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Entanglement-storage units

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Entanglement-storage units

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Abstract. We introduce a protocol based on optimal control to drive many-body quantum systems into long-lived entangled states, protected from decoherence by large energy gaps, without requiring any *a priori* knowledge of the system. With this approach it is possible to implement scalable entanglement-storage units. We test the protocol in the Lipkin–Meshkov–Glick model, a prototype many-body quantum system that describes different experimental setups, and in the ordered Ising chain, a model representing a possible implementation of a quantum bus.

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1. Introduction

Entanglement represents the manifestation of correlations without a classical counterpart and is regarded as the necessary ingredient at the basis of the power of quantum information processing. Indeed, quantum information applications such as teleportation,

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quantum cryptography or quantum computers rely on entanglement as a crucial resource [1]. Within the current state of the art, promising candidates for truly scalable quantum information processors are considered architectures that interface hardware components playing different roles such as, for example, solid-state systems as stationary qubits combined in hybrid architectures with optical devices [2]. In this scenario, the stationary qubits are a collection of engineered qubits with desired properties, as decoupled as possible from one another to prevent errors. However, this architecture is somehow unfavorable for the creation and conservation of entanglement. Indeed, it would be desirable to have hardware where entanglement is ‘naturally’ present and that can be prepared in a highly entangled state that persists without any external control: the closest quantum entanglement analogue of a classical information memory support, i.e. an *entanglement-storage unit* (ESU). Such hardware once prepared could be used at later times (alone or with duplicates)—once the desired kind of entanglement has been distilled—to perform quantum information protocols [1].

The biggest challenge in the development of an ESU is entanglement frailty: it is strongly affected by the detrimental presence of decoherence [1]. Furthermore, the search for a proper system to build an ESU is undermined by the increasing complexity of quantum systems with a growing number of components, which makes entanglement more frail, more difficult to characterize, to create and to control [3]. Moreover, given a many-body quantum system, the search for a state with the desired properties is an exponentially hard task in the system size. Nevertheless, in many-body quantum systems entanglement arises naturally: for example—when undergoing a quantum phase transition—in the proximity of a critical point the amount of entanglement possessed by the ground state scales with the size [3, 4]. Unfortunately, due to the closure of the energy gap at the critical point, the ground state is an extremely frail state: even very small perturbations might destroy it, inducing excitations toward other states. However, a different strategy might be successful, corroborated also by very recent investigations on the entanglement properties of the eigenstates of many-body Hamiltonians, where it has been shown that in some cases they are characterized by entanglement growing with the system size [5, 6].

In this paper, we show that by means of a recently developed optimal control technique [7, 8] it is possible to identify and prepare a many-body quantum system in robust, long-lived entangled states (ESU states). More importantly, we drive the system toward ESU states without the need for any *a priori* information on the system, either about the eigenstates or about the energy spectrum. Indeed, we do not first solve the complete spectrum and eigenstates, which is an exponentially difficult problem in the system size. Recently, optimal control was used to drive quantum systems in entangled states or to improve the generation of entanglement [9]. However, here we have in mind a different scenario: to exploit the control to steer a system into a highly entangled state that is stable and robust even after switching off the control (see figure 1). Moreover, we want to outline the fact that we do not choose the goal state, but only its properties. In the following, we show that ESU states are gap-protected entangled eigenstates of the system Hamiltonian *in the absence* of the control, and that for an experimentally relevant model it is indeed possible to identify and drive the system into the ESU states. We show that the ESU states, although not characterized by the maximal entanglement sustainable by the system, are characterized by entanglement that grows with the system size. Once a good ESU state has been detected, due to its robustness it can be stored, characterized and thus used for later quantum information processing.

