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Driving atoms into decoherence-free states

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Abstract. We describe the decoherence-free subspace of N atoms in a cavity, in which decoherence due to the leakage of photons through the cavity mirrors is suppressed. We show how the states of the subspace can be *entangled* with the help of weak laser pulses, using the high decay rate of the cavity field and strong coupling between the atoms and the resonator mode. The atoms remain decoherence-free with a probability which can, in principle, be arbitrarily close to unity.

1. Introduction

Following the theoretical formulation of quantum computing [1, 2] and the first algorithms for problems which can be solved more easily on a quantum computer than on a classical computer [3, 4] the practical implementation of such a device has become a challenging task. Initial steps have already been taken. Quantum bits (qubits) can be realized for instance by storing the information in a superposition of the internal states of two-level atoms. To provide the interaction between the atoms necessary to perform operations between the qubits, the coupling via vibrational modes [5]–[9] or via the single mode inside a cavity [10]–[12] can be used. In other proposals, level shifts due to dipole–dipole interaction [13]–[16] and due to light shifts [17, 18] have been considered.

The main limiting factor for quantum computing is decoherence. This normally limits factoring [3], for example, to small numbers [19]–[21] and demonstrates the necessity for error correcting codes [22]–[24]. However, even with the help of quantum error correction, it remains unknown whether decoherence will still destroy the quantum coherence too rapidly for any practical use if the number of qubits required is of the order of several hundreds or thousands. Indeed, a superposition of two quantum mechanical wavefunctions loses its coherence very rapidly with the ‘distance’ between the components involved [25].

However, it has recently become clear that decoherence-free subspaces (DFSs) of the total Hilbert space may exist, in which the states are in principle exempt from decoherence [26]–[29]. They arise if the coupling to the environment has a certain symmetry. The decoherence-free

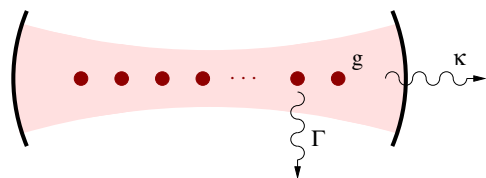


Figure 1. A schematic view of the system. The two-level atoms are held at fixed positions in the cavity sufficiently far apart that they can be addressed individually by laser beams.

(DF) states then all acquire the same phase factor, so that arbitrary superpositions of them remain intact in spite of the interaction with the environment [25]. DFSs are promising candidates for quantum computing. The dependence of quantum information processing on error correction schemes is substantially reduced [30]. While the underlying theoretical nature of DFSs has received much attention, far less is known about potential realizations (for examples see [31, 32]) and the manipulation of the states *inside* the DFS in general (see, however, [26], [33]–[36]). Their method for manipulating the states inside a certain DFS in [33, 34] is very different from the approach proposed here in that the state of their system always remains completely inside the DFS, requiring an exchange interaction that is not easily available in quantum optics.

In this paper we give an example of a DFS which can be implemented using present technology, at least for small numbers of qubits and we describe how to prepare and to manipulate the states *inside* a subspace. The system we discuss consists of N macroscopically separated metastable two-level atoms and is shown in figure 1. We generate an interaction between the atoms by placing them at fixed positions in a cavity which acts as a resonator for an electromagnetic field. The atoms can be stored between the cavity mirrors in a linear trap or in the nodes of a standing light field. The atomic transition is assumed to be in resonance with a single field mode in the cavity. The atoms should be strongly coupled to the field mode and the interaction between each atom and the field is given by the coupling constant g_i . As a simplification we assume that $g_i \equiv g$ for all i , but the ideas discussed here can also be carried over to the more general case.

The main source of decoherence in this system is that a photon can leak out through the cavity mirrors with a rate κ which is due to the coupling of the resonator mode to the free radiation field. Even if the cavity mode is empty, the atoms will in general transfer excitation into the resonator mode which then can be lost. As we will show later, this process does not take place if the cavity mode is empty and the atoms are prepared in a *trapped* state. As a result an example of a DFS is found. The trapped states of two two-level atoms in a cavity have been discussed in [37]–[40]. They belong to a two-dimensional Hilbert space which includes the ground state and the maximally entangled state $(|1\rangle_1|0\rangle_2 - |0\rangle_1|1\rangle_2)/\sqrt{2}$. We will show below that the trapped states of N atoms create a DFS of dimension

$$\binom{N}{N/2} \quad \text{or} \quad \binom{N}{(N+1)/2} \quad (1)$$

for odd and even numbers of atoms, respectively. For large N the dimension roughly equals $[2/(\pi N)]^{1/2} 2^N$ and therefore increases with N almost as fast as does the dimension of the whole state space, 2^N .

The distance between the atoms should be much larger than an optical wavelength. This allows us to address each atom individually by a single laser pulse. If their Rabi frequencies are much smaller than the constants g and κ , laser pulses can be used to prepare and to manipulate the states *inside* the DFS. The reason for this is a mechanism which strongly inhibits the transition from trapped to non-trapped states in this parameter regime and which can be understood with the help of the *quantum Zeno effect* [41]–[44]. We in fact profit from a high decay rate of the resonator field and the results do not depend on precise values of g and κ . Arbitrary unitary operations can be constructed in a DF qubit formed out of two states of two atoms. In particular we show how a maximally entangled Bell state of the two atoms can be generated out of the atomic ground state.

In the system we discuss here one source of decoherence remains. Even if the spontaneous decay rate of the atoms is decreased by the presence of the resonator, photons can still be emitted spontaneously into non-cavity field modes. We therefore propose to use metastable atoms, which have a very small decay rate Γ . Spontaneous emission can be neglected if the durations of the operations performed on the atoms are short relative to $1/\Gamma$. Therefore the applied laser pulses cannot be arbitrarily weak, as is necessary for the scheme to work. Care is thus needed to ensure that an overall advantage is obtained [20, 21]. Problems arising from this will be discussed in detail.

In principle, one could argue that an even larger Hilbert space of atomic states than the DFS considered here can be obtained by storing atoms (or ions) in free space without a surrounding cavity. For this, atomic decoherence is also due only to spontaneous emission. We should emphasize that the major advantage of the system discussed here is that two qubit entanglement operations can be performed with the help of laser pulses, while laser pulses cannot entangle atoms in the free space case using our approach.

