Supplementary Material: Turbulence in the Two-dimensional Fourier-truncated Gross-Pitaevskii Equation

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1. Figure F1

In figure F1 (F1), we have plotted the time evolution of the total number of vortices \( N_v \) for our DNS runs A1-A3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{(F1) Plots showing the time evolution of the total number of vortices \( N_v \) (both vortices and antivortices) from our DNS runs A1 (purple curve), A2 (green curve) and A3 (sky-blue curve).}
\end{figure}

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2. Figure F2

Figure 2 shows the time evolution of $k_{hd}(t)$ for the DNS runs A1-A3.

![Figure 2](image_url)

**Figure 2.** (F2) Plot showing the time evolution of the wave number $k_{hd}$ from our DNS runs A1 (purple curve), A2 (green curve) and A3 (sky-blue curve); here, $k_{hd} = 2\pi \lambda^{1/2} = N_v^{1/2} \Delta k$. A brown horizontal line represents the wave number $k = 1\Delta k$, which is related to the inverse of the system-size length scale.

3. Figure F3

Figure 3 shows the PDFs of $v_x$ and $v_y$, the Cartesian components of the velocity, for our DNS runs D10 ($E^{D10} > E_{BKT}^{128^2}$, see Table 4 in the main text) at $t = 9500$.

![Figure 3](image_url)

**Figure 3.** (F3) Semilog (base 10) plots of the PDFs of the $x$ (red circles) and $y$ (green squares) components of the velocity from our DNS run D10 ($N_c = 128$) at $t = 9500$, corresponding to the initial condition of type IC1 (see Table 4 in the main text). The dashed lines indicate power-law fits ($\sim v_i^{-\gamma}$) to the left (blue-dashed line, $\gamma = 2.6$) and right (orange-dashed line, $\gamma = 2.6$) tails of the PDFs, we show fits only for $i = x$. 
4. Video S1

This video illustrates the time evolutions, from our DNS run A1, of the following: [top left panel] semilog (base 10) plots of the PDFs of the $x$ (red circles) and $y$ (green squares) components of the velocity (cf. figure 2 (a)-(c)); [top right panel] pseudocolor plots of the vorticity $\omega = \nabla \times \mathbf{v}$ (with high-$k$ modes filtered out); [bottom left panel] log-log (base 10) plots of the spectra $E_{kin}^i(k)$, $E_{kin}^c(k)$, and $E_q(k) + E_{int}(k)$ (cf. figures 4 (a), 5 (a)-(c), and 7 (a)-(c); the orange-dashed line shows a $k$ power-law behaviour); [bottom right panel] log-log (base 10) plots of the spectra $n(k)$ (cf. figures 8 (a)-(c); a $k^{-1}$ power law is shown by the orange-dashed line).

5. Video S2

This video illustrates the time evolution of log-log (base 10) plots of the compensated, incompressible kinetic energy spectra $k^{5/3}E_{kin}^i(k)$ from our DNS runs A1 (purple curve), A2 (green curve), A3 (sky-blue curve), and A4 (brown curve); a $k^{-5/3}$ power law in $E_{kin}^i(k)$ is shown by the orange-dashed line to guide the eye (for uncompensated versions of these spectra see figures 4 (a)-(d)).

6. Video S3

This video illustrates the time evolution of log-log (base 10) plots of the spectrum $E_{kin}(k)$ from the following DNS runs: [panel V1] - A1 (purple curve), A2 (green curve), and A3 (sky-blue curve); [panel V2] A6 (purple curve), A9 (green curve), and A10 (sky-blue curve); [panel V3] A11 (purple curve), A12 (green curve), and A13 (sky-blue curve); [panel V4] A1 and A5-A8 (with $N_e^2 = 1024^2, 512^2, 256^2, 128^2, \text{and } 64^2$).