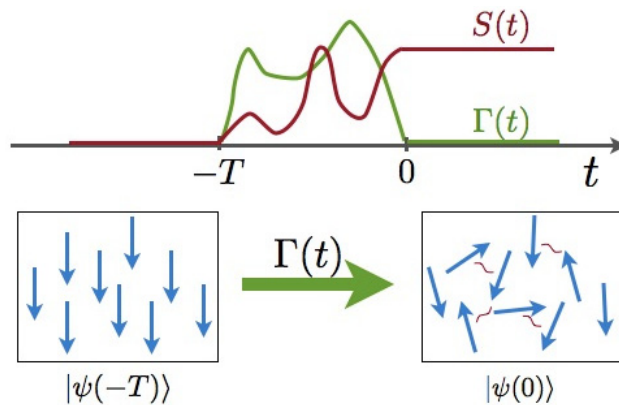


Figure 1. ESU protocol: a system is initially in a reference state $|\psi(-T)\rangle$, e.g. the ground state, and is optimally driven via a control field $\Gamma(t)$ in an entangled eigenstate $|\psi(0)\rangle$, protected from decoherence by an energy gap. $S(t)$ represents a generic measure of entanglement.

Here we provide an important example of this approach, based on the Lipkin–Meshkov–Glick (LMG) model [10], a system realizable in different experimental setups [2, 11]; we prepare an ESU maximizing the von Neumann entropy of a bipartition of the system and we model the action of the surrounding environment with noise terms in the Hamiltonian. However, our protocol is compatible with different entanglement measures and different models, such as the concurrence between the extremal spins in an Ising chain, see section 5. Note that with a straightforward generalization it can be adapted to a full description of open quantum systems [12].

The paper is organized as follows. In section 2, the general protocol to steer a system onto the ESU state is presented; in section 3, we consider the application of the protocol to the LMG model; in section 4, we discuss the effect of a telegraphic classical noise on the protocol; in section 5, we test the protocol into an Ising spin chain, and finally in section 6, we present the conclusions.

2. Entanglement-storage unit (ESU) protocol

As depicted in figure 1, we consider the general scenario of a system represented by a tunable Hamiltonian $H[\Gamma]$, where $\Gamma(t)$ is the control field, and initialized in a state $|\psi_{\text{in}}\rangle$ that can be easily prepared. We assume that the control field $\Gamma(t)$ can be modulated only in the finite time interval $[-T, 0]$; outside of this interval, for $t < -T$ and $0 < t$, we impose $\Gamma \equiv \tilde{\Gamma}$ (e.g. absence of control). According to our protocol, at the end of the control procedure, i.e. once the control field is brought back to the value $\tilde{\Gamma}$, the system has prepared been in a state with desired properties (for instance, high entanglement), stable in the absence of the control and robust against noise and perturbations.

Optimal control has already been used to enhance a given desired property without targeting an *a priori* known state; unfortunately, the results of such an optimization are usually fragile and ideally require a continuous application of the control in order to be stabilized [9]. However, in practical situations, a continuous application of control can be unrealistic, being

either simply impossible or too expensive in terms of resources. An example is the initialization of a quantum register that has to be physically moved into different spatial locations (such as a portable memory support), or if the control field used to initialize has to be switched on and off in order to manipulate different parts of the apparatus; in such situations, the register should indeed also be stable once disconnected from the device employed for its initialization. Consequently, in certain applications, a procedure capable of preparing quantum targets intrinsically stable even in the absence of sustained external manipulations is not only highly desirable but also crucial. The main contribution of our work is to move a step forward in this direction, proposing a flexible recipe to improve the stability of the outcome of a generic optimization process.

The simple idea behind our method is the following. As is well known, in a closed system the evolution of an arbitrary state is driven by the Schrödinger equation $i\hbar|\dot{\psi}(t)\rangle = H(t)|\psi(t)\rangle$. Assuming that, as in the absence of control, the Hamiltonian is constant $H(t) = H[\tilde{\Gamma}]$, we can evaluate the extent of deviation induced by the time evolution in an infinitesimal time dt after switching off the control [13]:

$$1 - |\langle\psi(t)|\psi(t+dt)\rangle|^2 = \Delta\tilde{E}^2 dt^2/\hbar^2 + O(dt^3), \quad (1)$$

where $\Delta\tilde{E} = \sqrt{\langle\psi(t)|H^2[\tilde{\Gamma}]|\psi(t)\rangle - \tilde{E}^2}$ and $\tilde{E} = \langle\psi(t)|H[\tilde{\Gamma}]|\psi(t)\rangle$ correspond, respectively, to the energy fluctuations and the energy of the Hamiltonian in the absence of control. Then from equation (1) it is clear that an arbitrary state is stabilized by minimizing the quantity $\Delta\tilde{E}$. In particular, by reaching the condition $\Delta\tilde{E} = 0$, the system is also prepared in an eigenstate of $H[\tilde{\Gamma}]$.