One method of entangling atoms via their interaction with a resonator mode, in which the atoms fly through a high finesse cavity, is discussed in [10, 12]. The duration for which the atoms interact with the field is fixed and determined by the atomic velocity. If the atoms leave the cavity their temporal evolution stops and the prepared state is stable. Using this idea to perform many operations in a sequence and to scale up the system by using many atoms becomes costly in both time and material. In our approach, once the system has been prepared in a state of the DFS, it does not change, because the interaction among the atoms, the cavity mode and the environment of the system is effectively switched off. The atoms can be stored in the cavity for long periods and arbitrarily many operations can be performed.

The paper is organized as follows. In the next section we give a detailed description of the physical system. In section 3 we review the quantum jump approach [45]–[47] employed to describe the dissipative dynamics. This approach is equivalent to the Monte Carlo wavefunction approach [48] and to quantum trajectories [49]. It also gives a simple criterion for a state to be DF. We construct the DFS for N atoms in section 4. How the states in the DFS can be manipulated is explained in the following two sections. We summarize our results in section 7.

2. A description of the physical system

The system considered here consists of N metastable two-level atoms (or ions) confined to fixed positions inside an optical cavity. In the following $|0\rangle_i$ and $|1\rangle_i$ denote the ground and the excited state of atom i , respectively. The Pauli operator $\sigma_i = |0\rangle_{ii}\langle 1|$ is the atomic lowering operator. The atoms with level separation $\hbar\omega_0$ are considered to be in resonance with a single mode of the electromagnetic field inside the cavity. The coupling strength for coupling of each atom

to the cavity mode g is taken to be real. The field annihilation operator for the cavity mode is denoted by b . In addition the atoms are weakly coupled to the free radiation field outside the cavity with a coupling constant $g_{\mathbf{k}\lambda}^{(i)}$ for the i th atom and a field mode with wavevector \mathbf{k} and polarization λ . The annihilation operator for this mode is $a_{\mathbf{k}\lambda}$. This free radiation field provides an environment for the atoms and is responsible for spontaneous emission. We also take into account the non-ideality of cavity mirrors by coupling the field inside the resonator to the outside with a strength $\tilde{g}_{\mathbf{k}\lambda}$, so that single photons can leak out. The annihilation operator of the free radiation field to which the cavity field couples is given by $\tilde{a}_{\mathbf{k}\lambda}$. Then, in the Schrödinger picture, the Hamiltonian of the system and its environment is given by

$$H = \sum_{i=1}^N \hbar\omega_0 \sigma_i^\dagger \sigma_i + \hbar\omega_0 b^\dagger b + \sum_{\mathbf{k}\lambda} \hbar\omega_{\mathbf{k}} (a_{\mathbf{k}\lambda}^\dagger a_{\mathbf{k}\lambda} + \tilde{a}_{\mathbf{k}\lambda}^\dagger \tilde{a}_{\mathbf{k}\lambda}) + i\hbar \sum_{i=1}^N g b \sigma_i^\dagger + i\hbar \sum_{i=1}^N \sum_{\mathbf{k}\lambda} g_{\mathbf{k}\lambda}^{(i)} a_{\mathbf{k}\lambda} \sigma_i^\dagger + i\hbar \sum_{\mathbf{k}\lambda} \tilde{g}_{\mathbf{k}\lambda} \tilde{a}_{\mathbf{k}\lambda} b^\dagger + \text{h.c.} \quad (2)$$

The first four terms give the interaction-free Hamiltonian and correspond to the free energy of the atoms, the resonant cavity mode and the electromagnetic fields outside the system. Going over to the interaction picture with respect to the interaction free Hamiltonian gives rise to the interaction Hamiltonian

$$H_I = i\hbar \sum_{i=1}^N g b \sigma_i^\dagger + i\hbar \sum_{i=1}^N \sum_{\mathbf{k}\lambda} g_{\mathbf{k}\lambda}^{(i)} a_{\mathbf{k}\lambda} \sigma_i^\dagger e^{i(\omega_0 - \omega_{\mathbf{k}})t} + i\hbar \sum_{\mathbf{k}\lambda} \tilde{g}_{\mathbf{k}\lambda} \tilde{a}_{\mathbf{k}\lambda} b^\dagger e^{i(\omega_0 - \omega_{\mathbf{k}})t} + \text{h.c.} \quad (3)$$

The first term contains the coupling of the atoms to the cavity mode. The second term describes the coupling of the atoms to the free radiation field and is responsible for spontaneous emission with a decay rate Γ (see figure 1), as will be shown in the next section. From the last term the damping of the cavity mode by leakage of photons through the cavity mirrors will arise. The decay rate of a single photon inside the resonator is κ and we assume here that

$$g \sim \kappa \quad (4)$$

i.e. g and κ are of the same order of magnitude.

To prepare and manipulate the states of the atoms inside the DFS, resonant laser pulses are applied, which address each atom individually. The Rabi frequency of the laser which interacts with atom i will be denoted by Ω_i . The Hamiltonian describing the effect of the laser in the rotating wave approximation and in the interaction picture chosen above is equal to

$$H_{\text{laser I}} = \frac{\hbar}{2} \sum_{i=1}^N \Omega_i \sigma_i + \text{h.c.} \quad (5)$$

We will assume here, for all $\Omega_i \neq 0$, that

$$\Gamma \ll |\Omega_i| \ll g. \quad (6)$$

Note that the frequencies Ω_i are in general complex numbers. Their phase factors cannot be compensated by changing the basis of the atomic states, because we have already chosen the coupling constants g_i to be the same for all atoms.

To increase the precision of the state preparation, detectors which continuously monitor the free radiation field outside the system could be used. If a photon is emitted spontaneously or leaks out through the cavity mirrors, one should stop the experiment and re-initiate the whole process. However, even without detectors the experiment can work, in principle, with an arbitrarily high success rate. We will show that the probability for the loss of a photon is negligible and only small errors are introduced if it is not recorded.

3. The conditional temporal evolution

One necessary requirement for quantum computing is the ability to manipulate the qubits in a controlled way. In any quantum algorithm, a system in an arbitrary pure state has to be transformed into another pure state by appropriate coherent unitary operations. In general the system considered here interacts with its environment, loses a photon stochastically and, after a short time, has to be described by a density matrix. To avoid this we consider in the following only the specific temporal evolution under the condition that no decay takes place, which can easily be determined from a quantum jump approach description [45, 46] of the system. In this section we summarize the main results of this approach.

