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2005 New J. Phys. 7 201

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Spacetime in string theory

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New Journal of Physics 7 (2005) 201

Received 13 October 2004

Published 29 September 2005

Online at <http://www.njp.org/>

doi:10.1088/1367-2630/7/1/201

Abstract. We give a brief overview of the nature of spacetime emerging from string theory. This is radically different from the familiar spacetime of Einstein's relativity. At a perturbative level, the spacetime metric appears as 'coupling constants' in a two-dimensional quantum field theory. Nonperturbatively (with certain boundary conditions), spacetime is not fundamental but must be reconstructed from a holographic, dual theory.

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1. Introduction

A hundred years ago, our view of space and time was dramatically changed by the introduction of special relativity. Ten years later, Einstein made spacetime dynamical in his general theory of relativity. It has long been expected that quantum gravity will cause an even more radical change in our view of spacetime. String theory is a promising approach to a consistent quantum theory of gravity. In the past few decades, a new picture of spacetime has been emerging from this theory. While this picture is far from complete, it is already clear that spacetime has many different features than it does in relativity. I will discuss some of these new features below.

(For a description of spacetime by another approach to quantum gravity, see [1].) This will be a nontechnical discussion focusing on the main ideas and results. More details can be found in the references, which hopefully provide an introduction to the (vast) literature (see also [2]).

String theory starts with the idea that fundamental particles are not point-like but excitations of a one-dimensional string. These strings have a tension which defines a new fundamental length scale in the theory ℓ_s . The first thing one notices when one quantizes a string in flat spacetime is that one needs more than four spacetime dimensions. The second thing is that the ground state of the string is a tachyon. To remove the tachyon, one adds fermions and requires that the string be supersymmetric. This superstring is consistent in 10 spacetime dimensions.

Thus string theory incorporates two major changes to the spacetime of general relativity that were proposed long before string theory became popular in the mid-1980s. The fact that spacetime might have more than four dimensions goes back to Kaluza and Klein in the 1920s. The standard explanation for why we have not seen these extra dimensions is that they are wrapped up in a small compact manifold. Supersymmetry is usually described as a symmetry between bosons and fermions, but it is much more than just another symmetry of matter. It is really an extension of the Poincaré symmetry of spacetime, and can be viewed as saying that there are fermionic directions as well as bosonic directions to spacetime. In this sense, it describes the first extension of spacetime since space and time were unified by Minkowski.¹

However, this is just the beginning of the story. Spacetime in string theory is not just the usual spacetime with a few extra (bosonic and fermionic) dimensions. As we will see, there are symmetries which equate geometrically different spacetimes and even topologically different spacetimes. There are ways to resolve spacetime singularities. There are also ways to decrease the dimension of spacetime and, in some cases, eliminate it altogether.

To begin our discussion, we remind the reader of a few basic facts about string theory.² If one quantizes a free relativistic (super)string in flat spacetime, one finds an infinite tower of modes of increasing mass. Let us assume that the string is closed, i.e., topologically a circle. Then, at the massless level, there is a symmetric traceless tensor mode which is identified with a linearized graviton. There is also an antisymmetric tensor mode $B_{\mu\nu}$ and a scalar ϕ called the dilaton. These states arise even without supersymmetry. Bilinears in the fermions produce additional massless bosonic states which are higher rank generalizations of Maxwell fields. They are described by p -forms $F_p = dA_{p-1}$.

Next, one postulates a simple splitting and joining interaction between strings. The strength of this interaction is given by a dimensionless coupling constant g_s (which is related to the dilaton). Newton's constant G is not an independent parameter, but given in terms of g_s and ℓ_s by $G \sim g_s^2 \ell_s^8$ in 10 dimensions. Remarkably, one can show that this simple splitting and joining interaction reproduces the perturbative expansion of general relativity. This was the earliest indication that string theory incorporated general relativity. But string theory is certainly not restricted to perturbing about flat spacetime. We will see how to recover the full vacuum Einstein equation, $R_{\mu\nu} = 0$, directly from string theory in section 2.

Over the past decade, it has become clear that string theory is much more than just a theory of strings. There are other extended objects called branes. The name comes from membranes that are two-dimensional, but branes exist in any dimension: 0-branes are point particles, 1-branes are strings, etc. Branes are nonperturbative objects in that their tension is inversely related to a

¹ Actually, supersymmetry was first developed in the context of two-dimensional string worldsheets and later extended to four-dimensional field theories and gravity.

² By now, there are several excellent textbooks on string theory, e.g. [3, 4].

power of the coupling g_s . The most common type of brane is called a D-brane and it has a tension $T \propto 1/g_s$. So one would never see these objects in perturbation theory in g_s . Even though they are very heavy, the gravitational field they produce is governed by $GT \sim g_s$; so as $g_s \rightarrow 0$, there should be a flat space description of these objects and it was found by Polchinski [5]. At weak coupling, a D-brane is a surface in Minkowski spacetime on which open strings can end. The D stands for ‘Dirichlet’ and refers to the boundary conditions on the ends of the open strings. These open strings move freely along the brane but cannot leave the brane unless the endpoints join and form a closed string. The massless states of an open string include a spin 1 particle, so every D-brane comes with a $U(1)$ gauge field. When N D-branes come together, the open strings stretching from one to another also become massless. This enhances the resulting gauge group from $U(1)^N$ to $U(N)$. These D-branes are also sources for the p -form fields F_p .