Our protocol relies on the use of optimal control implemented through the Chopped Random Basis (CRAB) technique [7, 8]. The CRAB method consists of expanding the control field onto a truncated basis (e.g. a truncated Fourier series) and in minimizing an appropriate cost function with respect to the weights of each component of the chopped basis (see [7, 8] for details of the method).

In particular, for the ESU protocol a CRAB optimization is performed with the goal of minimizing the cost function \mathcal{F} :

$$\mathcal{F}(\lambda)|_{\psi(0)} = -S + \lambda \frac{\Delta\tilde{E}}{\tilde{E}}, \quad (2)$$

where S represents a measure of entanglement, λ is a Lagrange multiplier and the cost function is evaluated on the optimized evolved state $|\psi(0)\rangle$ produced with a control process active in the time interval $[-T, 0]$. As discussed previously and shown in the following, the inclusion in \mathcal{F} of the constraint on energy fluctuations is the crucial ingredient to stabilize the result of the optimization also for times $t > 0$, that is, once the control has been switched off.

We conclude this section by stressing a couple of important advantages of our protocol with respect to other possible approaches to the problem, for instance evaluating all the eigenstates of the system and picking from among them the state(s) with the desired properties. First, in our protocol we never compute the whole spectrum of the system, but simply require evaluation of the energy and the energy fluctuations into the evolved state, see equation (2); therefore, our procedure can also be applied to situations in which it is not possible to compute all the eigenstates of the Hamiltonian (e.g. many-body non-integrable systems or DMRG simulations or experiments including a feedback loop). Furthermore, it can occur that none of the eigenstates of the system has the desired property we would like to enhance; then by simply considering the eigenstates one could not gain any advantage. In contrast, with our protocol in this situation

it is possible to identify states that, even though different from exact eigenstates, still show an enhanced robustness, such as the optimal state found in the considered scenario, see section 5.

3. ESU and the Lipkin–Meshkov–Glick model

We decided to apply the protocol to the Lipkin–Meshkov–Glick model [10] because it represents an interesting prototype of the challenge we address: it describes different experimental setups [2, 11], and the entanglement properties of the eigenstates are in general not known. Indeed, the entanglement properties of the eigenstates of one-dimensional many-body quantum systems have been related to the corresponding conformal field theories [5]; however, for the LMG model, to our knowledge, this study has never been performed and a conformal theory is not available [15]. Finally, the optimal control problem we address is highly non-trivial as the control field is global and space independent with no single-site addressability [9].

The LMG Hamiltonian describes an ensemble of spins with infinite-range interaction and is written as [14]

$$H = -\frac{C}{N} \sum_{i < j}^N \sigma_i^x \sigma_j^x - \Gamma(t) \sum_i^N \sigma_i^z, \quad (3)$$

where N is the total number of spins, σ_i^α 's ($\alpha = x, y, z$) are the Pauli matrices on the i th site and C is a constant measuring the intensity of the spin–spin interaction. By introducing the total spin operator $\vec{J} = \sum_i \vec{\sigma}_i/2$, the Hamiltonian can be rewritten, apart from an additive constant and a constant factor, as