With the help of the quantum jump approach one can obtain a conditional Hamiltonian H_{cond} , which describes the temporal evolution of the system provided that no photon is emitted, either by spontaneous emission or by leakage of photons through the cavity mirrors. This Hamiltonian can be evaluated by second-order perturbation theory from the expression

$$H - \frac{i}{\hbar} H_{\text{cond}} \Delta t = \langle 0_{\text{ph}} | U_I(\Delta t, 0) | 0_{\text{ph}} \rangle \quad (7)$$

using equations (3) and (5). Here $|0_{\text{ph}}\rangle$ is defined as the vacuum state of the free radiation fields outside the system. In a similar way to that used in [39], in which the case of two atoms in a cavity was discussed, one finds

$$H_{\text{cond}} = i\hbar g \sum_{i=1}^N b \sigma_i^\dagger + \text{h.c.} - i\hbar \Gamma \sum_{i=1}^N \sigma_i^\dagger \sigma_i - i\hbar \kappa b^\dagger b + H_{\text{laser I}}. \quad (8)$$

The corresponding conditional temporal development operator, $U_{\text{cond}}(t, 0) = \exp(-iH_{\text{cond}}t/\hbar)$, is non-unitary because H_{cond} is non-Hermitian. This leads to a decrease of the norm of the vector developing with U_{cond} and is connected to the waiting time distribution for emission of a (the next) photon. If at $t = 0$ the state of the system is $|\psi_0\rangle$, the state at time t is given by the normalized state [45, 46]

$$|\psi^0(t)\rangle = U_{\text{cond}}(t, 0)|\psi_0\rangle / \|\cdot\|. \quad (9)$$

The probability P_0 of observing *no* photon in $(0, t)$ with a broadband detector (over all space) is

$$P_0(t, \psi_0) = \|U_{\text{cond}}(t, 0)|\psi_0\rangle\|^2. \quad (10)$$

In a real experiment, the emitted photons are actually registered with an efficiency η smaller than 1, or even $\eta = 0$. Then the system is, in the case of no photon detection, prepared in a statistical mixture of the form

$$[P_0|\psi^0\rangle\langle\psi^0| + (1 - \eta)(1 - P_0)\rho_\perp] / \text{tr}(\cdot). \quad (11)$$

Here ρ_\perp describes the state of the system for the case of photon emissions, which is in general different from the state $|\psi^0\rangle$ we want to prepare.

4. Construction of the decoherence-free subspace

With the help of the quantum jump approach we easily find a necessary and sufficient criterion for establishing a decoherence free subspace (DFS). For all states $|\psi\rangle$ of a DFS, the probability of no photon emission for all times t has to remain unity, i.e.

$$P_0(t, \psi) \equiv 1 \forall t \geq 0. \quad (12)$$

This condition is fulfilled if the system effectively does not interact with the environment [27]. In addition, our criterion demands that the system's own temporal evolution does not move the state out of the DFS. In this section we neglect spontaneous emission ($\Gamma = 0$) and determine all states which satisfy condition (12). In the following $|n\rangle$ denotes a states with n photons in the cavity field mode, $|\varphi\rangle$ corresponds to a state of the atoms only and we define $|n\rangle \otimes |\varphi\rangle \equiv |n\varphi\rangle$.

Let us first investigate under what condition the probability density for the loss of a photon by a system in a state $|\psi\rangle$ is equal to zero. This is the case if $dP_0(t, \psi)/dt|_{t=0} = 0$ and leads, using equations (9) and (10), to the condition

$$\langle\psi|(H_{\text{cond}} - H_{\text{cond}}^\dagger)|\psi\rangle = -2i\kappa\langle\psi|b^\dagger b|\psi\rangle = 0. \quad (13)$$

Therefore each state of the DFS must be of the form

$$|\psi\rangle = |0\varphi\rangle. \quad (14)$$

As expected, only if the cavity mode is empty does no photon leak out through the resonator mirrors. However, condition (14) is not yet a sufficient criterion for the states of a DFS. To ensure that $P_0(t, \psi) \equiv 1$ for all times t , the cavity mode must never become populated. All matrix elements of the conditional Hamiltonian of the form $\langle n\varphi'|H_{\text{cond}}|0\varphi\rangle$ have to vanish for $n \neq 0$. Using equation (8) we find that this is the case, if and only if

$$J_-|\varphi\rangle \equiv \sum_{i=1}^N \sigma_i|\varphi\rangle = 0. \quad (15)$$

Under this condition the system's own temporal evolution does not drive the state out of the DFS. The states defined by equations (14) and (15) are also known in the literature as *trapped* states [37]–[40]. An explicit expression for the trapped states of $N = 2, 3$ and 4 atoms is given in [31].

Atomic states which fulfil condition (15) are well known in quantum optics as the Dicke states, of the form $|l, -l\rangle$ in the usual $|j, m\rangle$ notation [50]. They are eigenstates of the total Pauli spin operator. The quantum number l can take on the values $\frac{1}{2}, \frac{3}{2}, \dots, N/2$ for N odd and $0, 1, \dots, N/2$ for N even. The states $|l, -l\rangle$ are highly degenerate, namely $\binom{N}{N/2-l} - \binom{N}{N/2-l-1}$ -fold degenerate for $l \leq N/2 - 1$. Together with the single ground state $|N/2, -N/2\rangle$, the dimension of the total DFS sums up to the expression given in equation (1).

The Dicke states with a fixed quantum number l are also eigenstates of the operator $\sum_i \sigma_i^\dagger \sigma_i$ which measures the excitation n in the system [50]. The relation between n and l is given by $n = N/2 - l$. We describe now how an orthonormal basis for such a subset of states which are orthogonal to all other Dicke states can be found. Using the notation

$$|a_{ij}\rangle \equiv (|1\rangle_i|0\rangle_j - |0\rangle_i|1\rangle_j)/\sqrt{2} \quad (16)$$

and equation (15), it can be proven that each state of the form

$$|\varphi\rangle = |0\rangle_2 \otimes |a_{13}\rangle \otimes |a_{45}\rangle \otimes \dots \otimes |0\rangle_N \quad (17)$$

in which, for instance, the first and third atoms are in an antisymmetrical state, the second one is in the ground state and so on, is a Dicke state. Writing down all possible states in which n pairs of atoms are in the antisymmetrical state and all others in the ground state gives a subset of Dicke states. They all have the same excitation number n and cover uniformly the whole subspace of Dicke states $|l, -l\rangle$ with $n = N/2 - l$. Now these states can be orthogonalized. An orthonormal basis for the DFS of N atoms can be obtained by joining together all atomic sub-bases for fixed n combined with the vacuum state of the cavity field.

