Supplementary Information for “Interfacing GHz-bandwidth heralded single photons with a warm vapour Raman memory”

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1 March 2015

PACS numbers: 42.50.Ct, 42.50.Ex, 03.67.Lx, 42.65.Hw

Keywords: quantum memory, single photon storage, room-temperature, Raman interaction, quantum computer, quantum communication, temporal multiplexer


1. Storage with large time-bandwidth products versus multi-mode storage

In this paper, we refer to the capability of the memory to store and retrieve the outcomes of heralded probabilistic devices, such as single photon sources, as temporal multiplexing. This type of multiplexing allows to temporally synchronise these probabilistically operating devices by holding the outcome of one device and retrieving it on-demand upon successful operation of another device. For this multiplexing strategy the time-bandwidth product \(B\) is a key metric [1], because it determines the number of trials that can be conducted to obtain a successful outcome of the second device, while holding in storage the outcome of the first in the memory. The time each such trial takes can be assumed equal to the duration of the device’s output signal. In turn, this corresponds to the duration \(\Delta t\) of the control pulses for the Raman memory, which are inversely proportional to the memory acceptance bandwidth \(\delta\), set by the frequency bandwidth of the control. Using the memory storage time \(\tau\) as the duration over which information can be held in the memory, the maximum number of trials equals the time-bandwidth
product $B = \tau \delta$. Notably, the temporal synchronisation can be performed with memories that are only capable to store a single signal mode.

One has to distinguish from this type of temporal multiplexing the multi-mode storage capacity of a memory. In the temporal domain, multi-mode capacity refers to the number of subsequent signal pulses a memory is capable to store from a signal pulse train. Examples for such systems are rare-earth ion doped crystals, operated with the atomic-frequency comb protocol [2–5]. The advantage of such a multi-mode memory lies in speeding up entanglement distribution in a quantum repeater chain [6], which has recently been demonstrated for frequency-domain multiplexing [7]. While the Raman memory protocol is inherently single-mode, a chain of Raman memories could be used to store multiple signal modes, as recently proposed in [8].

2. Experimental set-up

Here we describe in detail the two setup configurations employed to operate the experiment with heralded single photon and coherent state inputs. While the system must be operated in feed-forward mode for the storage and retrieval of heralded single photons, experiments with coherent state input signals can be implemented by triggering the experiment off an internal photodiode of the Ti:Sa master laser [9, 10]. The latter is significantly simpler to implement, since it does not require simultaneous operation of the SPDC source, for which reason we use this configuration for the coherent state storage measurements. Fig. S1 (a) and (b) contrast the differences in the components and signal paths for both triggering methods. Additionally, Fig. S1 (c) illustrates the full experimental set-up, comprising all critical components for implementation of both triggering methods.