The idea that there can be extended objects with degrees of freedom stuck to the brane has given rise to an entirely new way to view extra spatial dimensions, called brane-worlds [6, 7]. Rather than imagine that the extra dimensions are very small, one can imagine that they are much larger and we live on a $(3 + 1)$ -dimensional brane in this higher dimensional space. In other words, all the observed particles (quarks, leptons, gauge bosons, etc) are confined to move on the brane and only gravity exists in the bulk since the graviton comes from closed strings. One important consequence is that the fundamental higher dimensional Planck scale could be of the order of 1 TeV, and we might see quantum gravity or string theory effects at the Large Hadron Collider at CERN. This would be tremendously exciting, but it is important to remember that this is only one of many possible scenarios and it is not a unique prediction of string theory.

The fact that the extra dimensions can be relatively large has motivated a study of black holes in more than four spacetime dimensions. This has resulted in a number of surprises including a demonstration that stationary solutions with event horizons are not characterized by their mass, charge and angular momentum [8]: there is a violation of black hole uniqueness in higher dimensions. Among the new stationary solutions that have recently been found, there are neutral black rings [8], supersymmetric charged black rings [9] and nonuniform black strings [10]. I will not discuss these further since they are basically a result of general relativity in higher dimensions. I want to focus on the more fundamental changes in spacetime that arise from string theory.

In section 2, I discuss the role of spacetime in perturbative string theory and indicate how different geometries and topologies can be equivalent. Section 3 contains a discussion of perhaps the most important lesson about spacetime that has emerged from nonperturbative string theory: holography. Section 4 contains some concluding remarks.

2. Perturbative probes of spacetime

At the perturbative level, one starts with a background spacetime metric $g_{\mu\nu}$ satisfying an equation to be specified shortly. A one-dimensional string traces out a two-dimensional worldsheet. So the dynamics of the string is described by a two-dimensional theory describing how this worldsheet moves in spacetime. It is convenient to introduce a metric on the worldsheet, q_{ab} . If σ^a are local coordinates on the worldsheet and X^μ are local coordinates on spacetime, the dynamics of a string in this background is described by a two-dimensional sigma model:

$$S = \ell_s^{-2} \int d^2\sigma \sqrt{-q} q^{ab} \nabla_a X^\mu \nabla_b X^\nu g_{\mu\nu}. \quad (2.1)$$

Classically, this action is invariant under conformally rescaling the worldsheet metric, q_{ab} . Quantum mechanically, there can be a conformal anomaly. We now demand that this conformal invariance be preserved quantum mechanically. In other words, we have a two-dimensional conformal field theory (CFT). The vanishing of the conformal anomaly imposes an equation on the spacetime metric, which becomes the field equation for $g_{\mu\nu}$. In a derivative expansion, this looks like Einstein's equation at leading order, but then has higher order terms involving powers and derivatives of the curvature multiplied by appropriate powers of the string scale ℓ_s .

Associated with every two-dimensional CFT is a number, c , called the central charge. For a free field theory, c is just the number of scalar fields. So, for a string in flat spacetime, c is the spacetime dimension. The presence of two-dimensional diffeomorphism invariance in string theory leads to the condition that $c = 26$. So, without fermions and supersymmetry, string theory is consistent in 26 spacetime dimensions. If one adds fermions, one only needs 10 dimensions, as mentioned earlier. We are thus led to the following general definition of perturbative string theory: perturbative string theory is equivalent to two-dimensional CFT with $c = 26$.

By 'perturbative', we do not mean that the sigma model (2.1) is quantized perturbatively. The complete quantum CFT is included in this statement, but this is analogous to first quantization. In the full second-quantized theory, different topologies for the two-dimensional worldsheet correspond to different orders in the quantum loop expansion. This is usually studied for static (or stationary) backgrounds where one can analytically continue to Euclidean signature. Then one can work with Euclidean worldsheet metrics and Riemann surfaces. Higher genus surfaces describe higher loop contributions. The CFT on S^2 describes the classical (tree level) theory, the CFT on T^2 describes the one loop correction, etc. Much less is known about this quantum loop expansion when the backgrounds are time-dependent.

Notice that the role of spacetime has changed significantly. The focus is no longer on fields propagating on spacetime, but rather on two-dimensional quantum field theories. The spacetime metric acts like 'coupling constants' on the two-dimensional fields. This has several far-reaching implications which we now discuss.

Since the string only senses the spacetime through this sigma model, two metrics that yield the same sigma model are indistinguishable in string theory. Apparently, trivial changes to the sigma model action can result in significant changes to the spacetime geometry. We give two examples. The first is called T-duality [11, 12]. Consider a metric with a Killing field $\partial/\partial Y$. For simplicity, we will also assume there is a reflection symmetry $Y \rightarrow -Y$, so the metric is $ds^2 = g_{ij} dX^i dX^j + f dY^2$, where g_{ij} and f can depend on X^i but are independent of Y . The two-dimensional sigma model is

$$S = \ell_s^{-2} \int [g_{ij} \nabla_a X^i \nabla^a X^j + f \nabla_a Y \nabla^a Y]. \quad (2.2)$$

The field equation for Y is $\nabla_a (f \nabla^a Y) = 0$, which implies $d(f^* dY) = 0$. Let us introduce a new field \tilde{Y} via $f^* dY = d\tilde{Y}$. Then, the action becomes³

$$S = \ell_s^{-2} \int [g_{ij} \nabla_a X^i \nabla^a X^j + f^{-1} \nabla_a \tilde{Y} \nabla^a \tilde{Y}]. \quad (2.3)$$

³ Strictly speaking, this 'derivation' of T-duality only works on Euclidean worldsheets since otherwise there is a minus sign in front of the second term in (2.3). A proper derivation involves gauging the symmetry associated with the Killing field and imposing a constraint that the associated field strength vanishes [12].