$$H = -\frac{1}{N} J_x^2 - \Gamma J_z \quad (4)$$

(from now on, we set $C = 1$ and $\hbar = 1$). The symmetries of the Hamiltonian imply that the dynamics is restricted to subspaces of fixed total magnetization J and fixed parity of the projection J_z ; a convenient basis for such subspaces is represented by the Dicke states $|J, J_z\rangle$ with $-J < J_z < J$ [16]². In the thermodynamical limit, the system undergoes a second-order QPT from a quantum paramagnet to a quantum ferromagnet at a critical value of the transverse field $|\Gamma_c| = 1$. There is no restriction on the reference value $\tilde{\Gamma}$ and on the initial state $|\psi_{\text{in}}\rangle$: we choose $\tilde{\Gamma} \gg 1$, corresponding to the paramagnetic phase, and as the initial state $|\psi_{\text{in}}\rangle$, the ground state of $H[\tilde{\Gamma}]$, i.e. the separable state in which all the spins are polarized along the positive z -axis [2]. A convenient measure of the entanglement in the LMG model is given by the von Neumann entropy $S_{L,N} = -\text{Tr}(\rho_{L,N} \log_2 \rho_{L,N})$ associated with the reduced density matrix $\rho_{L,N}$ of a block of L spins out of the total number N , which gives a measure of the entanglement present between two partitions of a quantum system [16]. In our analysis we consider two equal partitions, i.e. $S \equiv S_{N/2,N}$. Note that the maximally entangled state at a fixed size N is given by $\rho_M = \mathbf{1}/(N/2 + 1)$ and $S_{\rho_M} = \log_2(N/2 + 1)$ [16]. In figure 2, we report the entanglement $S_{N/2,N}$ of the eigenstates deep inside the paramagnetic phase at $\tilde{\Gamma} = 10$, for systems of different sizes. Clearly, also far from the critical point $\Gamma = 1$, many eigenstates possess a remarkable amount of entanglement that scales with the system size.

The effect is shown more clearly in figure 3, where the entanglement of the central eigenstate (red full circles) at $\tilde{\Gamma} = 10$ is compared with the entanglement of the ground state

² For the solution of the LMG model in the thermodynamical limit, see also Ribeiro *et al* [16].

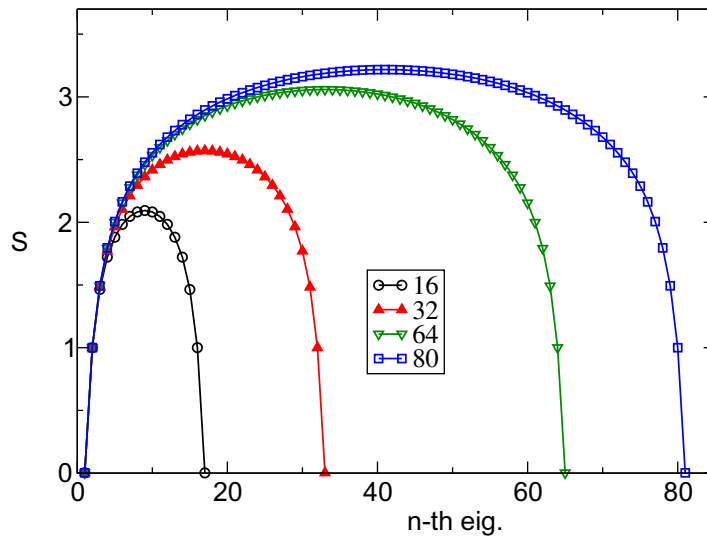


Figure 2. LMG model: static entanglement S of the eigenstates at $\tilde{\Gamma} = 10$ for different system sizes $N = 16, 32, 64, 80$. The eigenstates are ordered according to their energy, i.e. $n = 1$ corresponds to the ground state.

