Supplementary material for New Journal of Physics article entitled: “Carrier-induced refractive index change observed by a whispering gallery mode shift in GaN microrods”

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Abstract. In this supplementary material scanning electron microscopy measurements are performed on the microrod to determine the diameter and error range. Calculations have been carried out to confirm that a slight variation of the microrod diameter can not be responsible for the observed whispering gallery mode shift.

1. Diameter and error range of the microrod

Scanning electron microscopy (SEM) images have been recorded from the investigated microrod. The sample was tilted by 90° to have a perpendicular view to the rod axis as shown in figure 1(a). The magnification of ×9000 has the advantage that the total microrod is visible in the SEM image. Distance measurements at the top, center, and bottom have been performed in figure 1(b) showing a constant diameter of 1.74 µm. The measurements on the right show a pixel resolution of 0.02 µm. Two parallel lines reaching from top to the bottom are drawn in figure 1(c) and are already indicating the constant diameter along the microrod within the error range.

SEM images with a higher magnification with a perpendicular view on the rod axis turned out not to be reproducible. At the top, with vacuum behind the rod artificial squeezing arose (not shown here). The origin of this effect remains to be unclear. The squeezing effect was not visible at the bottom of the rod with GaN structures in the
background. Therefore, it was decided to measure a more precise diameter in a tilted view.

For high magnification \((\times 50000)\) images the sample was tilted by 60° with respect to the rod axis. In this arrangement, no artificial squeezing appeared. Sapphire in the background of the rod seems to prevent the squeezing effect. A series of SEM images in figure 2 show distance measurements at the top of the rod (a), upper central part of the rod (b), lower central part of the rod (c) and at the bottom of the rod (d). The distance measurements were divided into two sections each smaller than 1 \(\mu m\) because the software would display distance values larger than 1 \(\mu m\) in the units of \(\mu m\) (instead of nm) only with two numbers of decimal places.

A constant diameter of 1749 nm \((900 + 849 nm)\) can be measured along the rod (for the measurement on the top the focus had to be slightly adjusted which results in a distance measurement of 1748 nm). The SEM image (d) also shows the pixel resolution measurements. One pixel corresponds to 4 nm. Since the edge of the rod can not be exactly determined due to some blur at such a high resolution it is reasonable to increase the measuring error to \(\pm 0.01 \mu m\). The diameter of the microrod can be given

Figure 1. SEM images in perpendicular view to the rod axis: raw image in (a); diameter measurements at the top, center, and bottom and pixel measurements in (b); parallel lines at the edge of the microrod in (c).
Figure 2. SEM images in high magnification (×50000) from a 60° tilted microrod at the top (a), upper center (b), lower center (c), and bottom (d). Pixel measurements are included in (d).

as $1.75 \pm 0.01 \mu m$.

The SEM images give the impression, that the diameter along the rod is constant. However, a diameter variation of a few nm is still possible and can have a significant impact on the spectral position of a whispering gallery modes (WGMs).

2. Influence of the diameter on the spectral position of whispering gallery modes

In order to exclude that the diameter is responsible for the observed WGM shift it is important to check the influence of the diameter on the spectral position of WGMs. Calculations have been performed based on equations (1) and (2) of the main article on the diameter dependent spectral position of triangular transversal magnetic (TM) WGMs. Figure 3 is showing the results for triangular WGMs in the 2.00-3.36 eV spectral range and for rods having an inner diameter of 1000-3000 nm. Furthermore, $E_0$ in equation (2) was set to 3.429 eV. Each line represents a mode number $N$ (the line in the upper right corner represents $N = 50$). As visible, an increase of the diameter
would always lead to a mode shift towards lower energy. However, there is not only a $d \sim \lambda_{\text{WGM}} \sim 1/E_{\text{WGM}}$ dependence, which leads to a smaller slope at lower energies. The spectral energy dependent refractive index is responsible, that the slope is also decreasing close to the band gap of GaN at $\sim 3.4 \text{eV}$. To visualize this issue a black dashed line is drawn as a guide to the eye showing that the largest slope is between 3.2-2.8 eV. Outside this range the slope is smaller.

Concrete calculations have been carried out based on the investigated microrod: a microrod having an inner diameter of 1.780 µm (blue solid line in the graph) is assumed and compared with the calculated WGMs with a microrod having a diameter of 1.770 µm. As discussed above, a diameter change by 10 nm might not be visible by SEM measurements. The diameter change leads to a blue shift of 11, 13, 13 and 7 meV for the TM$_{19}$, TM$_{26}$, TM$_{30}$, and TM$_{34}$ WGM, respectively. Comparing these values with the experimental values in figure 3(d) of the main article it is clear, that a variation of the diameter along the rod can not be responsible for the WGM shift.

**Figure 3.** The graph shows the diameter dependent spectral position of triangular TM WGMs.