It turns out that the change of variables from Y to \tilde{Y} leaves the CFT invariant [13]. (More precisely, this is true if Y and \tilde{Y} are both periodically identified with inverse periods.) However, (2.3) describes a string moving in a geometrically different spacetime. We have replaced g_{YY} with $1/g_{YY}$. In the simplest case of flat spacetime with one direction compactified on a circle, this shows that a circle of radius R is equivalent to one with radius ℓ_s^2/R . Intuitively, the reason is that strings in this background have two types of states. If the string winds around the circle n times, its energy is nR/ℓ_s^2 , while if it is moving around the circle its energy is m/R for some integer m since momentum is quantized. Clearly, if we interchange R and ℓ_s^2/R , and m, n , the spectrum of states is unchanged. One can show that all string interactions are also invariant.

Another example involves strings moving on spacetimes of the form $M_4 \times K$, where K is a compact six-dimensional space. The condition of conformal invariance (and four-dimensional supersymmetry) requires that K be Ricci flat and Kahler, i.e., a Calabi–Yau space [14]. By changing a sign (of a left moving $U(1)$ charge) in the sigma model, one changes its interpretation from a string moving on K to a string moving on a geometrically and topologically different space, called the mirror of K . Since the sign is arbitrary from the CFT standpoint, strings on K are equivalent to strings on \tilde{K} [15]. This ‘mirror symmetry’ has been checked in a dramatic way. By performing a calculation on K and reinterpreting it in terms of \tilde{K} , Candelas *et al* [16] were able to calculate the number of rational curves of degree k in a particular Calabi–Yau space \tilde{K} . These are maps of S^2 into \tilde{K} described by equations of degree k . These numbers grow very quickly and look like

$$\begin{aligned} k = 1: & \quad 2875 \\ k = 2: & \quad 609\,250 \\ k = 3: & \quad 317\,206\,375 \\ k = 4: & \quad 242\,467\,530\,000 \end{aligned} \tag{2.4}$$

These numbers are very hard to calculate directly using traditional methods. At the time when [16] appeared, mathematicians had only been able to calculate the first two. Since then, new techniques were found to compute them directly and they all agree with the predictions of mirror symmetry [17].

Another consequence of the fact that strings see the spacetime only through the sigma model is that spacetimes which are singular in general relativity can be completely nonsingular in string theory. To see this, first note that the definition of a singularity in string theory is different from that in general relativity. In general relativity, we usually define a singularity in terms of geodesic incompleteness which is based on the motion of test particles. In string theory, we use the sigma model which describes test strings. So a spacetime is considered singular if test strings are not well behaved.⁴ A simple example of a spacetime which is singular in general relativity but not in string theory is the quotient of Euclidean space by a discrete subgroup of the rotation group. The resulting space, called an orbifold, has a conical singularity at the origin. Even though this leads to geodesic incompleteness in general relativity, it is completely harmless in string theory. This is essentially because strings are extended objects.

The orbifold has a very mild singularity, but even curvature singularities can be harmless in string theory. A simple example follows from applying T-duality to rotations in the plane. This results in the metric $ds^2 = dr^2 + (1/r^2) d\phi^2$ which has a curvature singularity at the origin. However, strings on this space are completely equivalent to strings in flat space.

⁴ Strictly speaking, one should also require that the other objects in the theory—branes—have well-behaved propagation.

As mentioned above, string theory has exact solutions which are the product of four-dimensional Minkowski space and a compact Calabi–Yau space. A given Calabi–Yau manifold usually admits a whole family of Ricci flat metrics. So one can construct a solution in which the four large dimensions stay approximately flat and the geometry of the Calabi–Yau manifold changes slowly from one Ricci flat metric to another. In this process, the Calabi–Yau space can develop a curvature singularity. In many cases, this can be viewed as arising from a topologically nontrivial S^2 or S^3 being shrunk down to zero area. It has been shown that when this happens, string theory remains completely well defined. The evolution continues through the geometrical singularity to a nonsingular Calabi–Yau space on the other side [18, 19].

The reason why this happens is roughly the following. There are extra degrees of freedom in the theory associated with branes wrapped around topologically nontrivial surfaces. As long as the area of the surface is nonzero, these degrees of freedom are massive, and it is consistent to ignore them. However, when the surface shrinks to zero volume, these degrees of freedom become massless, and one must include them in the analysis. When this is done, the theory is nonsingular.

The above singularities are all a product of time and a singular space. However, other singularities which involve time in a crucial way have also been shown to be harmless. Putting many branes on top of each other produces a gravitational field which often has a curvature singularity at the location of the brane. It has been shown that one can understand physical processes near this singularity in terms of excitations of the branes.