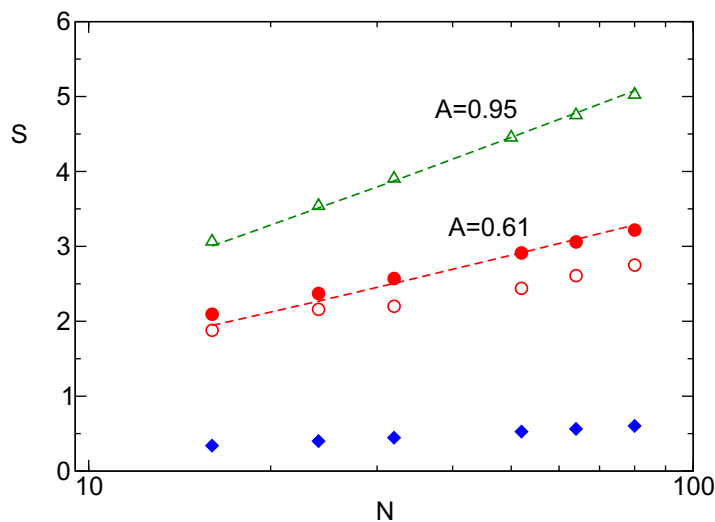


Figure 3. The LMG model: ground state entanglement at the critical point (full blue diamonds); the central eigenstate entanglement at $\tilde{\Gamma} = 10$ (full red circles); the maximal eigenstate entanglement obtained with the optimization for $\lambda \neq 0$ (empty red circles) and $\lambda = 0$ (green triangle). The red (green) dashed line is the numerical fit $A \log_2(N/2 + 1)$ with $A = 0.61$ ($A = 0.95$).

at the critical point (blue full diamonds). Both the sets of data show a logarithmic scaling with the size, but the entanglement of the central eigenstate is systematically higher and grows more rapidly.

Dynamics. We initialize the system in the non-entangled ground state of the Hamiltonian $H[\tilde{\Gamma}]$ with $1 \ll \tilde{\Gamma} = 10$ so that in the absence of control, i.e. for $\Gamma \equiv \tilde{\Gamma}$ independent of time, the

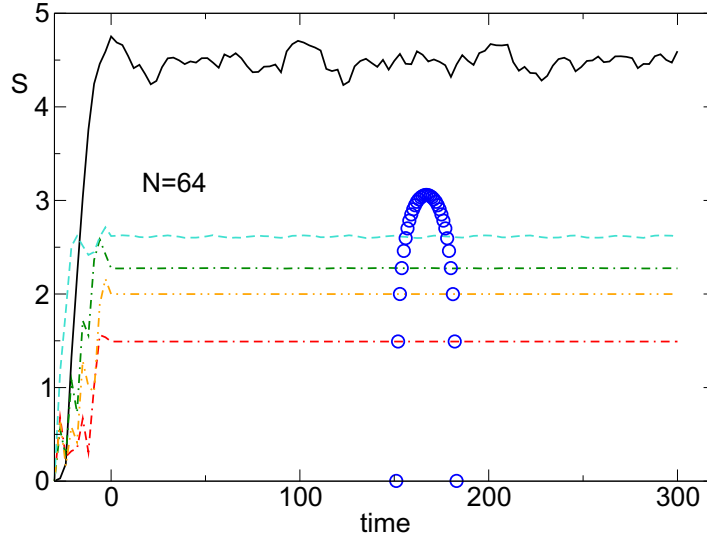


Figure 4. Entanglement entropy $S(t)$ as a function of time (time unit C^{-1}) for different λ values: $\lambda = 0$ (black) continuous, $\lambda = 5$ (red) dash-dash-dotted, $\lambda = 1.8$ (green) dot-dashed, $\lambda = 1.9$ (orange) dot-dot-dashed and $\lambda = 1.2$ (cyan) dashed line. Blue circles represent the entropy of the eigenstates for $N = 64$ and $\tilde{\Gamma} = 10$.