Common pulse preparation stage For both triggering mechanisms a mode-locked Ti:Sa oscillator (Spectra Physics Tsunami), emitting an 80 MHz train of 360 ps pulses at 852 nm with 1.2 W average power (IR), is used as master laser, whose frequency and beam pointing are actively stabilised. The Ti:Sa output (IR) is frequency-doubled in a 2 mm PPKPT crystal (SHG, violet panel) with $\eta_{\text{SHG}} \approx 0.005 \%$ efficiency to generate the UV pump at 426 nm (blue line) for the PPKTP waveguide SPDC source (yellow panel). The remaining, unconverted Ti:Sa pulses, at 852 nm, are recovered to provide the memory control pulses by separation from the SHG on a dichroic mirror. From this pulse train the control field is generated by picking two consecutive pulses for 12.5 ns storage time, which is done by a Pockels cell (PC, pink panel). Depending on the input signal type, the PC is triggered either on SPDC herald detection events or on a clock signal generated from an internal photodiode inside the Ti:Sa cavity. Pulses, selected by the PC, are subsequently coupled into 10 m long single mode fibre (SMF) with an efficiency of $\eta_{\text{control}} \approx 53 \%$ for achieving sufficient time delay to temporally overlap them with the SPDC signal pulses in the Cs cell.
**Single-photon source** In the case of heralded SPDC photon storage with feed-forward memory operation (Fig. S1 (a)), the UV pump pulses, with \( \approx 1 \text{ mW} \) average power, are coupled into the \( 3 \mu m \times 6 \mu m \times 2 \text{ cm} \) PPKTP waveguide with a coupling efficiency \( \eta_C \approx 10\% \). The PPKTP waveguide (AdvR) produces near-degenerate, orthogonally polarised signal and idler photon pairs at 852 nm with a phasematching bandwidth of \( \sim 0.1 \text{ nm} \) (\( \sim 40 \text{ GHz} \)). SPDC signal (purple) and idler (red) photons are SMF coupled behind the waveguide output with \( \eta_{\text{signal}} = 72 \pm 2\% \) and \( \eta_{\text{herald}} = 73 \pm 2\% \) efficiency, respectively. Subsequently, the idler is frequency-filtered (purple panel) using a sequence of three air-gap Fabry-Perot (FP) etalons and a 0.01 nm bandwidth volume holographic grating filter (ONDAX). Two of the etalons have a free-spectral range (FSR) of 18.2 GHz, whereby the third has an FSR = 103 GHz and is used in a double-pass configuration (the entire sequence is labelled as herald filter in Fig. 1 (a) of the main text). Setting the filter resonance to \( \Delta = +24.4 \text{ GHz} \) detuning from the \( 6^2S_{1/2} F = 3 \rightarrow 6^2P_{3/2} \) transition projects the SPDC signal into two-photon resonance with the control (see Fig. 1 (e) in the main text). The peak transmission of the herald filter stage is \( T = 31 \pm 2\% \). It is measured by 360 ps Ti:Sa laser pulses, which are resonant with the etalons and employed for alignment. Its FWHM filter bandwidth is \( \sim 0.7 \text{ GHz} \).

**Feed-forward operation** Idler detection on a single-photon counting module (Perkin Elmer), SPCM D\(_T\) in Fig. S1 and Fig. 1 (a) of the main text, generates electronic trigger pulses (thin red line Fig. S1 (c)), which are sent to a digital delay generator unit (SRS DG535). In turn, the digital delay generator sends appropriately delayed gating pulses to the FPGA and TAC/MCA modules for coincidence counting with the signal SPCMs D\(_H\) and D\(_V\), as well as the Pockels cell trigger unit. The delay generator provides fine adjustment for pulse picking with the PC to synchronise the arrival times between SPDC signal photons and the control read-in pulse in the memory cell. We fix the herald detection rate between 5.3 – 7.3 kHz, using an UV pump power of \( \sim 1 \text{ mW} \) for the SPDC source. This limit on the experimental repetition rate is due to the PC (see below), leading to a detected signal-idler coincidence rate of \( \sim 30 \text{ Hz} \) (see tables 1 and 2). Meanwhile, the heralded SPDC signal photons propagate in an 83 m and a 7.97 m long single-mode fibre (SMF), before being combined with the control field on a PBS in front of the Cs cell. Both SMFs provide sufficient temporal delay to compensate the electronic delays of SPCM D\(_T\), the digital delay generator and the PC.

