Supplemental material for
‘Effective Floquet-Gibbs states for dissipative quantum systems’

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1. Truncation dependence

In Fig. 1, we show the dependence of the probability difference \( \Delta \text{Prob} \equiv \Delta \text{Prob}^{[0]} \) on the truncation frequency \( \omega_{\text{trunc}} \). The probability differences are stable against the change of \( \omega_{\text{trunc}} \). Although setting \( \omega_{\text{trunc}} = 1 \) gives well approximated values of \( \Delta \text{Prob} \), in this work we adopt \( \omega_{\text{trunc}} = 10 \).
2. Time-independence of $\Delta \text{Prob}^{[t]}$

In Fig. 2 we show the time dependence for the $\Delta \text{Prob}^{[t]}$ plotted in Figs. 4 and 6 in the main text. Since the quantity $\Delta \text{Prob}^{[t]}$ is time-independent, $\Delta \text{Prob}^{[0]}$ forms a good representative of the trace distance; i.e., $\Delta \text{Prob}^{[t]} = \Delta \text{Prob}^{[0]} \equiv \Delta \text{Prob}$.

**Figure 2.** Time dependence of probability differences $\Delta \text{Prob}^{[t]}$ for $\hbar \Omega = 4.6 \hbar \omega$. Left figure shows data for $\lambda^2 = 10^{-6}$ (●), $\lambda^2 = 2^6 \cdot 10^{-6}$ (○), and $\lambda^2 = 2^{14} \cdot 10^{-6}$ (*) at $h \omega_c = 100 \hbar \omega$. Right figure shows data for $h \omega_c = 0.4 \hbar \omega$ (●), $h \omega_c = 3.2 \hbar \omega$ (○), and $h \omega_c = 100 \hbar \omega$ (*) at $\lambda^2 = 0.01$. Thus, the probability differences are almost independent of $t$. 