state $|\psi_{\text{in}}\rangle$ does not evolve apart from a phase factor. After the action of the CRAB-optimized driving field $\Gamma(t)$ for $t \in [-T, 0]$ the state is prepared in $|\psi(0)\rangle$ (a typical optimal pulse is shown in the inset of figure 5), and we observe the evolution of the state over times $t > 0$. The behavior of the entanglement is shown in figure 4 for different values of the weighing factor λ and $N = 64$. For $\lambda = 0$ highly entangled states are produced; however, the entanglement $S(t)$ oscillates indefinitely with the time. In contrast, if the energy fluctuations are included in the cost function ($\lambda \neq 0$), the optimal driving field steers the system into entangled eigenstates of $H[\tilde{\Gamma}]$, as confirmed by the absence of the oscillations in the entanglement and by the entanglement eigenstate reference values (blue empty circles). These results are confirmed by the survival probability in the initial state $P(t) = |\langle\psi(0)|\psi(t)\rangle|^2$ reported in figure 5: the state prepared with $\lambda = 0$ decays over very fast time scales τ_0 , while for $\lambda \neq 0$ it remains close to unity for very long times $\tau_\lambda \gg \tau_0$. The small residual oscillations for $N = 64$ and $\lambda = 1.2$ are due to the fact that in this case the optimization leads to a state corresponding to an eigenstate up to 98%. We repeated the optimal preparation for different system sizes and initial states, and show the entanglement of the optimized states for $\lambda = 0$ (green empty triangles) and $\lambda \neq 0$ ($\Delta\tilde{E}/\tilde{E} < 0.05$, $P > 95\%$ red empty circles) for different system sizes in figure 3. In all cases a logarithmic scaling with the size is achieved.

4. Random telegraph noise

A reliable ESU should be robust against external noise and decoherence even when the control is switched off, in such a way that it could be used for subsequent quantum operations. In order to test the robustness of the optimized states, we model the effect of decoherence by adding a random telegraph noise and we monitor the time evolution in such a noisy environment [1].

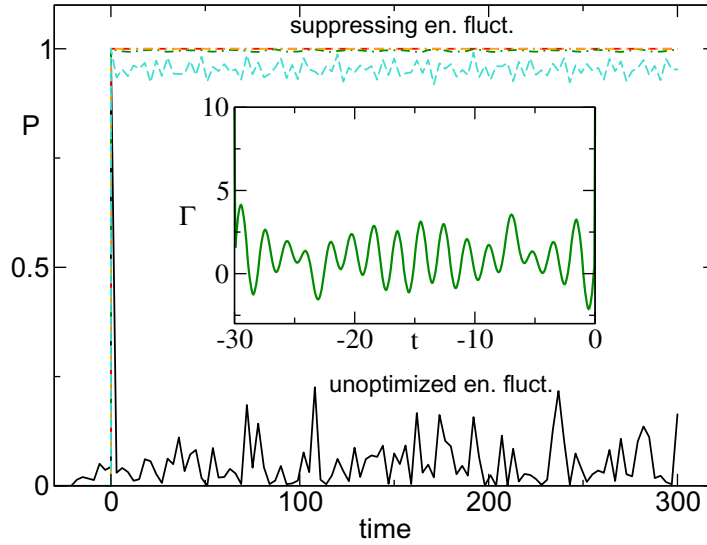


Figure 5. Survival probability $P(t)$ as a function of time (time unit C^{-1}) for different λ values: $\lambda = 0$ (black) continuous, $\lambda = 5$ (red) dash-dash-dotted, $\lambda = 1.8$ (green) dot-dashed, $\lambda = 1.9$ (orange) dot-dot-dashed and $\lambda = 1.2$ (cyan) dashed line. Inset: optimal driving field $\Gamma(t)$ for $\lambda = 1.8$ and $N = 64$.

In particular, we study the evolution induced by the Hamiltonian

$$H = -\frac{1}{N}[1 + I_\alpha \alpha(t)]J_x^2 - \tilde{\Gamma}[1 + I_\beta \beta(t)]J_z, \quad (5)$$