**Coherent state inputs** In order to switch between heralded SPDC photons and coherent states as memory input signals, a PBS is used to couple the coherent state pulses into the second signal SMF. Notably, only one input type is applied at any one time. Thus, for heralded SPDC storage experiments, the coherent state arm is blocked and vice versa. Coherent-state input signal pulses are produced by re-directing a small fraction of the picked control beam through an electro-optic modulator (EOM), generating 9.2 GHz sidebands on the first (write) pulse (beige panel). An air-spaced
etalon with an FSR = 38.86 GHz is used to isolate the red-detuned sideband required for two photon resonance. Here the digital delay generator is triggered by the Ti:Sa intra-cavity photodiode signal (Fig. S1 (b)). Consequently, switching the experimental control electronics between the configurations for heralded SPDC photon and coherent state input signals only requires a change in the settings of the digital delay generator. Additionally the digital delay generator also switches the EOM. Using a fast rf-switch (6 ns rise time), the EOM is only supplied with the required 9.2 GHz modulation frequency for the input time bin. As a result, coherent state input signal pulses are only present in the read-in time bin, whereas the control pulses for storage and retrieval exist for both time bins [9, 10].

**Memory preparation** In both configurations, the signal is stored in a 7.5 cm long Cs cell, which is surrounded by a single layer magnetic shield [9, 10] (orange panel). The Cs vapour is heated to 70 °C, with a cell cold spot at 67.5 °C. The atoms are initially prepared in the $6^2S_{1/2} F = 4$ hyperfine manifold by optical pumping with a CW frequency-stabilised external cavity diode laser (grey panel). Its beam, with $\sim 3$ mW average power, is counter-propagating along the control beam path and is resonant with the $6^2S_{1/2} F = 3 \leftrightarrow 6^2P_{3/2} D_2$-transition (the upper hyperfine structure is not resolved due to Doppler broadening). We measure a state preparation efficiency $\geq 95\%$, which agrees well with our theoretical model. The diode laser is on continuously during the heralded single photon storage experiment. For coherent state storage, the optical pumping is turned off by an acusto-optic modulator (AOM), also gated by the digital delay generator. This difference results from the long AOM switching times in our setup configuration of $\sim 1 \mu$s, which exceeds the available SMF time delay of the heralded SPDC signal photons. For coherent state storage this limitation does not arise. Here, in contrast to the probabilistic occurrences of SPDC herald detection events, the Ti:Sa photodiode provides a deterministic repetition rate of 80 MHz. Thus the AOM switch-off can be postponed by one repetition cycle to turn off the optical pumping for the following storage event. To this end the 5.72 kHz repetition rate for memory experiments leaves 170 $\mu$s between storage events, which suffices for AOM switching. Notably, while the presence of the optical pumping slightly reduces the overall memory efficiency $\eta_{\text{tot}}$ for heralded SPDC photons (see below), we have certified that this difference in the optical pumping timing has no influence on the photon statistics. It neither affects the $g^{(2)}$ of the retrieved signal, nor the $g^{(2)}$ of the noise in both time bins.

**Signal detection** After passing through the memory cell, the control is separated from the signal first by polarisation filtering on a calcite polarising beam displacer (PBD), providing control attenuation of $\geq 40$ dB. Conveniently the PBD is also used for insertion of the optical pump into the Cs cell. Once transmitted through the PBD, the signal is spatially filtered using a SMF, with a coupling efficiency of $\eta_{\text{sig.filt.}} \approx 88\%$, and frequencyfiltered by a sequence four air-gap Fabry-Perot etalons (green panel, labelled as signal filter in Fig. 1 (a) of the main text). The first three etalons have
an FSR = 18.2 GHz, whereby the last etalon has an FSR = 103 GHz and is again operated in double-pass. The etalon chain suppresses any control leakage through the PBD into the signal mode by 90 dB with respect to the signal, which has an on-resonance transmission of $T \approx 10\%$ measured with 360 ps pulses from the Ti:Sa laser. Notably, the filtering also removes any collisional fluorescence noise generated by the control fields. For measuring the heralded autocorrelation function $g^{(2)}$, the signal is thereafter split 50 : 50 into two spatial modes on a PBS. Each mode is coupled into multi-mode fibre (MMF), leading to two single-photon counting modules, SPCMs $D_H$ and $D_V$ (Perkin Elmer). Detection events on both SPCMs are counted by a field-programmable gate array (FPGA) in coincidence with output events of the digital delay generator. This way, the detection is independent of the specific experimental triggering mechanism. Furthermore, a time-to-amplitude-converter (TAC) - multi-channel-analyser (MCA) system is used to record arrival time histograms between trigger events from the digital delay generator and signal photons, registered on $D_H$ in the horizontally-polarised detection arm. Binning the time differences between events registered by both detectors yields the histograms shown in Fig. 2 of the main text.