Let us define analogously to equation (16)

$$\begin{aligned} |s_{ij}\rangle &\equiv (|1\rangle_i|0\rangle_j + |0\rangle_i|1\rangle_j)/\sqrt{2} & |g_{ij}\rangle &\equiv |0\rangle_i|0\rangle_j & |e_{ij}\rangle &\equiv |1\rangle_i|1\rangle_j \\ |xy\rangle &\equiv |x_{12}y_{34}\rangle. \end{aligned} \quad (18)$$

Then, for instance, an orthonormal basis of the trapped states of *four* atoms can be obtained by orthogonalizing the states $|g_{12}g_{34}\rangle$, $|g_{12}a_{34}\rangle$, $|g_{13}a_{24}\rangle$, $|g_{14}a_{23}\rangle$, $|g_{23}a_{14}\rangle$, $|g_{24}a_{13}\rangle$, $|g_{34}a_{12}\rangle$ and $|a_{12}a_{34}\rangle$ and one finds

$$|gg\rangle, |ga\rangle, |ag\rangle, |aa\rangle, |x_1\rangle \equiv (|sg\rangle - |gs\rangle)/\sqrt{2} \quad |x_2\rangle \equiv (|eg\rangle + |ge\rangle - |ss\rangle)/\sqrt{3}. \quad (19)$$

An orthonormal basis state for the Dicke states within the DFS of *two* atoms is $\{|g_{12}\rangle, |a_{12}\rangle\}$.

In general, to obtain a simple form of the states which form the DF qubits, one can combine the atoms into pairs. The ground states and the antisymmetrical states of each pair can then form one qubit. Thus for instance the first four states in equation (19) could be used to obtain two qubits. In this way we find $N/2$ qubits for an even number of atoms. They belong to a $2^{N/2}$ -dimensional subspace of the total DFS. The additional states can serve as auxiliary levels that can be used to realize certain logical operations.

5. Manipulation of the DF states of two atoms

We now know how DF qubits can be constructed resulting from the states of N atoms in a cavity. However, to do quantum computing one also has to be able to perform operations *inside* the DFS. In this section we discuss using the example of *two* atoms how DF states can be manipulated. To do so a weak laser pulse is applied to create Rabi frequencies Ω_1 and Ω_2 which obey condition (6). We discuss the effect of the pulse on the system with the help of a quantum jump approach description (see section 3) which also gives the probability of no photon emission, i.e. the success rate of the proposed experiment. It will be shown that the atoms remain DF with a success rate which can, in principle, be arbitrarily close to 1. This is due to a mechanism which decouples trapped states from non-trapped ones, which we will explain in detail. A generalization of the scheme to higher numbers of atoms is given in the next section.

In the following we use the same notation as that given in equations (16) and (18), but suppress the index 12 for simplicity. As was shown above the two trapped states of two atoms are $|g\rangle$ and $|a\rangle$. The states $|s\rangle$ and $|e\rangle$ complete a basis for the atomic states. From equation (8) and with the abbreviation

$$\Omega_{\pm} \equiv (\Omega_1 \pm \Omega_2)/(2\sqrt{2}) \quad (20)$$

the conditional Hamiltonian, which describes the temporal evolution of the system under the condition of no photon losses, becomes during the laser interaction

$$\begin{aligned} H_{\text{cond}} &= -i\hbar g \sum_{n=0}^{\infty} [2(n+1)]^{1/2} (|n+1g\rangle\langle ns| + |n+1s\rangle\langle ne| - \text{h.c.}) \\ &+ \hbar \sum_{n=0}^{\infty} \Omega_+ (|ng\rangle\langle ns| + |ns\rangle\langle ne| + \text{h.c.}) + \Omega_- (|ng\rangle\langle na| - |na\rangle\langle ne| + \text{h.c.}) \\ &- i\hbar \sum_{n=0}^{\infty} \Gamma (|na\rangle\langle na| + |ns\rangle\langle ns|) + 2\Gamma |ne\rangle\langle ne| - i\hbar \sum_{n=1}^{\infty} \sum_x n\kappa |nx\rangle\langle nx|. \end{aligned} \quad (21)$$

The first term describes the exchange of excitation between the field mode and the atoms, while the laser pulses change only the atomic states, as shown in figure 2. Terms proportional to Γ and

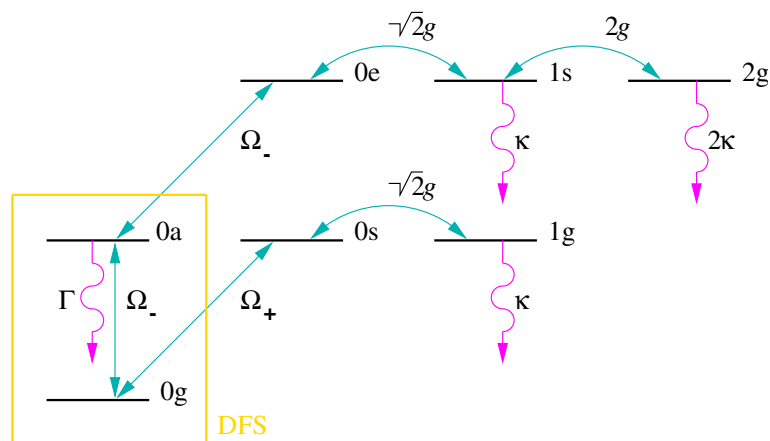


Figure 2. The level scheme of the two two-level atoms and the cavity mode showing the most important possible transitions inside the system. The DFS contains the states $|0g\rangle$ and $|0a\rangle$. Two weak lasers excite the transition inside the DFS and couple it to the states $|0e\rangle$ and $|0s\rangle$ with Rabi frequencies Ω_- and Ω_+ , respectively. Owing to the presence of the cavity mode, transitions between the states $|0s\rangle$, $|0e\rangle$, $|1g\rangle$, $|1s\rangle$ and $|2g\rangle$ take place with a rate g . If the cavity mode becomes populated a photon can leak out with a rate κ .

κ are responsible for a decrease in the norm of the state vector, if higher modes of the cavity are populated or spontaneous emission of the atoms can take place.