where $\alpha(t)$, $\beta(t)$ are random functions of the time with a flat distribution in $[-I_j, I_j]$ ($j = \alpha, \beta$), changing the random value every typical time $1/\nu$. The case $I_\alpha = I_\beta = 0$ corresponds to a noiseless evolution. The first important observation is that the frequency ν of the signal fluctuations is crucial in determining its effects [17]. Indeed in figure 6, the survival probability $P(t)$ is plotted as a function of the time in the presence of a strong noise, $I_\alpha = I_\beta = 0.2$, for a system of $N = 64$ spins and for a given initial optimal state obtained with $\lambda = 1.8$ (see figure 4). When ν is either too low (empty circles) or too high (full diamonds) the effect of the noise is reduced; however, around a resonant frequency ν_R (dashed line with crosses) its effect is enhanced and the state is quickly destroyed. We checked that the resonant frequency is the same for different eigenvalues, different sizes and different noise strengths (data not shown), reflecting the fact that in the paramagnetic phase ($\tilde{\Gamma} \gg 1$) the gap separating the eigenstates is proportional to $\tilde{\Gamma}$ independently of the size of the system and of the state itself, see equation (4). Therefore, we analyze this worst-case scenario, setting $\nu = \nu_R$ from now on. In figure 7, we compare the survival probability $P(t)$ for three instances of the disorder at the resonant frequency with an intensity of the disorder $I_\alpha = I_\beta = 0.01$. The noise-induced dynamics of the states obtained optimizing only with respect to the entanglement (i.e. setting $\lambda = 0$, full symbols in figure 7) drastically depends on the (in general unknown) details of the noise affecting the system; thus, such states cannot be used as ESU. Conversely, the states prepared with $\lambda \neq 0$ (empty symbols in figure 7) turn out to be stable, noise-independent and long-living entanglement. Finally, in figure 8 we study the decay times of the survival probability $P(t)$ studying the time $T_{0.8}$ needed to drop below a given threshold $P_{\min} = 0.8$ as a function of the system size N and of the intensity

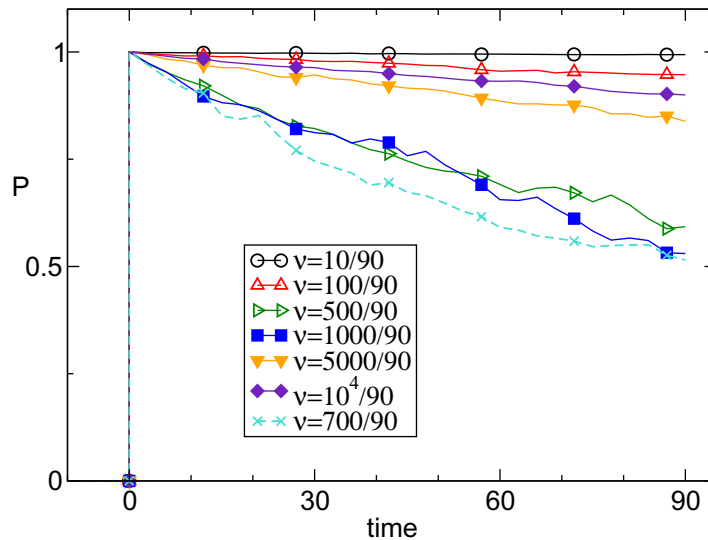


Figure 6. Survival probability $P(t)$ as a function of time (time unit C^{-1}), averaged over 30 noise instances for $I_\alpha = I_\beta = 0.2$, $N = 64$, $\lambda = 1.8$ and different noise frequencies. The worst case (dashed line with crosses) is for $\nu_R = 700/(90 C^{-1}) = 7.8 C$.