However, it is simply not true that all singularities are removed in string theory. Consider the plane wave:

$$ds^2 = -2 du dv + dx_i dx^i + h_{ij}(u) x^i x^j du^2. \quad (2.5)$$

If h_{ij} is traceless, this metric is Ricci flat. The u dependence is arbitrary. It is easy to show that all the higher-order perturbative corrections to Einstein’s equation vanish since the curvature is null [20]. In fact, one can show that (2.5) defines an exact CFT [21], so this is an exact solution to string theory. If h_{ij} diverges as $u \rightarrow u_0$, then the plane wave is singular. One can study string propagation in this background and show that, in some cases, the string does not have well-behaved propagation through this curvature singularity. The divergent tidal forces cause the string to become infinitely excited [20].

Despite all this progress, we still do not have a good understanding of the most important types of singularities: those arising from gravitational collapse or cosmology. This remains an area of active investigation.

We have focused on the metric, but classical backgrounds for string theory can also include some matter fields. The massless states of the 10-dimensional superstring are in one-to-one correspondence to the fields of supergravity, so these are the allowed matter fields. Their classical field equations look like the supergravity equations plus higher derivative corrections.

We have mainly discussed 10-dimensional backgrounds since this is the critical dimension for the superstring. But there are ways around this restriction. Recall that the fundamental requirement is that the central charge c is fixed. If the other matter fields are allowed to be nonzero at infinity, the dimension of spacetime can be reduced keeping the central charge unchanged. For example, if the dilaton grows linearly in a spatial direction, then the dimension of spacetime can be decreased to as low a value as two [22]! Also, if the three-form $H = dB$ is nonzero at infinity, the dimension is also changed. This arises naturally if the spacetime is a product of time and a group manifold [23].

There are more subtle ways to lower the spacetime dimension. One of these is called an asymmetric orbifold [24]. Suppose one wants a consistent background with $n < 10$ dimensions. One starts with a 10-dimensional flat spacetime. Since the fields X^μ satisfy a wave equation on the worldsheet, they can be divided into a left-moving and a right-moving part $X^\mu = X_L^\mu + X_R^\mu$. Now, one takes $10 - n$ of these fields and makes different identifications on X_L and X_R . It is as if X_L and X_R are living on different orbifolds. Clearly, it no longer makes sense to view these fields as coordinates on space. The physical space is now lower dimensional, and the central charge is made up of some fields on the worldsheet that do not have a spacetime interpretation.

One can take this idea even further. So far, we have mainly discussed CFTs which come from a sigma model. Even though spacetime has unusual properties, at least there is a spacetime. These examples can thus be viewed as the geometric phase of string theory. However, string theory is not restricted to sigma models. As we have seen, one can consider any two-dimensional CFT with $c = 26$. Some of these can be viewed as describing strings moving in spacetimes where the curvature is of the order of the string scale and not really well defined.

3. Nonperturbative lesson: holography

Our knowledge of nonperturbative string theory is still incomplete, but it has already yielded a radical new view of spacetime called holography. Roughly speaking, holography is the idea that physics in a region can be described by fundamental degrees of freedom living on the boundary of this region. The idea that quantum gravity might be holographic was first suggested by 't Hooft [25] and Susskind [26], motivated by the fact that black hole entropy is proportional to its horizon area.

The most concrete form of this idea is due to Maldacena [27] and called the AdS (anti-de Sitter)/CFT correspondence (although it is really a conjecture). Maldacena considered a stack of N parallel D 3-branes on top of each other. As mentioned earlier, the strength of the gravitational field this produces is governed by $g_s N$. When $g_s N \ll 1$, the spacetime is nearly flat and there are two types of string excitations. There are open strings on the brane whose low-energy modes are described by a $U(N)$ gauge theory. There are also closed strings away from the brane. When $g_s N \gg 1$, the backreaction is important and the metric describes an extremal black 3-brane. This is a generalization of a black hole appropriate for a three-dimensional extended object. It is extremal with respect to the charge carried by the 3-branes, which sources the five-form F_5 . Near the horizon, the spacetime becomes a product of S^5 and five-dimensional AdS space. (This is directly analogous to the fact that near the horizon of an extremal Reissner–Nordstrom black hole, the spacetime is $AdS_2 \times S^2$.) String states near the horizon are strongly red-shifted and have very low energy as seen asymptotically. In a certain low-energy limit, one can decouple these strings from the strings in the asymptotically flat region. At weak coupling, $g_s N \ll 1$, this same limit decouples the closed strings from the excitations of the 3-branes. Thus we get a connection between a gauge theory at weak coupling, and strings in $AdS_5 \times S^5$ at strong coupling. But both of these theories are (in principle) defined for all values of the coupling. Maldacena suggested that they were in fact equivalent physical descriptions. More precisely, String theory with $AdS_5 \times S^5$ boundary conditions is equivalent to a four-dimensional $\mathcal{N} = 4$ supersymmetric $U(N)$ gauge theory.

The gauge theory is conformally invariant and hence is a four-dimensional CFT. More generally, starting with other branes (or several different types of branes), one is led to the following statement: AdS/CFT correspondence: string theory on spacetimes which asymptotically

approach the product of AdS and a compact space is completely described by a CFT ‘living on the boundary at infinity’.