Let us assume that the system is in the ground state $|0g\rangle$ at time $t = 0$ when a laser pulse of length T is applied. The *unnormalized* state of the system under the condition of no photon losses $|\psi^0(t)\rangle$ at time t is denoted in the following by

$$|\psi^0(t)\rangle = \sum_{n,x} c_{nx}(t) |nx\rangle. \quad (22)$$

To describe the temporal evolution of the coefficients c_{nx} we obtain from the time dependent Schrödinger equation $i\hbar d/dt |\psi^0(t)\rangle = H_{\text{cond}} |\psi^0(t)\rangle$ a system of differential equations,

$$\begin{aligned} \dot{c}_{ng} &= -i\Omega_- c_{na} - i\Omega_+ c_{ns} - (2n)^{1/2} g c_{n-1s} - n\kappa c_{ng} \\ \dot{c}_{na} &= -i\Omega_-^* c_{ng} + i\Omega_- c_{ne} - (\Gamma + n\kappa) c_{na} \\ \dot{c}_{ns} &= -i\Omega_+^* c_{ng} - i\Omega_+ c_{ne} - (2n)^{1/2} g c_{n-1e} + [2(n+1)]^{1/2} g c_{n+1g} - (\Gamma + n\kappa) c_{ns} \\ \dot{c}_{ne} &= i\Omega_-^* c_{na} - i\Omega_+^* c_{ns} + [2(n+1)]^{1/2} g c_{n+1s} - (2\Gamma + n\kappa) c_{ne} \end{aligned} \quad (23)$$

which will be solved to a good approximation in the following.

5.1. A simplified discussion

First we discuss the case in which the spontaneous emission by the atoms can be neglected and we set $\Gamma = 0$. The simplified calculation given in this subsection describes already the main behaviour of the system due to the laser interaction, namely the one-qubit rotation.

As shown in figure 2, only the amplitudes c_{0g} and c_{0a} change slowly in time, on a time scale proportional to $1/|\Omega_-|$. Here we are interested in exactly this temporal evolution. All other levels change on a time scale $1/g$ and $1/\kappa$ which is much shorter due to condition (6).

If the system is initially in a DF state the laser pulse excites also the states $|0s\rangle$ and $|0e\rangle$. Then the excitation of these levels is transferred with the rate g into states in which the cavity mode is populated. These states are immediately emptied by one of the following two mechanisms. One possibility is that a photon leaks out through the cavity mirrors. However, as long as the population of the cavity field is small, the leakage of a photon through the cavity mirrors is very unlikely to take place. With a much higher probability the excitation of the cavity field vanishes during the conditional temporal evolution due to the last term in the conditional Hamiltonian in equation (21). No population can accumulate in non-DF states and we can assume that $c_{nx} \equiv 0$ for all states outside the DFS and to zeroth order the differential equation (23) simplifies to

$$\begin{aligned}\dot{c}_{0g} &= -i\Omega_- c_{0a} \\ \dot{c}_{0a} &= -i\Omega_-^* c_{0g}.\end{aligned}\quad (24)$$

This equation describes the temporal evolution of the DF states to a very good approximation.

If the trapped states are populated once only, the system remains inside the DFS. It behaves like a two-level system with the states $|g\rangle$ and $|a\rangle$ driven by a laser with Rabi frequency $2\Omega_-$. If the system is initially, when the laser pulse of length T is applied, in the ground state $|0g\rangle$, the atomic state at the end of the pulse is given by

$$|\psi^0(T)\rangle = \cos(|\Omega_-|T)|0g\rangle - i\frac{\Omega_-^*}{|\Omega_-|} \sin(|\Omega_-|T)|0a\rangle.\quad (25)$$

By varying the length T of the laser pulses and control over the phase of Ω_- any arbitrary rotation between the two states $|0g\rangle$ and $|0a\rangle$ can be realized. Owing to equations (10) and (25), the probability of finding no photon, $P_0(T, \psi_0)$, is unity. Note that the qualitative behaviour is independent of the Rabi frequencies Ω_1 and Ω_2 , as long as $\Omega_1 \neq \Omega_2$. To a very good first approximation the atomic states do not move out of the DFS. The quantitative behaviour of the atoms does not depend on the precise values of g and κ , which simplifies possible realizations of the proposed experiment.

The mechanism which decouples the DFS of the two atoms from the other states works better the larger the parameters g and κ are relative to Ω_{\pm} , which is why condition (6) has been chosen. In addition, we assumed κ and g to be of the same order of magnitude (see equation (4))[†]. Here we use the presence of *leaky* cavity mirrors to ensure that no photon is emitted while the laser pulse is applied! The cavity mode does not become populated during the process which entangles the two atoms with each other and prepares them in the entangled state (25). Another example in which the no-photon temporal evolution has been used to entangle atoms without a coupling between them via a populated field mode is described in [39]. In [51] this basic approach is used to describe how the state of an atom in a cavity can be teleported to an atom inside another distant cavity solely by observing emitted photons.

5.2. A more detailed discussion

In this subsection we discuss the effect of the laser pulse in more detail and assume again that $\Gamma \neq 0$. To solve the differential equations (23) we make use of an adiabatic elimination suggested by the separation of the frequency scales (4) and (6). Again, equation (23) shows that the only

[†] If the decay rate of the cavity mode is much larger than g , the states with photons in the cavity are emptied quickly and the transition between the states with $n = 0$ and $n = 1$ is inhibited by the same mechanism as that explained in the previous paragraph.

coefficients that do *not* evolve on the fast time scale g or κ are c_{0g} and c_{0a} . They change with the small rates Ω_{\pm} and Γ . Their temporal evolution is given by

$$\begin{aligned}\dot{c}_{0g} &= -i\Omega_-c_{0a} - i\Omega_+c_{0s} \\ \dot{c}_{0a} &= -i\Omega_-^*c_{0g} + i\Omega_-c_{0e} - \Gamma c_{0a}.\end{aligned}\quad (26)$$

