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Quantum gyroscopes and Gödel's universe: entanglement opens a new testing ground for cosmology

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Abstract. Some exact solutions of Einstein's field equations represent a rotating universe. One example is Gödel's cosmological model. Bianchi solutions generalize the Gödel metric and include the expansion of the universe. We propose a measurement of the cosmic rotation using a light or matter wave interferometer based on the Sagnac effect. Entanglement between the quanta employed in this quantum gyroscope enhances the accuracy, thereby coming closer to the more-than-challenging requirements of such experiments.

1. From Foucault via Thirring and Lense to Gödel

A central problem of classical mechanics is the measurement of the rotation of a coordinate system. Foucault's pendulum [1] hanging in the large dome of the Panthéon in Paris in 1850 was one of the first mechanical devices to be used for observing the rotation of the Earth. It is interesting to note that a similar experiment had already been performed in 1661 by V Viviani, a student of E Galileo and G Torricelli, in Florence. In 1925 Michelson and Gale [2] used a rectangular interferometer of size 2010×1113 feet at Clearing, Illinois, to perform this task optically. Their device made use of the Sagnac effect [3] discovered in 1913, in an attempt to verify experimentally the existence of the ether. The Thirring–Lense effect [4]–[6], predicted in 1918, is a closely related phenomenon. Indeed, according to the theory of general relativity, the rotation of the Earth creates a gravitational field in addition to the familiar radially symmetric Newtonian one. Since this field is analogous to the magnetic

field obtained from the rotation of a uniformly charged sphere, it carries the name gravito-magnetic field.

The Thirring–Lense effect is considered to be a crucial test of general relativity [7]. Unfortunately, the gravito-magnetic field of the Earth is extremely weak. Many suggestions for measuring the resulting force have been made [8]. One such proposal [9]—the gravity probe B experiment [10]—is based on the precession of mechanical gyroscopes. It has been prepared since 1963 and is now scheduled for launch on 30 October 2002 at 8:14 pm. Another approach relies on the precession of two LAGEOSs (Laser Geodynamics Satellite) [11] in appropriate orbits. This experiment has already provided the first empirical indication of the Thirring–Lense field [12]. Moreover, due to the enormous progress in the production of cold atoms [13], the European HYPER (hyper-precision cold-atom interferometry in space) Collaboration [14] has suggested installing an atom interferometer in a satellite to map out the Thirring–Lense field via the Sagnac effect. In the present paper we go one step further and use the Sagnac effect to put an upper bound on the rotation of the universe. The basis of our study is the cosmological model of Gödel [15]. We show that a quantum gyroscope [16], that is a Sagnac interferometer with entangled atoms or photons, makes such an experiment feasible.

2. Gyroscopes and cosmology

In the most elementary explanation of the Sagnac effect we consider two counter-propagating light pulses on a rotating platform. There is a time delay between the two pulses returning to their common launching point. The pulse that propagates against the rotation arrives before the pulse that has to catch up with the starting point. This time delay is proportional to the rotation rate. When we use standing waves rather than pulses, the time delay translates into a phase or frequency shift that can easily be measured. The Sagnac effect is the basis of the navigational systems of ring laser gyroscopes that are now in every aeroplane. The use of a Sagnac interferometer to test general relativity was proposed [17, 18] in the early 1980s. However, at that time the accuracy of these devices was not sufficient for measuring the Thirring–Lense effect.

The Sagnac effect is not limited to light waves but occurs also for matter waves. It has been observed for neutrons [19], electrons [20] and, most recently, for atoms [21]. Matter waves have an enormous advantage over light waves since the Sagnac phase shift increases with decreasing wavelength. In the European ATOPIIS (atom optics and interferometry in space) project [29], atom interferometers will serve in the International Space Station to probe the foundations of general relativity. Here and in the HYPER satellite, Gödel's prediction of a rotating universe opens a new testing ground for this theory.

In 1917, Einstein found a cosmological model according to which the universe is expanding. In order to prevent this feature he also introduced the cosmological constant Λ . At the same time de Sitter showed that an isotropic homogeneous and static universe needs a non-vanishing cosmological constant. In 1930, Eddington remarked that a static universe with a positive cosmological constant is unstable. Consequently, Einstein and de Sitter postulated $\Lambda = 0$ and claimed the universe to be flat. In 1935, H P Robertson and A D Walker proved that the Friedmann–Lemaître line element is the most general solution of the Einstein field equations assuming an isotropic and homogeneous universe.