We will focus on the first form of this correspondence, since this is the best understood case. At first sight, this conjecture seems unbelievable. How could an ordinary field theory describe all of string theory? These two theories certainly look different at weak coupling, but a crucial aspect of this correspondence is that when the string theory is weakly coupled, the gauge theory is strongly coupled and vice versa. This is because the radius of curvature of AdS_5 and S^5 are both given by

$$\ell = (4\pi g_s N)^{1/4} \ell_s. \quad (3.1)$$

The Yang–Mills coupling g_{YM} is related to the string theory coupling by $g_{YM}^2 = 2\pi g_s$. The effective coupling in a large N gauge theory is the ’t Hooft coupling, $g_{YM}^2 N$, so this must be large in order for $\ell \gg \ell_s$, which is necessary for even a spacetime interpretation to be valid in the string theory.

One sign that this is not completely crazy comes from comparing the symmetries. $\mathcal{N} = 4$ $U(N)$ super-Yang–Mills has a gauge field, four (Weyl) fermions and six scalars φ^i , all in the adjoint representation of the gauge group. It has an $SO(4, 2)$ symmetry coming from conformal invariance and an $SO(6)$ symmetry coming from rotation of the scalars. This agrees with the geometric symmetries of $AdS_5 \times S^5$. Thus all spacetimes which asymptotically approach $AdS_5 \times S^5$ have an asymptotic symmetry group which agrees with the gauge theory. If the gauge theory is on $S^3 \times \mathbf{R}$ with the radius of the three-sphere also given by ℓ , then the global time translation in the bulk agrees with time translation in the field theory and hence the energies of states in the field theory and string theory should agree.

One cannot prove the AdS/CFT correspondence since we do not have an independent nonperturbative definition of string theory to compare it with. In fact, since the gauge theory is defined nonperturbatively, one can view this as a nonperturbative and (mostly) background-independent definition of string theory. A background spacetime metric only enters under the boundary conditions at infinity. Of course, this only makes sense if the gauge theory can reproduce what is known about string theory. There are many checks one can make, and so far the AdS/CFT correspondence has survived all of them. I will now describe some of these tests. (For a more complete discussion, see [28].)

The initial checks concerned perturbations of $AdS_5 \times S^5$. It was shown that all linearized supergravity states (massless string modes) have corresponding states in the gauge theory with the same energy [29]. It was also shown that some interactions agree [30]. For a long time, it was difficult to give a precise description of the massive excited string states in terms of the strongly coupled gauge theory. However, considerable progress has been made recently for a class of states with large angular momentum on the S^5 or AdS_5 [31, 32]. In some cases, one can even construct a two-dimensional sigma model directly from the gauge theory and show that it agrees (in a certain approximation) to the sigma model describing strings moving in $AdS_5 \times S^5$ [33].

For perturbations of $AdS_5 \times S^5$, one can reconstruct the background spacetime from the gauge theory as follows. Fields on S^5 can be decomposed into spherical harmonics, which can be described as symmetric traceless tensors on \mathbf{R}^6 : $T_{i\dots j} X^i \dots X^j$. Restricted to the unit sphere one gets a basis of functions. Recall that the gauge theory has six scalars and the $SO(6)$ symmetry of rotating the φ^i . So the operators $T_{i\dots j} \varphi^i \dots \varphi^j$ give information about position on S^5 . The field theory lives on $S^3 \times \mathbf{R}$, which can be viewed as the boundary at infinity of AdS_5 . So the only remaining direction is the radial direction. This is believed to correspond to an energy scale in the gauge theory. Large radii correspond to high energy or short distance in the gauge theory.

For example, a fundamental string stretched into a circle of large radius in AdS_5 corresponds to a thin flux tube in the gauge theory. The fact that the flux tube naturally expands corresponds to the fundamental string collapsing down to smaller radii.

Evidence for the AdS/CFT correspondence goes far beyond these perturbative checks. Recently, a detailed map has been found between a class of nontrivial asymptotically $AdS_5 \times S^5$ supergravity solutions and a class of states in the gauge theory [34]. These states and geometries both preserve half of the supersymmetry of $AdS_5 \times S^5$ itself. On the field theory side, one restricts to fields that are independent of S^3 and hence reduce to $N \times N$ matrices. In fact, all the states are created by just one scalar field, so it can be described by a single matrix model. This theory can be quantized exactly and the states can be labelled by arbitrary closed curves on a plane. On the gravity side, one considers solutions to 10-dimensional supergravity involving just the metric and (selfdual) five-form F . The field equations are simply $dF = 0$ and

$$R_{\mu\nu} = F_{\mu\alpha_1\cdots\alpha_4} F_{\nu}^{\alpha_1\cdots\alpha_4}. \quad (3.2)$$

There exists a large class of stationary solutions to (3.2) which have an $SO(4) \times SO(4)$ symmetry, and can be obtained by solving a linear equation. These solutions are nonsingular, have no event horizons, but can have a complicated topology. They are also labelled by arbitrary closed curves on a plane. This provides a precise way to map states in the field theory into bulk geometries [34]. Only for some ‘semi-classical’ states is the curvature below the Planck scale everywhere. It is natural to conclude that the full quantum gravity description of this sector of the theory is just given by the matrix model.