The amplitudes of all other states, which evolve on the fast time scale g or κ , follow the slowly varying coefficients c_{0g} and c_{0a} . Therefore we can neglect their derivatives relative to the fast rates g and κ . By setting the derivatives of c_{0s} , c_{0e} , c_{1g} , c_{1s} and c_{2g} in equation (23) equal to zero we obtain the equations

$$\begin{aligned}0 &= -i\Omega_+^*c_{0g} - i\Omega_+c_{0e} + \sqrt{2g}c_{1g} - \Gamma c_{0s} \\ 0 &= i\Omega_-^*c_{0a} - i\Omega_+^*c_{0s} + \sqrt{2g}c_{1s} - 2\Gamma c_{0e} \\ 0 &= -i\Omega_-c_{1a} - i\Omega_+c_{1s} - \sqrt{2g}c_{0s} - \kappa c_{1g} \\ 0 &= -i\Omega_+^*c_{1g} - i\Omega_+c_{1e} - \sqrt{2g}c_{0e} + 2gc_{2g} - (\Gamma + \kappa)c_{1s} \\ 0 &= -i\Omega_-c_{2a} - i\Omega_+c_{2s} - 2gc_{1s} - 2\kappa c_{2g}.\end{aligned}\quad (27)$$

From figure 2 and equation (23) we can see that all other coefficients corresponding to non-DF states are smaller by at least one factor of $|\Omega_{\pm}|/g$, because they can be excited only via driving with the weak laser pulse if the states $|1s\rangle$ and $|2g\rangle$ are populated. The amplitudes of these higher states can therefore be neglected in equation (27) and we obtain a closed set of equations which can be solved easily for the coefficients of the DF states. We find

$$\begin{pmatrix} \dot{c}_{og} \\ \dot{c}_{oa} \end{pmatrix} = - \begin{pmatrix} k_1 & i\Omega_- \\ i\Omega_-^* & k_2 \end{pmatrix} \begin{pmatrix} c_{og} \\ c_{oa} \end{pmatrix} \equiv -M \begin{pmatrix} c_{og} \\ c_{oa} \end{pmatrix}\quad (28)$$

with

$$k_1 \equiv \frac{|\Omega_+|^2\kappa}{2g^2} \quad k_2 \equiv \frac{|\Omega_-|^2(2g^2 + \kappa^2)}{2g^2\kappa} + \Gamma.\quad (29)$$

The eigenvalues of M are

$$\lambda_{1,2} = \frac{k_1 + k_2}{2} \pm i|\Omega_-| \left[1 - \left(\frac{k_1 - k_2}{2|\Omega_-|} \right)^2 \right]^{1/2} \equiv \frac{k_1 + k_2}{2} \pm iS.\quad (30)$$

Making use of the formula

$$e^{-Mt} = \frac{M - \lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 t} + \frac{M - \lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2 t}\quad (31)$$

which can be checked by applying it to the eigenvectors of M (for the general case see [52]), we find

$$\begin{pmatrix} c_{og}(t) \\ c_{oa}(t) \end{pmatrix} = e^{-Mt} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = e^{-(k_1+k_2)t/2} \left[\begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos(St) - \frac{1}{2S} \begin{pmatrix} k_1 - k_2 \\ 2i\Omega_- \end{pmatrix} \sin(St) \right]\quad (32)$$

which are the coefficients of the DF states at time T under the condition of no photon emission.

After the laser pulse is turned off at time T the excitation of all non-DF states vanishes during a short transition time of the order $1/g$ and $1/\kappa$ due to the conditional temporal evolution. Therefore the state of the atoms shortly after T and under the condition that no photon was emitted can be obtained by normalizing the state $c_{0g}(T)|0g\rangle + c_{0a}(T)|0a\rangle$. It is given by

$$|\psi^0(T)\rangle = \left[\left(\cos(ST) - \frac{k_1 - k_2}{2S} \sin(ST) \right) |0g\rangle - i \frac{\Omega_-^*}{S} \sin(ST) |0a\rangle \right] / \|\cdot\|.\quad (33)$$

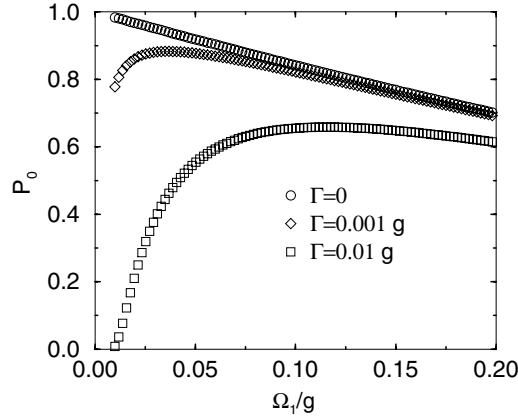


Figure 3. The probability of successful preparation of the maximally entangled DF state $|0a\rangle$ as a function of the Rabi frequency Ω_1 for $\Omega_2 = -\Omega_1$, $\kappa = g$ and three values of Γ .

The probability of a successful operation is given by the probability of no photon emission in $(0, T)$. According to equation (10) it is given by $|c_{0g}(T)|^2 + |c_{0a}(T)|^2$ and leads to

$$P_0(T, g) = e^{-(k_1+k_2)T} \left[1 - \frac{k_1 - k_2}{S} \sin(ST) \cos(ST) + \frac{(k_1 - k_2)^2}{2S^2} \sin^2(ST) \right]. \quad (34)$$

The state $|\psi^0(T)\rangle$ belongs to the DFS. Using equations (8), (14) and (15) one can show that $H_{\text{cond}}|\psi^0(T)\rangle = 0$ and $|\psi^0(T)\rangle$ is now (without the laser interaction) stable in time. If one neglects again all terms proportional to Γ and $|\Omega_{\pm}|/g$, equation (34) agrees with the result given in equation (25). The laser pulse performs a rotation on the DF qubit. As can be seen from equation (34), the sum $k_1 + k_2$ can be interpreted as the decay rate of the system. As long as this rate is much smaller than $1/T$ the probability of a successful preparation is close to 1.

5.3. Preparation of a maximally entangled state of the atoms

Finally, we discuss as an example the preparation of the maximally entangled atomic state $|a\rangle$ while the cavity is empty. Owing to equation (33) this can be done by choosing the length of the laser pulse equal to

$$T = \frac{1}{S} \text{arccot} \left(\frac{k_1 - k_2}{2S} \right) \approx \frac{\pi}{2|\Omega_-|}. \quad (35)$$

Figure 3 shows the success rate P_0 for this scheme and results from a numerical solution of equation (23). The result agrees very well with $P_0(T, 0g)$ given in equation (34) in the chosen parameter regime. For zero spontaneous emission, success rates arbitrarily close to unity can be achieved by reducing the Rabi frequency Ω_1 . However, for $\Gamma \neq 0$ this is not possible. If the laser pulse becomes very long the probability of the occurrence of spontaneous emission of a photon increases and is no longer negligible. For finite values of Γ there is an optimal value of Ω_1 for which the success rate of the preparation scheme has a maximum.