3. Heralding efficiency

The heralding efficiency of $\eta_{\text{herald}} = 0.22$, quoted in the main text, represents the probability of observing a signal photon at the memory input facet, upon detection of a herald photon. In other words, this corresponds to the number of photons sent into the memory when detecting a herald photon, which triggers a memory experiment by picking a control pulse sequence with the Pockels cell. The heralding efficiency is recorded by turning off the memory interaction (blocking the control), while still optically pumping the Cs atoms. The accumulated coincidences between the herald and each of the signal detectors $c_{\text{her,sig}} = c_{\text{her,H}} + c_{\text{her,V}}$ are normalised by the number $c_{\text{her}}$ of counts registered on the herald detector $D_T$. This ratio is corrected for the combined transmission of the Cs cell, the optics behind the memory output and the transmission of the signal filter stage, $T_{\text{tot}} \approx 10\%$, as well as the detection efficiency of the SPCMs $D_H$, $D_V$, which are assumed as $\eta_{\text{SPCM}} \approx 50\%$. From these numbers the heralding efficiency is obtained by

$$\eta_{\text{herald}} = \frac{c_{\text{her,sig}}}{c_{\text{her}} \cdot T_{\text{tot}} \cdot \eta_{\text{SPCM}}}.$$ 

Notably this definition assumes negligible contributions from higher order SPDC emissions, which is justified in our case and can be seen by the low $g^{(2)} = 0.016 \pm 0.004$ measured for heralded SPDC input signals (main text).

4. Memory efficiency

The memory efficiency is calculated from the coincidence count rates between signal and herald SPCMs for the memory input and output time bins, $c_{\text{in/out}}^{\text{her,H/V}}$. These are measured
for 4 different input field combinations \((l)\), referred to as experimental settings, to access all contributions to the detected signal (see also Fig. 2 in the main text):

(i) Memory on \((scd)\): signal \((s)\), control \((c)\) and optical pumping \((d)\) are sent into the Cs cell (blue lines in Fig. 2).

(ii) Input signal \((sd)\): control is blocked (green lines in Fig. 2).

(iii) Noise \((cd)\): input signal is blocked (red lines in Fig. 2).

(iv) Optical pumping background \((d)\), i.e. signal and control are blocked.
For the arrival time histograms the $c_{\text{in/out}}$ are the integrated pulses, normalised by the sum of all herald trigger events $c_{\text{her}}$. The memory efficiency is given by

$$\eta_{\text{tot}} = \frac{(c_{\text{sd}}^{\text{out}} - c_{\text{cd}}^{\text{out}} - (c_{\text{sd}}^{\text{out}} - c_{\text{d}}^{\text{out}}))}{(c_{\text{sd}}^{\text{in}} - c_{\text{d}}^{\text{in}})},$$  \hspace{1cm} (1)

whereby the error on $\eta_{\text{tot}}$ derives from Gaussian error propagation using Poissonian count rate errors on the $c_{\text{in/out}}$. Notably, $c_{\text{d}}^{\text{in/out}}$ is negligible and has thus been omitted in Fig. 2. The memory efficiency corresponds to the product of the read-in efficiency

$$\eta_{\text{in}} = \frac{c_{\text{sd}}^{\text{in}} + c_{\text{cd}}^{\text{in}} - c_{\text{d}}^{\text{in}} - c_{\text{scd}}^{\text{in}}}{c_{\text{sd}}^{\text{in}} - c_{\text{d}}^{\text{in}}}$$

and the retrieval efficiency $\eta_{\text{retr}} = \frac{\eta_{\text{tot}}}{\eta_{\text{in}}}$. For heralded single photons and coherent states, with similar input photon number of $N_{\text{in}} = 0.23$ photons per pulse, the following efficiency values are obtained:

<table>
<thead>
<tr>
<th>Signal type</th>
<th>$\eta_{\text{in}}$</th>
<th>$\eta_{\text{retr}}$</th>
<th>$\eta_{\text{mem}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heralded single photons</td>
<td>39 ± 3%</td>
<td>54 ± 7%</td>
<td>21 ± 2%</td>
</tr>
<tr>
<td>Coherent states</td>
<td>51 ± 2%</td>
<td>57 ± 3%</td>
<td>28 ± 2%</td>
</tr>
</tbody>
</table>

The difference between both signal types is firstly due to a residual mode mismatch between the spectra of heralded SPDC photons and the control pulses, which leads to a decrease in memory read-in efficiency of SPDC signal photons compared to storage of coherent state inputs. The mismatch arises from filtering of the SPDC idler photons, which does not perfectly project the signal into the spectral mode of the control. Secondly the presence of optical pumping during storage for heralded single photons also contributes by slightly reducing the retrieval efficiency when compared to measurements on coherent states, where the diode laser is turned off during the storage time.

A crude model of the memory, as a beam splitter interaction [11] followed by a two-mode squeezer [12], provides a conservative estimate of the gain experienced by the signal field due to FWM. We obtain $\eta_{\text{tot}} = \eta_{\text{FWM}} = G \cdot \eta_{\text{tot,0}}$, where $\eta_{\text{tot,0}}$ is the efficiency of the beam splitter interaction describing a noise-free memory, and where $G = 1 + \epsilon_{\text{out}}$ is the gain of a two-mode squeezer that produces $\epsilon_{\text{out}}$ spontaneous noise photons per pulse. For our experiment, with $\epsilon_{\text{out}} = 0.15 \pm 0.05$ noise photons per pulse, this would correspond to an inflation of our measured efficiencies by a factor of 1.15 over the noise-free case. However, the FWM interaction is not a pure two-mode squeezer and the gain is likely to be smaller than this.

### 5. FWM as the important noise source

In our analysis FWM is the solely important noise source. In general one could expect spontaneous Raman scattering (SRS) from residual incoherent population, left in the storage state by imperfect optical pumping, to contribute as an additional noise source besides FWM. In practice this effect is small. SRS, generated by the control from such
residual incoherent population, is isotropic and very little of it couples into the SMF used to collect the signal at the memory output. Notably, there is a 0.5 m distance between the Cs cell and the lens used to collimate the signal for SMF coupling. So the collected solid angle is small. To certify this, we have conducted the following tests:

- SRS can, for instance, be distinguished from FWM by observation of the FWM anti-Stokes channel, which is located at $\Delta_{AS} = 24.4$ GHz detuning from the Cs resonance (see Fig. 1 (f) of the main text). While SRS only emits noise into the Stokes channel, FWM necessitates the simultaneous emission of anti-Stokes noise. We observe noise in both channels and obtain a ratio between Stokes and anti-Stokes noise as predicated by our model, assuming solely FWM noise, i.e., using perfect state preparation ($w = 1$ in Appendix B of the main text).

- Moreover, placing the cell in a solenoid and applying a magnetic field along the optical axis allows to distinguish SRS from FWM. Since FWM relies on a spin-wave coherence, just as the Raman memory does, a magnetic field leads to dephasing and noise reduction. SRS on the other hand remains unaffected by the magnetic field. We have observed and verified this feature experimentally.