Understanding the Hawking–Bekenstein entropy of black holes in terms of quantum states had been a longstanding problem. However, the AdS/CFT correspondence provides a natural solution. A black hole in AdS_5 is described by the Schwarzschild AdS geometry

$$ds^2 = - \left(\frac{r^2}{\ell^2} + 1 - \frac{r_0^2}{r^2} \right) dt^2 + \left(\frac{r^2}{\ell^2} + 1 - \frac{r_0^2}{r^2} \right)^{-1} dr^2 + r^2 d\Omega_3. \quad (3.3)$$

Denoting the Schwarzschild radius by r_+ , the Hawking temperature of this black hole is

$$T_H = \frac{\ell^2 + 2r_+^2}{2\pi r_+ \ell^2}. \quad (3.4)$$

When $r_+ \gg \ell$, the Hawking temperature is large, $T_H \sim r_+/\ell^2$. This is quite different from a large black hole in asymptotically flat spacetime which has $T_H \sim 1/r_+$. The gauge theory description is just a thermal state at the same temperature T_H . Let us compare the entropies. It is difficult to calculate the field theory entropy at strong coupling, but at weak coupling, we have of order N^2 degrees of freedom, on a three-sphere of radius ℓ at temperature T_H and hence

$$S_{YM} \sim N^2 T_H^3 \ell^3. \quad (3.5)$$

On the string-theory side, the solution is the product of (3.3) and an S^5 of radius ℓ . So recalling that $G \sim g_s^2 \ell_s^8$ in 10 dimensions and dropping factors of order unity, the Hawking–Bekenstein entropy of this black hole is

$$S_{BH} = \frac{A}{4G} \sim \frac{r_+^3 \ell^5}{g_s^2 \ell_s^8} \sim \frac{T_H^3 \ell^{11}}{g_s^2 \ell_s^8} \sim N^2 T_H^3 \ell^3, \quad (3.6)$$

where we have used (3.1) in the last step. The agreement with (3.5) shows that the field theory has enough states to reproduce the entropy of large black holes in AdS_5 . Putting in all the numerical factors, one finds that $S_{BH} = \frac{3}{4}S_{YM}$ [35]. Since S_{BH} is a measure of the number of states at strong coupling, and S_{YM} has been calculated at weak coupling, it should not be surprising that they are not precisely equal. We do not yet understand why they are related by a simple factor of 3/4.

There is another test one can perform with the gauge theory at finite temperature. At long wavelengths, one can use a hydrodynamic approximation and think of this as a fluid. It is then natural to ask: what is the speed of sound waves? Conformal invariance implies that the stress energy tensor is traceless, so $p = \rho/3$ which implies that $v = 1/\sqrt{3}$. The question is: can you derive this sound speed from the AdS side? This would seem to be difficult since the bulk does not seem to have any preferred speed other than the speed of light. But recent work has shown that the answer is yes [36]. Using the correspondence, one can also compute other hydrodynamic quantities such as the shear viscosity, but these are hard to check since they are difficult to calculate directly in the strongly coupled thermal gauge theory. There is also a field theory interpretation of black hole quasinormal modes. A perturbation of the black hole decays with a characteristic time set by the imaginary part of the lowest quasinormal mode. This should correspond to the timescale for the gauge theory to return to thermal equilibrium. One can show that the quasinormal mode frequencies are poles in the retarded Green's function of a certain operator in the gauge theory. The particular operator depends on the type of field used to perturb the black hole.

In quantum field theory, there is a standard procedure for integrating out high energy degrees of freedom and obtaining an effective theory at low energy. This is known as renormalization group (RG) flow. If one starts with a CFT at high energy, the RG flow is trivial. The low-energy theory looks the same as the high-energy theory. This is because there is no intrinsic scale. But if you perturb the theory by adding mass terms to certain fields, the RG flow is nontrivial and one obtains a different theory at low energies. Since the energy scale corresponds to radius, this RG flow in the boundary field theory corresponds to radial dependence in the bulk. Turning on mass terms corresponds to changing the boundary conditions for certain matter fields in the bulk. These new boundary conditions require that the matter fields are nonzero, so AdS is no longer a solution. By solving the Einstein equation with these new boundary conditions, one obtains a solution which approaches AdS (with, in general, a different radius of curvature) at small radius. By comparing the small r behaviour with the endpoint of the RG flow, one finds detailed numerical agreement [37]. So the classical Einstein equation knows a lot about RG flows in quantum field theory! Since we start by adding mass terms to the field theory, this also shows that the AdS/CFT correspondence can be extended to some nonconformal field theories. It is not yet clear how many quantum field theories have dual descriptions in terms of string theory. It has been suggested that this should be true for all theories with a large N limit.

4. Discussion

It should be clear that CFTs play a central role in our current understanding of string theory. Two-dimensional CFTs with the right central charge describe classical string solutions and the quantum-loop expansion. Other CFTs with (typically) different spacetime dimensions are believed to provide a complete nonperturbative description of string theory with asymptotically AdS boundary conditions. There is even one version of the AdS/CFT correspondence in which the

dual CFT is two-dimensional. This arises when the spacetime is asymptotically $AdS_3 \times S^3 \times T^4$. One thus has the confusing situation that a two-dimensional CFT describes perturbative string excitations about this space, and a different two-dimensional CFT describes the entire theory. What is even more surprising is that the first CFT describes loop corrections by changing the space it is defined on to higher genus Riemann surfaces, while the second CFT provides a complete nonperturbative description keeping the space it is defined on fixed (just $S^1 \times \mathbf{R}$)! In this case one again has a detailed map between certain supersymmetric states in the (second) CFT and nontrivial gravity solutions [38]. This case has at least one advantage over the $AdS_5 \times S^5$ case discussed above. The entropy of large black holes can now be reproduced exactly, including the numerical coefficient. This is related to the fact that a black hole in AdS_3 is a BTZ black hole which is locally AdS_3 everywhere. Thus when one extrapolates to small coupling, one does not modify the geometry with higher curvature corrections.