3. Sagnac effect of Gödel's universe

On the occasion of Einstein's 70th birthday, Gödel [15] proposed a new solution of Einstein's field equations. In his paper he had looked for and found solutions that have fewer symmetry assumptions than the Robertson–Walker metric. We quote from the introduction of his paper:

'All cosmological solutions with non-vanishing density of matter known at present have the common property, that in a certain sense, they contain an 'absolute' time-coordinate owing to the fact that there exists a one-parametric system of three-spaces everywhere orthogonal on the world lines of matter. It is easily seen that the non-existence of such a system of three-spaces is equivalent with the rotation of matter relative to the compass of inertia. In this paper I am proposing a solution (with the cosmological term $\neq 0$) which exhibits such a rotation'.

Indeed, his metric

$$ds^2 = a^2(dx_0^2 - dx_1^2 + \frac{1}{2}e^{2x_1} dx_2^2 - dx_3^2 + 2e^{x_1} dx_0 dx_2), \quad (1)$$

which contains the single free parameter a , involves a cross-term between the time and a space coordinate. Such an off-diagonal element indicates a rotation of the coordinate system and gives rise [17, 18, 22] to the Sagnac effect. In the limit of weak gravitational fields the Sagnac phase shift reads

$$\Delta\varphi = -\frac{2}{\lambda} \oint_C dx^i h_{0i} \quad (2)$$

where C denotes a closed path of the matter or light wave with reduced wavelength λ . In this limit the metric tensor $g_{\mu\nu}$ takes the form $g_{\mu\nu} \cong \eta_{\mu\nu} + h_{\mu\nu}$ where $\eta_{\mu\nu}$ is the metric tensor of flat spacetime and $h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$ represents the correction due to gravity.

A typical matter or light wave interferometer is of the size of a laboratory. On this scale the metric only shows small deviations from flat spacetime. Therefore, it is natural to consider the weak-field limit of Gödel's metric. Moreover, in order to read off the rotation rate of the universe it is useful to express the metric in cylindrical coordinates. We recall that Gödel has already introduced the new dimensionless coordinates (t, r, y, ϕ) in which the line element equation (1) reads

$$ds^2 = 4a^2(dt^2 - dr^2 - dy^2 + (\sinh^4 r - \sinh^2 r) d\phi^2 + 2\sqrt{2} \sinh^2 r d\phi dt). \quad (3)$$

Since the coordinates (x_0, x_1, x_2, x_3) used in equation (1) are also dimensionless and the line element ds has the unit of a length, the parameter a has the unit of a length. It is instructive to cast equation (3) into a more convenient form using the coordinates $\hat{t} \equiv 2at/c$, $\hat{r} \equiv 2ar$, $\hat{y} \equiv 2ay$, $\hat{\phi} \equiv \phi$ which yields

$$ds^2 = c^2 d\hat{t}^2 - d\hat{r}^2 - d\hat{y}^2 + 4a^2 \left[\sinh^4 \frac{\hat{r}}{2a} - \sinh^2 \frac{\hat{r}}{2a} \right] d\hat{\phi}^2 + 4\sqrt{2}ca \sinh^2 \frac{\hat{r}}{2a} d\hat{\phi} d\hat{t}. \quad (4)$$

This metric reduces to

$$ds^2 \cong c^2 d\hat{t}^2 - d\hat{r}^2 - d\hat{y}^2 - \hat{r}^2 d\hat{\phi}^2 + 2\hat{r}^2 \frac{1}{\sqrt{2}} \frac{c}{a} d\hat{\phi} d\hat{t} \quad (5)$$

in the case of $\hat{r} \ll a$. When we now compare this expression with the metric

$$ds^2 = [1 - (r\Omega_S/c)^2]c^2 dt^2 - dr^2 - dz^2 - r^2 d\phi^2 - 2r^2\Omega_S d\phi dt \quad (6)$$

of a coordinate system rotating with the rate Ω_S we find the rotation rate

$$\Omega_U \equiv \frac{1}{\sqrt{2}} \frac{c}{a} \quad (7)$$

of the universe together with the off-diagonal elements $h_{0\hat{\phi}} = -\hat{r}^2\Omega_U/c$ and $h_{0\hat{r}} = h_{0\hat{y}} = 0$.