6. $g^{(2)}$ measurement

6.1. Measurement procedure for $g^{(2)}$

To determine the $g^{(2)}$-values, presented in Fig. 3 of the main text, with sufficient precision, we need to collect count rate statistics on the coincidence probabilities $p_{\text{in/out}}^{\text{her,H/V}} = \frac{c_{\text{in/out}}^{\text{her,H/V}}}{c_{\text{her}}}$ and triple coincidence probabilities $p_{\text{her,H,V}}^{\text{in/out}} = \frac{c_{\text{in/out}}^{\text{her,H,V}}}{c_{\text{her}}}$ over long time scales. Long measurement times are necessary due to a technical limitation on the experiment repetition rate $f_{\text{rep}}$, which arises from the Pockels cell. At too high a repetition rate, the Pockels cell pulse-picking extinction ratio, i.e. the amplitude ratio of unpicked pulses with respect to picked pulses, degrades, for which reason experiments are conducted with $f_{\text{rep}} = 5.3 - 7.3$ kHz. Notably $f_{\text{rep}}$ equals the number of herald detection events $c_{\text{her}}$ and thus sets an upper boundary on the brightness requirements of the SPDC source.

The resulting limitation on detected coincidence counts $c_{\text{her,H/V}}^{\text{in/out}}$ and $c_{\text{her,H,V}}^{\text{in/out}}$ between the herald SPCM and the H-, V-signal SPCMs ($D_T$, $D_H$ and $D_V$ in Fig. 1) make it necessary to run the experiment over several days for each input signal type (heralded SPDC photons, coherent states at different mean photon numbers). Table 1 and 2 show the total measurement time $\Delta t_{\text{meas}}$ for each input signal type and the average detected coincidence count rates for double and triple coincidences.

Comparability between different measurement days is ensured by following a procedure, where a set sequence of experiment runs is conducted during each day. In each run the $g^{(2)}$ of the signal input is measured first for $\approx 10$ min. by blocking the control
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Table 1. Observed average coincidence rates $c^\text{in}_{\text{her,H}}$ between $D_T$, $D_H$ and coincidence rates $c^\text{in}_{\text{her,V}}$ between $D_T$, $D_V$. Numbers are shown for heralded SPDC (SPDC $scd$) and coherent state (Coh. $scd$) input signals with the memory interaction active, as well as for FWM noise only, i.e. detected noise when no signal is sent into the memory (Noise $cd$).

<table>
<thead>
<tr>
<th>Signal type</th>
<th>$c^\text{in}_{\text{her,H}}$ [Hz]</th>
<th>$c^\text{in}_{\text{her,V}}$ [Hz]</th>
<th>$c^\text{out}_{\text{her,H}}$ [Hz]</th>
<th>$c^\text{out}_{\text{her,V}}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPDC $scd$ $N_{in} = 0.22$</td>
<td>28.37 ± 0.08</td>
<td>28.6 ± 0.08</td>
<td>29.1 ± 0.1</td>
<td>29.4 ± 0.1</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 0.23$</td>
<td>26.49 ± 0.05</td>
<td>25.8 ± 0.05</td>
<td>32.49 ± 0.07</td>
<td>32.31 ± 0.07</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 0.49$</td>
<td>51.91 ± 0.09</td>
<td>51.4 ± 0.1</td>
<td>34.4 ± 0.1</td>
<td>34.1 ± 0.1</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 0.91$</td>
<td>80.8 ± 0.15</td>
<td>80.6 ± 0.17</td>
<td>55.9 ± 0.2</td>
<td>55.5 ± 0.2</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 1.66$</td>
<td>127.7 ± 0.35</td>
<td>130 ± 0.4</td>
<td>61.7 ± 0.2</td>
<td>62.6 ± 0.2</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 2.16$</td>
<td>169.4 ± 0.4</td>
<td>170 ± 0.4</td>
<td>80 ± 0.3</td>
<td>80.4 ± 0.2</td>
</tr>
<tr>
<td>Noise $cd$</td>
<td>7.58 ± 0.02</td>
<td>7.26 ± 0.02</td>
<td>19.13 ± 0.05</td>
<td>18.95 ± 0.05</td>
</tr>
</tbody>
</table>

Table 2. Total measurement times and observed average triple coincidence rates $c^\text{out}_{\text{her,H,V}}$ between $D_T$, $D_H$ and $D_V$, respectively. Signal type notation is the same as in Table 1.

