-energy model in disguise, rather than a modification of gravity; however, we should keep in mind that  $U(\psi)$  is not really the potential for a dynamical scalar field, but rather a nonlinear function of the matter variables. This is something of a matter of taste, and we will not pursue this possibility in the remainder of the paper.

We proceed to study structure formation in this model. We start with a Harrison-Zel'dovich scale-invariant spectrum of perturbations normalized to cosmic background explorer (COBE) by  $\Delta_\zeta^2 = (5.07 \times 10^{-5})^2$ , (where  $\Delta^2$  is the dimensionless power spectrum defined as  $\Delta^2 \equiv k^3 P(k)/2\pi^2$ ), and evolve through to today using a numerical code that includes both radiation

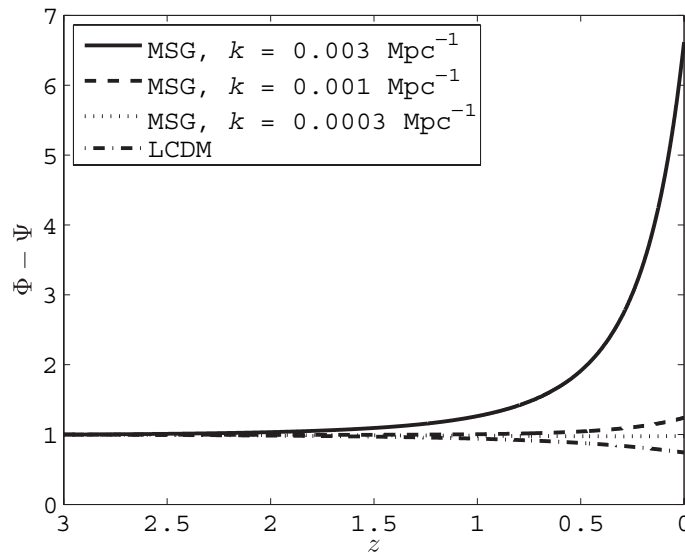


**Figure 4.** The dimensionless matter power spectrum for MSG and  $\Lambda$ CDM. As expected, in MSG the growth-rate is proportional to  $k$  and the power-spectrum increases exponentially with  $k$  for modes within the horizon. Non-linear scales are reached at scales of 240 Mpc ( $k = 0.004 \text{ Mpc}^{-1}$ ), compared to 10 Mpc  $\equiv 8h^{-1} \text{ Mpc}$  for  $\Lambda$ CDM. The linear growth can only be trusted for  $k < 0.004 \text{ Mpc}^{-1}$ . For smaller scales, the theory should return to a GR-like behaviour.

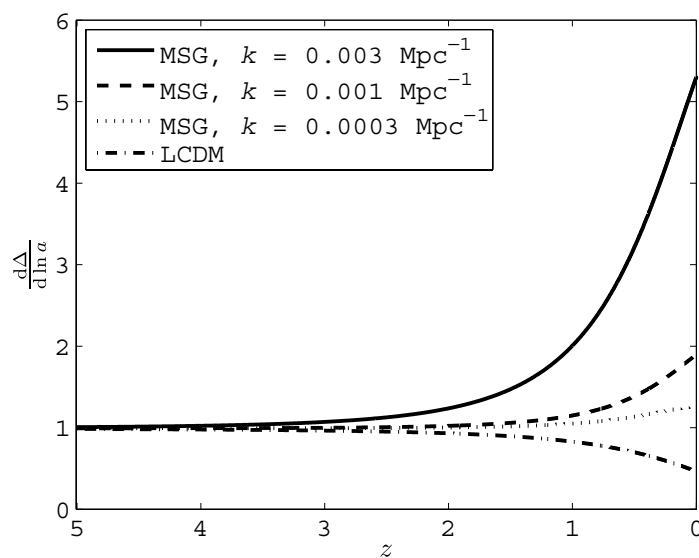
and dark matter. Since we expect small scales to be nonlinear, it is necessary to find the scale at which linear perturbation theory ceases to be valid. These results are shown in figure 4.

As expected, the nonlinear terms are important at much larger scales than in  $\Lambda$ CDM: approximately 240 Mpc compared to 10 Mpc for  $\Lambda$ CDM. Once the perturbations are nonlinear, perturbation theory breaks down and an N-body simulation with potentials depending on local densities must be used to obtain results. (Nonlinearities become important when  $\Delta \sim 1$ ; the potentials  $\Phi$  and  $\Psi$  are safely smaller at that time.) However, since for high-enough densities the equations of motion return to GR, we expect that the cosmology should behave in a GR-like manner for sufficiently high values of  $\delta$ .

The evolution of modes at very large scales, for which linear perturbation theory is valid throughout the history of the universe, is presented in figures 5 and 6. Structure growth is scale dependent and runaway, as predicted by the approximation in (55), with the rate of growth increasing as the universe departs from conventional matter-dominated behaviour. Such rapid structure formation drives the growth of gravitational potentials, which also increase rapidly during the acceleration era in a scale-dependent manner. This behaviour would significantly enhance the integrated Sachs-Wolfe (ISW) effect, at least at the lowest multipoles which are sensitive to only the largest scales, which remain linear. In addition, the fact that the gravitational potentials are growing at the same time as the density contrasts are increasing would lead to a galaxy-ISW correlation of opposite sign to that expected in  $\Lambda$ CDM; whereas these are correlated for  $\Lambda$ CDM, here they would be anti-correlated.