There are other approaches to nonperturbative string theory such as string field theory. I have focused on the AdS/CFT correspondence since it is the most extensively studied and also contains the most far reaching lesson about the nature of spacetime. It provides answers to some longstanding questions about quantum gravity. For example, it has often been suggested that space and time should be derived quantities in quantum gravity. But the problem has always been: if space and time are not fundamental, what replaces them? Here, the answer is that there is an auxiliary spacetime metric which is fixed by the boundary conditions at infinity. The CFT uses this metric, but the physical spacetime metric is a derived quantity. It is important to emphasize that the spacetime is not emerging from ‘strings’. In this approach, the so-called fundamental strings of string theory are also derived quantities. Both the strings and spacetime are constructed from the CFT.

As another example, consider the formation and evaporation of a small black hole in a spacetime which is asymptotically $AdS_5 \times S^5$. By the AdS/CFT correspondence, this process is described by ordinary unitary evolution in the CFT. So black hole evaporation does not violate quantum mechanics.

However, there remain many open questions. The dictionary relating spacetime concepts in the bulk and field theory concepts on the boundary is very incomplete and still being developed. For example, while we know how to translate certain states of the CFT into bulk geometries, we do not yet know the general condition on the state in order for a semiclassical spacetime to be well defined. Another class of questions concerns how to formulate holography for other boundary conditions. Can holography be extended to asymptotically flat or cosmological spacetimes? If so, will the dual description again be given in terms of a local quantum field theory? One extension that has already been carried out is to plane wave spacetimes. Penrose has shown that every spacetime has a plane wave as a limit. The idea is to blow up the geometry in a small neighbourhood of a null geodesic. Consider a null geodesic which stays at the origin of the AdS_5 but circles around the equator of the S^5 . Taking the Penrose limit yields a particularly simple plane wave

$$ds^2 = -2du dv + dx_i dx^i - x^2 du^2 \quad (4.1)$$

which has an $SO(8)$ symmetry. Berenstein *et al* [31] showed what this Penrose limit corresponds to the dual gauge theory. In this way, one can extend the AdS/CFT correspondence to strings in a plane wave background. One advantage is that excited string states can now be studied more easily and the entire string spectrum has been shown to agree with states in the dual theory. Even some interactions have been shown to agree [39].

The AdS/CFT correspondence can also be used to gain information about strongly coupled gauge theories. Certain calculations are easier to do on the gravity side and then translated into new field theory results. Although this has not been our focus here, there has been considerable effort in this direction motivated by a desire to better understand the strong interactions. Already, people have found geometrical analogues of confinement and chiral symmetry breaking [40].

The picture of spacetime emerging from string theory certainly seems bizarre and unconventional. But the notion of curved spacetime must have seemed equally bizarre and unconventional to the physicists of the early 20th century. It is clear that we do not yet have the final story. The picture is still emerging. Hopefully we will have the answer well before the 200th anniversary of spacetime. One can hardly imagine what physics will look like then.

Acknowledgments

I would like to thank O Aharony and D Marolf for discussions which helped clarify the above presentation. This work was partially supported by NSF grant PHY-0244764.

References

- [1] Ashtekar A 2005 *New J. Phys.* **7** 198
- [2] Marolf D 2004 Resource letter: the nature and status of string theory *Am. J. Phys.* **72** 730 (*Preprint hep-th/0311044*)
- [3] Polchinski J 1998 *String Theory* 2 vols (Cambridge: Cambridge University Press)
- [4] Zweibach B 2004 *A First Course in String Theory* (Cambridge: Cambridge University Press)
- [5] Polchinski J 1995 Dirichlet-branes and Ramond–Ramond charges *Phys. Rev. Lett.* **75** 4724 (*Preprint hep-th/9510017*)
- [6] Arkani-Hamed N, Dimopoulos S and Dvali G R 1998 The hierarchy problem and new dimensions at a millimeter *Phys. Lett. B* **429** 263 (*Preprint hep-ph/9803315*)
- [7] Randall L and Sundrum R 1999 An alternative to compactification *Phys. Rev. Lett.* **83** 4690 (*Preprint hep-th/9906064*)
- [8] Emparan R and Reall H S 2002 A rotating black ring in five dimensions *Phys. Rev. Lett.* **88** 101101 (*Preprint hep-th/0110260*)
- [9] Elvang H, Emparan R, Mateos D and Reall H S 2004 A supersymmetric black ring *Preprint hep-th/0407065*
- [10] Wiseman T 2003 Static axisymmetric vacuum solutions and non-uniform black strings *Class. Quantum Grav.* **20** 1137 (*Preprint hep-th/0209051*)
- [11] Buscher T H 1987 A symmetry of the string background field equations *Phys. Lett. B* **194** 59
- [12] Giveon A, Porrati M and Rabinovici E 1994 Target space duality in string theory *Phys. Rep.* **244** 77 (*Preprint hep-th/9401139*)
- [13] Rocek M and Verlinde E 1992 Duality, quotients, and currents *Nucl. Phys. B* **373** 630 (*Preprint hep-th/9110053*)
- [14] Candelas P, Horowitz G T, Strominger A and Witten E 1985 Vacuum configurations for superstrings *Nucl. Phys. B* **258** 46
- [15] Greene B R and Plesser M R 1990 Duality in Calabi–Yau moduli space *Nucl. Phys. B* **338** 15
Candelas P, Lynker M and Schimmrigk R 1990 Calabi–Yau manifolds in weighted P(4) *Nucl. Phys. B* **341** 383
- [16] Candelas P, De La Ossa X C, Green P S and Parkes L 1991 A pair of Calabi–Yau manifolds as an exactly soluble superconformal theory *Nucl. Phys. B* **359** 21
- [17] Givental A 1998 The mirror formula for quintic threefolds *Preprint math.AG/9807070*
- [18] Aspinwall P S, Greene B R and Morrison D R 1994 Calabi–Yau moduli space, mirror manifolds and spacetime topology change in string theory *Nucl. Phys. B* **416** 414 (*Preprint hep-th/9309097*)