<table>
<thead>
<tr>
<th>Signal type</th>
<th>$\Delta t_{\text{meas}}$ [min.]</th>
<th>$c^\text{in}_{\text{her,H,V}}$ [Hz]</th>
<th>$c^\text{out}_{\text{her,H,V}}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPDC $scd$ $N_{in} = 0.22$</td>
<td>295</td>
<td>0.126 ± 0.003</td>
<td>0.229 ± 0.004</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 0.23$</td>
<td>345</td>
<td>0.166 ± 0.003</td>
<td>0.309 ± 0.004</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 0.49$</td>
<td>160</td>
<td>0.567 ± 0.008</td>
<td>0.325 ± 0.006</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 0.91$</td>
<td>171</td>
<td>1.29 ± 0.01</td>
<td>0.84 ± 0.01</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 1.66$</td>
<td>116</td>
<td>3.11 ± 0.03</td>
<td>0.95 ± 0.01</td>
</tr>
<tr>
<td>Coh. $scd$ $N_{in} = 2.16$</td>
<td>147</td>
<td>5.38 ± 0.03</td>
<td>1.54 ± 0.02</td>
</tr>
<tr>
<td>Noise $cd$</td>
<td>1702</td>
<td>0.017 ± 0.0004</td>
<td>0.12 ± 0.001</td>
</tr>
</tbody>
</table>

field ($scd$). Subsequently, the $g^{(2)}$ for the transmitted and retrieved signal with active memory interaction ($scd$) is recorded for $\approx 30$ min. Finally, the $g^{(2)}$ of the FWM noise is measured for $\approx 30$ min. by blocking the signal input to the memory ($cd$).

For each run, we also calculate the memory efficiency using Eq. 1. Notably the counts $c^{\text{in/out}}$, entering Eq. 1, are scaled according to the measurement times of each setting $l$. The quoted efficiencies $\eta_{\text{tot}}$ represent the average memory efficiencies over all runs. Between each measurement run, the spatial overlap between signal, control and optical pumping is optimised for maximum memory efficiency. Furthermore, the filter stage alignment as well as the source heralding efficiency are inspected and re-optimised, if required. This procedure minimises systematic drifts in the experimental apparatus.
6.2. Calculation of $g^{(2)}$

In the calculation of the $g^{(2)}$-values for all input signal types, as shown by the datapoints in Fig. 3 of the main text, with the exception of the input $g^{(2)}$ for coherent states, we use Poissonian counting statistics and sum the observed count rates over all measurement runs $j$. Consequently the herald counts $c_{\text{her}}$, the coincidence counts $c_{\text{in/out}}^{\text{her},H/V}$ as well as the triple coincidence counts $c_{\text{in/out}}^{\text{her},H,V}$ are given by

$$c_{k,l}^{t,j} = \sum_{N_r=1}^{N_r} c_{t,j,k,l},$$

with $k \in \{(\text{her}), (\text{her}, H), (\text{her}, V), (\text{her}, H, V)\}$ and the total number of measurement runs $N_r$ for each setting $l \in \{\text{scd, sd, cd}\}$ and time bin $t \in \{\text{in, out}\}$. The detection probabilities $p_{t,k,l}=c_{t,k,l}/c_{\text{her},l}$, with $k \in \{(\text{her}, H), (\text{her}, V), (\text{her}, H, V)\}$, and in turn also $g^{(2)}_{l,t}$ are obtained using the summed coincidence counts $c_{t,k,l}^{t,j}$. The errors are given by Gaussian error propagation, using the Poissonian errors for the summed coincidence counts $\Delta c_{t,k,l}^{t,j} = \sqrt{c_{t,k,l}^{t,j}}$. This procedure is justified as the observed count rates for each type of input signal (i.e. heralded SPDC or coherent state with fixed photon number) are similar for each measurement day.