If all outgoing photons are registered and the experiment is repeated in the case of an *emission*, the fidelity of the prepared state can, for a very wide parameter regime, be very close to 1. For the parameters given in figure 3 it is always higher than 99%. If the photons

are registered only with an efficiency η smaller than 1, this fidelity has to be multiplied by $P_0/[1 - \eta(1 - P_0)]$ as can be seen from equation (11), to then give the fidelity of the prepared state in the case of no photon *detection*.

6. Manipulation of the DFS in general

In the last section we have shown that a weak enough laser pulse does not move the state of the system of two atoms out of the DFS. In this section we want to point out a physical principal behind this fact which allows a straightforward generalization of the preparation scheme to higher numbers of atoms in the cavity and other kinds of interaction. To do so we briefly review the quantum Zeno effect [41]. We also derive an *effective* Hamiltonian to describe the effect of a weak interaction in general.

The quantum Zeno effect [41] is a theoretical prediction for the behaviour of a system under rapidly repeated *ideal* measurements. It is a consequence of the projection postulate of von Neumann and Lüders [53, 54] which describes the effect of a single measurement and predicts that the probability of measuring whether the state of a system belongs to a certain subspace of states is given by its overlap with the subspace. If the outcome of the measurement is ‘yes’, the state of the system changes during the measurement process. It becomes projected onto the subspace. The quantum Zeno effect predicts that, if the time between subsequent measurements equals zero, the outcome of each following measurement is the same, even if an additional interaction which is intended to move the system into a complementary subspace is applied. The system can change only *inside* the subspace.

We now reconsider the system of N atoms inside the cavity and assume first that no laser pulse is applied to the atoms. Let us define ΔT as a time in which a photon is emitted with probability very close to unity, if the system is prepared in a non-DF state. Then the observation of the free radiation field outside the system over a time interval of the length ΔT can be interpreted as a measurement of whether the system is DF. If a photon is emitted, the system has not been in a DF state. Otherwise, its state belongs to the DFS. In the presence of a laser pulse the state of the system can be driven out of the DFS during ΔT , but as long as

$$|\Omega_i| \ll 1/\Delta T \quad (36)$$

this effect can be neglected and the observation of the free radiation field over a time interval ΔT can still be interpreted as a measurement of whether the atoms are DF to a very good approximation. This is the case in the scheme we discuss here. As has been shown in the previous section, ΔT has to be at least of the order $1/g$ and $1/\kappa$ and condition (36) leads to condition (6) given in the introduction.

In the scheme we propose the free radiation field outside the cavity is observed continuously, i.e. the time between two subsequent measurements is zero. Therefore the quantum Zeno effect can be used to predict the effect of the laser pulse on the temporal evolution of the system. It suggests that the system always remains DF if it is once prepared in a state of the DFS.

Generalization of the proposed scheme to other forms of state manipulation is straightforward. As long as the interaction is weak enough the state of the system does not move out of the DFS. The interpretation of the behaviour of the system with the help of the quantum Zeno effect can also be used to derive an *effective* Hamiltonian H_{eff} which describes the effect of a weak laser pulse on the system. We know that the state of the system can change only *inside* the DFS due to rapidly repeated measurements irrespective of whether the system

is still DF. Therefore the time development operator for a short time interval ΔT is to a good approximation given by

$$U_{\text{eff}}(\Delta T, 0) = \mathbb{P}_{\text{DFS}} U_{\text{cond}}(\Delta T, 0) \mathbb{P}_{\text{DFS}} \quad (37)$$

where \mathbb{P}_{DFS} is the projector onto the DFS. This leads to the effective Hamiltonian

$$H_{\text{eff}} = \mathbb{P}_{\text{DFS}} H_{\text{cond}} \mathbb{P}_{\text{DFS}}. \quad (38)$$

If we assume that spontaneous emission by the atoms is negligible ($\Gamma = 0$) the definition of the DF state by equations (14) and (15) allows us to simplify this equation. From equation (8) we find

$$H_{\text{eff}} = \mathbb{P}_{\text{DFS}} H_{\text{laser I}} \mathbb{P}_{\text{DFS}} \quad (39)$$

where $H_{\text{laser I}}$ describes the laser interaction and is given in equation (5). The effect of the laser on the system considered here is very different from its effect on atoms in free space. It confines the system inside the DFS and can be used to generate entanglement between the atoms in the cavity. The effective Hamiltonian for a single laser pulse depends on N different Rabi frequencies which can be chosen arbitrarily. This allows us to perform a wide range of operations such as implementation of the CNOT quantum gate between the qubits of a DFS. A concrete proposal for quantum computation using dissipation which is based on the idea discussed here in detail can be found in [55].

In the case of *two* atoms, which has been discussed in the previous section, the effective Hamiltonian (39) equals

$$H_{\text{eff}} = \frac{\hbar}{2\sqrt{2}} (\Omega_1 - \Omega_2) |0g\rangle \langle 0a| + \text{h.c.} \quad (40)$$

and leads directly to equation (24) in the previous section. The DFS of *four* atoms is six-dimensional. Using the notation given in equation (19) we find

$$\begin{aligned} H_{\text{eff}} = \frac{\hbar}{2\sqrt{2}} \left\{ (\Omega_1 + \Omega_2 - \Omega_3 - \Omega_4) \left[\frac{1}{\sqrt{2}} |0gg\rangle \langle 0x_1| + \left(\frac{2}{3}\right)^{1/2} |0x_1\rangle \langle 0x_2| \right] + \text{h.c.} \right. \\ \left. + (\Omega_1 - \Omega_2) \left(|0gg\rangle \langle 0ag| + |0ga\rangle \langle 0aa| - \frac{1}{\sqrt{3}} |0ag\rangle \langle 0x_2| \right) + \text{h.c.} \right. \\ \left. + (\Omega_3 - \Omega_4) \left(|0gg\rangle \langle 0ga| + |0ag\rangle \langle 0aa| - \frac{1}{\sqrt{3}} |0ga\rangle \langle 0x_2| \right) + \text{h.c.} \right\}. \quad (41) \end{aligned}$$

7. Conclusions

We have given an example of a DFS suitable for quantum computing and have identified a mechanism for the manipulation of states *within* the DFS which can be understood in terms of the quantum Zeno effect and allows generalization to other forms of manipulation. This concept was demonstrated in detail for the example of two two-level atoms, which led to an efficient method of entangling them and was generalized to N two-level atoms.