- [19] Strominger A 1995 Massless black holes and conifolds in string theory *Nucl. Phys. B* **451** 96 (*Preprint hep-th/9504090*)
Greene B R, Morrison D R and Strominger A 1995 Black hole condensation and the unification of string vacua *Nucl. Phys. B* **451** 109 (*Preprint hep-th/9504145*)
- [20] Horowitz G T and Steif A R 1990 Space-time singularities in string theory *Phys. Rev. Lett.* **64** 260
- [21] Amati D and Klimcik C 1989 Nonperturbative computation of the Weyl anomaly for a class of nontrivial backgrounds *Phys. Lett. B* **219** 443
- [22] Myers R C 1987 New dimensions for old strings *Phys. Lett. B* **199** 371
- [23] Gepner D and Witten E 1986 String theory on group manifolds *Nucl. Phys. B* **278** 493
- [24] Narain K S, Sarmadi M H and Vafa C 1987 Asymmetric orbifolds *Nucl. Phys. B* **288** 551
- [25] 't Hooft G 1993 Dimensional reduction in quantum gravity *Preprint gr-qc/9310026*
- [26] Susskind L 1995 The world as a hologram *J. Math. Phys.* **36** 6377 (*Preprint hep-th/9409089*)
- [27] Maldacena J M 1998 The large N limit of superconformal field theories and supergravity *Adv. Theor. Math. Phys.* **2** 231
Maldacena J M 1999 *Int. J. Theor. Phys.* **38** 1113 (*Preprint hep-th/9711200*)
- [28] Aharony O, Gubser S S, Maldacena J M, Ooguri H and Oz Y 2000 Large N field theories, string theory and gravity *Phys. Rep.* **323** 183 (*Preprint hep-th/9905111*)
- [29] Witten E 1998 Anti-de Sitter space and holography *Adv. Theor. Math. Phys.* **2** 253 (*Preprint hep-th/9802150*)
- [30] Lee S M, Minwalla S, Rangamani M and Seiberg N 1998 Three-point functions of chiral operators in $D = 4, N = 4$ SYM at large N, *Adv. Theor. Math. Phys.* **2** 697 (*Preprint hep-th/9806074*)
- [31] Berenstein D, Maldacena J M and Nastase H 2002 Strings in flat space and pp waves from $N = 4$ super Yang–Mills *J. High Energy Phys.* JHEP0204(2002)013 (*Preprint hep-th/0202021*)
- [32] Gubser S S, Klebanov I R and Polyakov A M 2002 A semi-classical limit of the gauge/string correspondence *Nucl. Phys. B* **636** 99 (*Preprint hep-th/0204051*)
- [33] Tseytlin A A 2004 Semiclassical strings and AdS/CFT *Preprint hep-th/0409296*
- [34] Lin H, Lunin O and Maldacena J 2004 Bubbling AdS space and 1/2 BPS geometries *Preprint hep-th/0409174*
- [35] Gubser S S, Klebanov I R and Peet A W 1996 Entropy and temperature of black 3-branes *Phys. Rev. D* **54** 3915 (*Preprint hep-th/9602135*)
- [36] Policastro G, Son D T and Starinets A O 2002 From AdS/CFT correspondence to hydrodynamics. II: sound waves *J. High Energy Phys.* JHEP0212(2002)054 (*Preprint hep-th/0210220*)
- [37] Freedman D Z, Gubser S S, Pilch K and Warner N P 1999 Renormalization group flows from holography supersymmetry and a c-theorem *Adv. Theor. Math. Phys.* **3** 363 (*Preprint hep-th/9904017*)
- [38] Lunin O, Mathur S D and Saxena A 2003 What is the gravity dual of a chiral primary? *Nucl. Phys. B* **655** 185 (*Preprint hep-th/0211292*)
Lunin O, Maldacena J and Maoz L 2002 Gravity solutions for the D1–D5 system with angular momentum *Preprint hep-th/0212210*
- [39] Pearson J, Spradlin M, Vaman D, Verlinde H and Volovich A 2003 Tracing the string: BMN correspondence at finite J^2/N , *J. High Energy Phys.* JHEP0305(2003)022 (*Preprint hep-th/0210102*)
- [40] Klebanov I R and Strassler M J 2000 Supergravity and a confining gauge theory: duality cascades and chiral symmetry breaking resolution of naked singularities *J. High Energy Phys.* JHEP0008(2000)052 (*Preprint hep-th/0007191*)