We test the validity of this procedure by also determining the $g^{(2)}_{j,l,t}$ values obtained for each measurement run $j$, by

$$g^{(2)}_{j,l,t} = \frac{c_{j,(\text{her}, H,V)}^{t} \cdot c_{j,\text{her}}^{t}}{c_{j,(\text{her}, H)}^{t} \cdot p_{(\text{her},V),l}^{t}},$$

(2)

Performing a double-sided, one-sample Student T-test on the $g^{(2)}_{j,l,t}$ with $g^{(2)}_{l,t}$ from Eq. 2 as the assumed population mean, yields that the Null hypothesis $(H_0)$ of $g^{(2)}_{l,t}$ being the mean of the $g^{(2)}_{j,l,t}$ cannot be rejected for any input signal type $l$ and time bin $t$ with $\geq 95\%$ confidence. The applicability of the T-test is also tested by performing a Shapiro-Wilk test on the $g^{(2)}_{j,l,t}$, where the Null hypothesis of the $g^{(2)}_{j,l,t}$ being normally distributed cannot be rejected with $\geq 95\%$ confidence for all input signal types $l$ and time bins $t$.

To obtain $g^{(2)}_{\text{in}} = 1.01 \pm 0.01$ for coherent states inputs, i.e. the $g^{(2)}$ when the control field is blocked (sd), we combine the data for all measured input photon numbers $N_{\text{in}}$ and calculate a weighted average over the $g^{(2)}_{j,l,t}$, with the total measurement time $\Delta t_{\text{meas}}$ for each input photon number $N_{\text{in}}$ as a weighting factor. Using the weighted mean over the $g^{(2)}_{j,l,t}$ for all individual measurement runs $j$, instead of the sum over all $j$, is required as the $c_{j,k,l}^{t}$ differ between the different input photon numbers $N_{\text{in}}$ (see table 1).

6.3. Statistical significance of $g^{(2)}$ difference between heralded SPDC photons and coherent states

To evaluate the statistical significance of the drop in $g^{(2)}$ between coherent states at $N_{\text{in}} = 0.23$ and heralded single photons at $\eta_{\text{herald}}$ retrieved from the memory (Fig. 3 in the main text), we perform a one-sided, two-sample Welch test on the individual
\( g_j^{(2)} = g_{j,\text{scel.out}}^{(2)} \) for both input signal types. A Welch test is chosen since the \( g_j^{(2)} \) for coherent states and SPDC signal photons have unequal sample sizes, are drawn from different populations and have different variances. Testing the Null hypothesis \( (H_0) \) that the \( g_j^{(2)} \)-samples for coherent states and heralded single photons have the same population mean, we obtain a rejection of \( H_0 \) with a confidence level of \( \geq 99.7\% \) \( (p\text{-value} = 8.7 \cdot 10^{-4}) \), which corresponds to a significance of \( \geq 3 \) standard deviations. Similar results are obtained when replacing the \( g_j^{(2)} \) of the coherent state signal with those of the FWM noise. To complete the argument we furthermore test the \( g_j^{(2)} \) for coherent states at \( N_{\text{in}} = 0.23 \) against those of the FWM noise, again under \( H_0 \) that the population means are equal. In this case, we cannot reject \( H_0 \) with any reasonable level of confidence \( (p\text{-value} = 0.967) \).

In conclusion, we can thus state that there is a statistically significant difference between the \( g_j^{(2)} \)-values of retrieved heralded single photons with respect to those of retrieved coherent states at equal photon number, as well as with respect to the \( g_j^{(2)} \) of the FWM noise. In contrast, there is no significant difference between the coherent state and FWM \( g_j^{(2)} \)-values.

References