**Figure 5.** The evolution of the quantity  $\Phi - \Psi$ , observable through the ISW effect, for modes close enough to the horizon to remain linear until today. In MSG, this combination of potentials grows during dark-energy domination, increasingly rapidly for higher  $k$ . This would lead to a significant increase in the ISW signal, at least for the lowest multipoles, which are sensitive only to the linear modes at largest scales. In  $\Lambda$ CDM this evolution is scale free.



**Figure 6.** Exponent of growth rate for comoving density contrast,  $(d\Delta/d \ln a)$ . In  $\Lambda$ CDM, it is scale-independent once radiation no longer dominates; structure growth slows down during dark-energy domination. For MSG, structure growth accelerates.

## 7. Conclusions

We have considered a class of modified-gravity models in which only the constraint equation of GR is modified, thereby introducing no new propagating degrees of freedom. These ‘MSG’ models allow for cosmological dynamics in which the universe self-accelerates, without the need for dark energy. We demonstrate that there exists a class of such theories with a consistent cosmic expansion history and which naturally satisfies all Solar System tests of gravity.

As with any explanation for cosmic acceleration, it is important to understand how it might be tested and distinguished from other models. To this end, we have studied the onset of structure formation in MSG using linear perturbation theory. Fixing parameters so that the background cosmology is well described by MSG, we have compared the rate of growth of different Fourier modes of the density perturbations with those predicted in the  $\Lambda$ CDM model. We find that, for a given  $k$ -mode, growth is more rapid in MSG than in  $\Lambda$ CDM, and therefore that perturbation theory breaks down at a correspondingly higher redshift.

To make detailed progress beyond this point would require an N-body simulation with potentials depending on local densities. However, since MSG is constructed so as to yield dynamics indistinguishable from GR at high enough densities, we expect this rapid growth to cease and to once again resemble that found in  $\Lambda$ CDM for sufficiently high values of  $\delta_m$ .

Particularly interesting is the evolution of modes on scales large enough that linear perturbation theory is valid throughout the history of the universe. The rapid structure formation on these scales drives the growth of gravitational potentials, which increase rapidly during the acceleration era in a scale-dependent manner. This behaviour enhances the ISW effect at the lowest multipoles. Since the gravitational potentials are growing at the same time as the density contrasts are increasing this should lead to a galaxy-ISW anti-correlation, in contrast with that expected in  $\Lambda$ CDM. A natural next step is to attempt to understand the growth of structure in the nonlinear regime.

A primary motivation for this work has been to understand the way in which modified gravity can be distinguished from dark energy. Currently, the leading candidate for a modified theory of cosmological gravity is the DGP model [2]–[4], despite lingering fundamental issues with the theory [51, 52]. Much effort has gone into understanding the evolution of cosmological perturbations in DGP gravity, with an emerging consensus that the gravitational potentials decay more rapidly at late times in DGP than they do in  $\Lambda$ CDM [26, 27], [30]–[34], [38, 39]. In MSG, in contrast, it appears as if the potentials generically grow at late times with respect to their conventional behaviour. It therefore seems to be difficult to imagine a model-independent prediction for the way in which modified gravity can be distinguished from theories of dynamical dark energy, although simultaneous measurements on the expansion history and the evolution of structure do of course provide stringent constraints on any specific model. It is clearly important to continue to explore the theoretical consequences of modifying GR on large scales, to better understand what clues observers should be looking for in the quest to solve the puzzle of the accelerating universe.