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References

- [1] Deutsch D 1985 *Proc. R. Soc. A* **400** 97
- [2] Deutsch D 1989 *Proc. R. Soc. A* **425** 73
- [3] Shor P W 1994 Algorithms for quantum computation: discrete log and factoring *Proc. 35th Ann. Symp. on the Foundations of Computer Science* ed S Goldwasser (Los Alamitos, CA: IEEE Computer Society) p 124
- [4] Grover L K 1997 *Phys. Rev. Lett.* **79** 325
- [5] Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **74** 4091
- [6] Poyatos J F, Cirac J I and Zoller P 1998 *Phys. Rev. Lett.* **81** 1322
- [7] Sørensen A and Mølmer K 1999 *Phys. Rev. Lett.* **82** 1971
- [8] Sørensen A and Mølmer K 2000 *Phys. Rev. A* **62** 022311
- [9] Schneider S, James D F V and Milburn G J 2000 *J. Mod. Opt.* **47** 499
- [10] Domokos P, Raimond J M, Brune M and Haroche S 1995 *Phys. Rev. A* **52** 3554
- [11] Pellizzari T, Gardiner S A, Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **75** 3788
- [12] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 2000 *Phys. Rev. Lett.* **83** 5166
- [13] Brennen G K, Caves C M, Jessen F S and Deutsch I H 1999 *Phys. Rev. Lett.* **82** 1060
- [14] Beige A, Huelga S F, Knight P L, Plenio M B and Thompson R C 2000 *J. Mod. Opt.* **47** 401
- [15] Jaksch D, Briegel H J, Cirac J I, Gardiner C W and Zoller P 1999 *Phys. Rev. Lett.* **82** 1975
- [16] Jaksch D, Cirac J I, Zoller P, Rolston S L, Cote R and Lukin M D 2000 *Preprint quant-ph/0004038*
- [17] Jonathan D, Plenio M B and Knight P L 2000 *Phys. Rev. A* at press, *quant-ph/0002092*
- [18] Steane A, Roos C F, Stevens D, Mundt A, Leibfried D, Schmidt-Kaler F and Blatt R 2000 *Preprint quant-ph/0003087*
- [19] Chuang I L, Laflamme R, Shor P W and Zurek W H 1995 *Science* **270** 1633
- [20] Plenio M B and Knight P L 1996 *Phys. Rev. A* **53** 2986
- [21] Plenio M B and Knight P L 1997 *Proc. R. Soc. A* **453** 2017
- [22] Steane A M 1996 *Proc. R. Soc. A* **452** 2251
- [23] Shor P W 1995 *Phys. Rev. A* **52** R2493
- [24] Calderbank A R and Shor P W 1996 *Phys. Rev. A* **54** 1098
- [25] Zurek W H 1993 *Prog. Theor. Phys.* **89** 281
- [26] Palma G M, Suominen K A and Ekert A K 1996 *Proc. R. Soc. A* **452** 567
- [27] Lidar D A, Chuang I L and Whaley K B 1998 *Phys. Rev. Lett.* **81** 2594
- [28] Duan L M and Guo G C 1997 *Phys. Rev. Lett.* **79** 1953
- [29] Zanardi P and Rasetti M 1997 *Phys. Rev. Lett.* **79** 3306
- [30] Lidar D A, Bacon D and Whaley K B 1999 *Phys. Rev. Lett.* **82** 4556
- [31] Duan L M and Guo G C 1998 *Phys. Rev. A* **58** 3491
- [32] Zanardi P and Rossi F 1999 *Phys. Rev. B* **59** 8170
- [33] Bacon D, Kempe J, Lidar D A and Whaley K B 2000 *Phys. Rev. Lett.* at press; *Preprint quant-ph/9908064*
- [34] Kempe J, Bacon D, Lidar D A and Whaley K B 2000 *Preprint quant-ph/0004064*
- [35] Viola L and Lloyd S 1998 *Phys. Rev. A* **58** 2733
- [36] Dugić M 2000 *Preprint quant-ph/0001009*
- [37] Radmore P M and Knight P L 1982 *J. Phys. B: At. Mol. Phys.* **15** 561
- [38] Meyer G M and Yeoman G 1997 *Phys. Rev. Lett.* **79** 2650
- [39] Plenio M B, Huelga S F, Beige A and Knight P L 1999 *Phys. Rev. A* **59** 2468
- [40] Yang G J, Zobay O and Meystre P 1999 *Phys. Rev. A* **59** 4012
- [41] Misra B and Sudarshan E C G 1977 *J. Math. Phys.* **18** 756

- [42] Itano W M, Heinzen D J, Bollinger J J and Wineland D J 1990 *Phys. Rev. A* **41** 2295
- [43] Beige A and Hegerfeldt G C 1996 *Phys. Rev. A* **53** 53
- [44] Beige A and Hegerfeldt G C 1997 *J. Phys. A: Math. Gen.* **30** 1323
- [45] Hegerfeldt G C and Wilser T S 1992 Classical and quantum systems *Proc. 2nd Int. Wigner Symp. (July 1991)* ed H D Doebner *et al* (Singapore: World Scientific) p 104.
- [46] Hegerfeldt G C and Sondermann D G 1996 *Quantum Semiclassical Opt.* **8** 121
- [47] Plenio M B and Knight P L 1998 *Rev. Mod. Phys.* **70** 101
- [48] Dalibard J, Castin Y and Mølmer K 1992 *Phys. Rev. Lett.* **68** 580
- [49] Carmichael H 1993 *An Open Systems Approach to Quantum Optics* (Berlin: Springer)
- [50] Mandel L and Wolf E 1995 *Optical Coherence and Quantum Optics* (Cambridge: Cambridge University Press)
- [51] Bose S, Knight P L, Plenio M B and Vedral V 1999 *Phys. Rev. Lett.* **83** 5158
- [52] Gantmacher F R 1986 *Matrizentheorie* (Berlin: Springer)
- [53] von Neumann J 1932 *Mathematische Grundlagen der Quantenmechanik* (Berlin: Springer)
- [54] Lüders G 1951 *Ann. Phys. Lp z.* **8** 323
- [55] Beige A, Braun D, Tregenna B and Knight P L 2000 *Phys. Rev. Lett.* at press *Preprint* quant-ph/0004043