We substitute these expressions for h_{0i} into equation (2), assume a circular path of radius \hat{R} around the origin for the wave and find the Sagnac phase shift

$$\Delta\varphi = \frac{4A\Omega_U}{\lambda c} \quad (8)$$

of Gödel's universe. Here $A = \pi\hat{R}^2$ denotes the area enclosed by the two counter-propagating waves.

4. Mass density determines rotation rate

According to equation (7) the rotation rate of the universe is inversely proportional to a . But what is the physical meaning of Gödel's parameter a ?

In order to answer this question we recall how Gödel derived his solution. From the ansatz equation (1) he calculated the Ricci tensor $R_{\mu\nu}$ and the curvature scalar R which determine the left-hand side of the Einstein field equation [23]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -\frac{8\pi G}{c^2}T_{\mu\nu} - \Lambda g_{\mu\nu}. \quad (9)$$

Here G denotes Newton's gravitational constant, and $T^{\mu\nu} = \rho u^\mu u^\nu$ is the energy-momentum tensor of a mass distribution ρ with (dimensionless) four-velocity u^μ .

In order to satisfy these equations, Gödel had to relate his model parameter a to the mass density ρ and the cosmological constant Λ , yielding the identifications

$$\frac{1}{a^2} = \frac{8\pi G}{c^2}\rho \quad (10)$$

and

$$\Lambda = -\left(\frac{1}{\sqrt{2}a}\right)^2. \quad (11)$$

Indeed, comparing equation (7) with (10) we obtain the explicit expression

$$\Omega_U \equiv \frac{1}{\sqrt{2}}(8\pi G\rho)^{1/2} \quad (12)$$

for the rotation rate of the universe, in complete agreement with Gödel:

'Matter everywhere rotates relative to the compass of inertia with the angular velocity: $2(\pi\kappa\rho)^{1/2}$, where ρ is the mean density of matter and κ is Newton's gravitational constant'.

5. Compass of inertia

But what does ‘compass of inertia’ mean? The answer to this question seems to be related to ‘Mach’s principle’ [24].

‘Mystic and murky is the measure many make of the meaning of Mach’.

With these words, Isenberg and Wheeler [25] start their paper on Mach’s principle, defending their idea that ‘inertia here is fixed by mass–energy there’. Indeed, Einstein was motivated by Mach’s idea that inertia is induced by cosmic matter. The original paper of Thirring [4] even quotes the relevant paragraph of Einstein’s paper. Mach’s principle motivated Thirring’s analysis of the forces acting on a test particle in a rotating mass shell: he found centrifugal and Coriolis forces. Later Lense and Thirring [5] calculated the gravitational field of a rotating sphere and studied its influence on the orbit of a satellite. However, these inertial forces do not confirm the validity of Mach’s principle within Einstein’s theory: the Thirring–Lense effect results from their choice of the coordinate system; the inertial mass is not changed at all by the rotation of the shell.

One possibility for experimentally defining the so-called ‘compass of inertia’, a concept put forward by Weyl [26] in 1924, makes use of the Foucault pendulum. In order to avoid geophysical complications we consider the pendulum at the north or south pole of the Earth. The plane of the swinging pendulum defines this compass directly. Another possibility is using the path of a free test particle as suggested in [23].

6. A need for quantum gyroscopes

We start this section by presenting some estimates for the rotation rate Ω_U determined by equation (12). When we assume for the mass density of the universe $\rho = 2 \times 10^{-31} \text{ g cm}^{-3}$ [27] and $G = 6.67 \times 10^{-8} \text{ cm}^3 \text{ g}^{-1} \text{ s}^{-2}$ we arrive at

$$\Omega_U \cong 4 \times 10^{-19} \text{ rad s}^{-1}. \quad (13)$$

It is interesting to compare this value to the rotation rates $\Omega_{\oplus} \cong 10^{-4} \text{ rad s}^{-1}$ of the Earth or $\Omega_{TL} \cong 10^{-14} \text{ rad s}^{-1}$ corresponding to the Thirring–Lense effect. Thus, the rotation rate of the universe is five decimal orders slower than the one corresponding to the Thirring–Lense effect.