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## References

- [1] Will C M 2001 *Living Rev. Rel.* **4** 4 (Preprint [gr-qc/0103036](#))
- [2] Dvali G R, Gabadadze G and Porrati M 2000 *Phys. Lett. B* **485** 208 (Preprint [hep-th/0005016](#))
- [3] Deffayet C 2001 *Phys. Lett. B* **502** 199 (Preprint [hep-th/0010186](#))
- [4] Deffayet C, Dvali G R and Gabadadze G 2002 *Phys. Rev. D* **65** 044023 (Preprint [astro-ph/0105068](#))
- [5] Freese K and Lewis M 2002 *Phys. Lett. B* **540** 1 (Preprint [astro-ph/0201229](#))
- [6] Dvali G and Turner M S 2003 Preprint [astro-ph/0301510](#)
- [7] Carroll S M, Duvvuri V, Trodden M and Turner M S 2003 Preprint [astro-ph/0306438](#)
- [8] Capozziello S, Carloni S and Troisi A 2003 Preprint [astro-ph/0303041](#)
- [9] Vollick D N 2003 *Phys. Rev. D* **68** 063510 (Preprint [astro-ph/0306630](#))
- [10] Flanagan E E 2004 *Phys. Rev. Lett.* **92** 071101 (Preprint [astro-ph/0308111](#))
- [11] Flanagan E E 2003 *Class. Quantum Grav.* **21** 417 (Preprint [gr-qc/0309015](#))
- [12] Vollick D N 2004 *Class. Quantum Grav.* **21** 3813 (Preprint [gr-qc/0312041](#))
- [13] Soussa M E and Woodard R P 2004 *Gen. Rel. Grav.* **36** 855 (Preprint [astro-ph/0308114](#))
- [14] Nojiri S and Odintsov S D 2003 Preprint [hep-th/0308176](#)
- [15] Carroll S M, De Felice A, Duvvuri V, Easson D A, Trodden M and Turner M S 2004 *Phys. Rev. D* **71** 063513 (Preprint [astro-ph/0410031](#))
- [16] Arkani-Hamed N, Cheng H C, Luty M A and Mukohyama S 2004 *J. High Energy Phys.* JHEP05(2004)074 (Preprint [hep-th/0312099](#))
- [17] Gabadadze G and Shifman M 2004 *Phys. Rev. D* **69** 124032 (Preprint [hep-th/0312289](#))
- [18] Moffat J W 2004 Preprint [astro-ph/0403266](#)
- [19] Clifton T, Mota D F and Barrow J D 2004 Preprint [gr-qc/0406001](#)
- [20] Chiba T 2003 *Phys. Lett. B* **575** 1 (Preprint [astro-ph/0307338](#))
- [21] Navarro I and Van Acoleyen K 2005 *Phys. Lett. B* **622** 1 (Preprint [gr-qc/0506096](#))
- [22] Mena O, Santiago J and Weller J 2005 Preprint [astro-ph/0510453](#)
- [23] Navarro I and Van Acoleyen K 2005 Preprint [gr-qc/0511045](#)
- [24] De Felice A, Hindmarsh M and Trodden M 2006 Preprint [astro-ph/0604154](#)
- [25] Calcagni G, de Carlos B and De Felice A 2006 Preprint [hep-th/0604201](#)
- [26] Lue A, Scoccimarro R and Starkman G D 2004 *Phys. Rev. D* **69** 124015 (Preprint [astro-ph/0401515](#))
- [27] Knox L, Song Y S and Tyson J A 2005 Preprint [astro-ph/0503644](#)
- [28] Ishak M, Upadhye A and Spergel D N 2005 Preprint [astro-ph/0507184](#)
- [29] Linder E V 2005 *Phys. Rev. D* **72** 043529 (Preprint [astro-ph/0507263](#))
- [30] Lue A 2006 *Phys. Rep.* **423** 1 (Preprint [astro-ph/0510068](#))
- [31] Sawicki I and Carroll S M 2005 Preprint [astro-ph/0510364](#)
- [32] Koyama K and Maartens R 2006 *J. Cosmol. Astropart. Phys.* JCAP01(2006)016 (Preprint [astro-ph/0511634](#))
- [33] Koyama K 2006 *J. Cosmol. Astropart. Phys.* JCAP03(2006)017 (Preprint [astro-ph/0601220](#))
- [34] Song Y S 2006 Preprint [astro-ph/0602598](#)
- [35] Bertschinger E 2006 Preprint [astro-ph/0604485](#)
- [36] Yamamoto K, Bassett B A, Nichol R C and Suto Y 2006 Preprint [astro-ph/0605278](#)
- [37] Uzan J P 2006 Preprint [astro-ph/0605313](#)
- [38] Sawicki I, Song Y-S and Hu W 2006 Preprint [astro-ph/0606285](#)
- [39] Song Y-S, Sawicki I and Hu W 2006 Preprint [astro-ph/0606286](#)
- [40] Tessandier P and Tourrenc P 1983 *J. Math. Phys.* **24** 2793
- [41] Magnano G and Sokolowski L M 1994 *Phys. Rev. D* **50** 5039 (Preprint [gr-qc/9312008](#))

- [42] Wands D 1994 *Class. Quantum Grav.* **11** 269
- [43] Bertotti B, Iess L and Tortora P 2003 *Nature* **425** 374
- [44] Faraoni V 2006 *Preprint* [gr-qc/0607016](#)
- [45] Capozziello S and Troisi A 2005 *Phys. Rev. D* **72** 044022 (*Preprint* [astro-ph/0507545](#))
- [46] Capozziello S, Stabile A and Troisi A 2006 *Preprint* [gr-qc/0603071](#)
- [47] Sotiriou T P and Liberati S 2006 *Preprint* [gr-qc/0604006](#)
- [48] Astier P *et al* 2006 *Astron. Astrophys.* **447** 31–48 (*Preprint* [astro-ph/0510447](#))
- [49] Spergel D N *et al* 2006 *Preprint* [astro-ph/0603449](#)
- [50] Dodelson S *Modern Cosmology* (New York Academic)
- [51] Luty M A, Porrati M and Rattazzi R 2003 *J. High Energy Phys.* JHEP09(2003)029 (*Preprint* [hep-th/0303116](#))
- [52] Adams A, Arkani-Hamed N, Dubovsky S, Nicolis A and Rattazzi R 2006 *Preprint* [hep-th/0602178](#)