We now turn to the accuracy of atom gyroscopes. Nowadays [28] these devices have a short-term rotation-rate sensitivity of $6 \times 10^{-10} \text{ rad s}^{-1}$ over one second of integration. For rubidium atoms this corresponds to measuring rotation rates of $2.18 \times 10^{-12} \text{ rad s}^{-1}$ at 10^9 atoms and 3 s drift time. When this measurement is repeated every three seconds the device will reach after 2 h an accuracy of $10^{-14} \text{ rad s}^{-1}$. In the course of one year [14] it would be possible to achieve an accuracy of $10^{-16} \text{ rad s}^{-1}$ which is about a hundredth of the expected effect.

In these interferometers the atoms are uncorrelated. However, a new situation arises when the atoms are entangled with each other. In this case we insert [16] into the two input ports of the interferometer the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left[\left| \frac{N+1}{2} \right\rangle_1 \left| \frac{N-1}{2} \right\rangle_2 - \left| \frac{N-1}{2} \right\rangle_1 \left| \frac{N+1}{2} \right\rangle_2 \right] \quad (14)$$

where $|N\rangle_j$ denotes a Fock state of N particles in the $j = 1, 2$ input mode. Here the particles can be photons or atoms. Even gyroscopes with Bose–Einstein condensates have been discussed.

Indeed, it has been shown [16] that such a quantum gyroscope ought to be about 10^8 times more sensitive to rotations than the standard one.

A similar enhancement factor due to entanglement has been proposed [30] for photons in the context of quantum lithography and has been verified experimentally [31]. Obviously for a large number of particles such entangled states are difficult to obtain. However, recently methods of achieving an amplification of entangled states, so-called massive qubits, have been demonstrated experimentally [32, 33].

Moreover, the Sagnac effect can also be interpreted as a consequence of synchronization of clocks along a closed path [34]. Since entanglement has already led to an improvement in quantum clock synchronization [35], it confirms the enhancement result of [16] in an independent way.

7. Gödel-like models with expansion

In his memoirs, Einstein dismissed Gödel's solution on 'physical grounds': indeed, the metric equation (1) predicts various unusual features. The most surprising one is that it allows for time travel. To again quote Gödel:

'... a temporal orientation is defined for the world line of every (real or possible) particle of matter or light, i.e., it is determined for any two neighboring points on it which one is earlier. On the other hand, however, no uniform temporal ordering of all point events agreeing in direction with all these individual orderings exists ... there also exist closed time-like lines ... and it is theoretically possible in these worlds to travel into the past, or otherwise to influence the past'.

The reason for this phenomenon lies in the globally non-stratified spacetime. For a more detailed discussion of this point we refer the reader to [36, 37].

Today we know that the universe is expanding. In contrast, Gödel's non-vanishing cosmological constant prevents any expansion. However, there exist generalizations of Gödel's universe containing a rotation as well as an expansion. Indeed, Gödel himself in 1950 suggested such an extension [38]. Moreover, there are generalizations [24, 39] of Bianchi-type VII_h and IX. The isotropy of the cosmic microwave background radiation measured by the Boomerang (balloon observations of millimetric extragalactic radiation and geomagnetic) experiment [40] clearly indicates that the rotation must be negligibly small. Indeed, the cosmic background explorer (COBE) has put the upper limit [41] $(\omega/H)_0 \lesssim 10^{-6}$ on the rotation rate. With the help of the Hubble constant $H_0 = 2.2 \times 10^{-18} \text{ s}^{-1}$ we arrive at the upper bound $\omega < 2.2 \times 10^{-24} \text{ s}^{-1}$ on the vorticity ω of the universe.

We conclude by summarizing our main results. Gödel's cosmological model assumes a rotation of the universe. The Sagnac effect allows us to measure rotations. Indeed, a matter or light wave gyroscope capitalizing on entangled atoms may reach the accuracy necessary to test Gödel's hypothesis.

In this context it is worthwhile mentioning that [42] discusses the motion of a test particle and the precession of an angular momentum, that is a mechanical gyroscope in Gödel's metric. However, no estimate for a possible experiment has been given.

Einstein seemed to dislike Gödel's model. Moreover, he certainly was not fond of entanglement in quantum mechanics as represented by the Einstein–Podolsky–Rosen situation. Therefore, it would be amusing if entanglement could provide some insight into this somewhat unusual model of the universe based on Einstein's theory of the relativity.

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