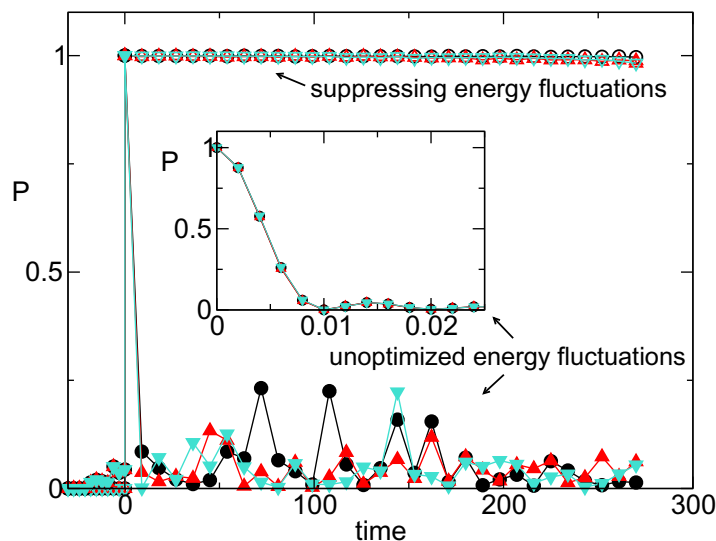


Figure 7. Survival probability $P(t)$ as a function of time (time unit C^{-1}) for three realizations of the noise with $I_\alpha = I_\beta = 0.01$ at frequency ν_R , $N = 64$, and $\lambda = 1.8$ (empty symbols) or $\lambda = 0$ (full symbols). Inset: blow-up of the region around $t = 0$ for the $\lambda = 0$ case.

of the disorder $I = I_\alpha = I_\beta$ (inset). These results clearly show that $T_{0.8}$ for ESU states is almost independent of the system size, reflecting the fact that the energy gaps in this region of the spectrum are mostly size independent. Note that, in contrast, $T_{0.8}$ for maximally entangled states decays linearly with the system size and that there are more than four orders of magnitude of difference in the decay times τ_λ and τ_0 . Finally, the inset of figure 8 shows that the scaling of

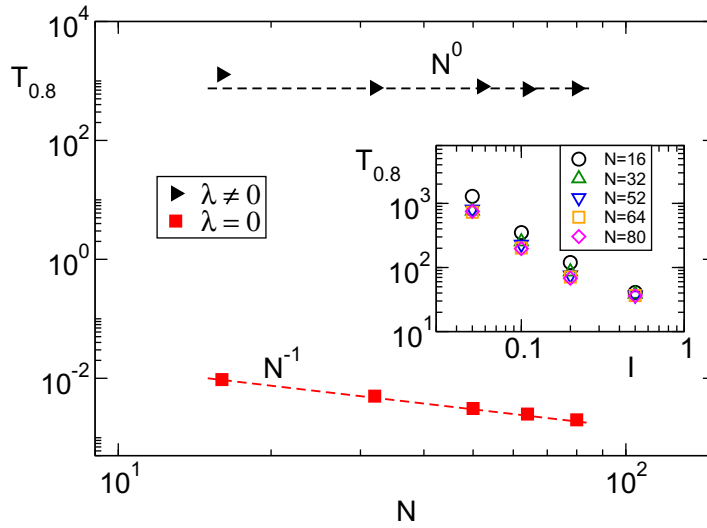


Figure 8. The time $T_{0.8}$ required to reduce the survival probability P below 0.8 for different prepared states $|\psi(0)\rangle$ with $\lambda = 0$ (red squares) and $\lambda \neq 0$ corresponding to the second eigenstate of the even parity sector of J_z (black triangles) as a function of the system size N . The dashed lines are fits of the four rightmost points (the biggest system sizes) resulting in $N^{-0.97}$ and $N^{-0.03}$, respectively. Inset: the time $T_{0.8}$ as a function of the intensity $I = I_\alpha = I_\beta$ of the disorder for different system sizes N .

$T_{0.8}$ with the noise strength for ESU states is approximately a power law and again depends very weakly on the system size N .

5. The Ising model: concurrence between extremal spins

In our previous discussion, we focused our attention on the optimization of the von Neumann entropy of eigenstates other than the ground state of the LMG model, in order to show the effectiveness of our protocol in controlling the dynamics and unexplored properties of many-body systems.

However, aiming at demonstrating the generality of the method, in this section we would like to present briefly the application of our protocol to a different situation, closer to the typical problems encountered in quantum information: in particular, we show how it is possible to stabilize the concurrence between the extremal spins of an open Ising chain.

The Hamiltonian of the ordered one-dimensional Ising model with nearest-neighbor interaction is given by

$$H = -C \sum_i^{N-1} \sigma_i^x \sigma_{i+1}^x - \Gamma(t) \sum_i^N \sigma_i^z, \quad (6)$$

where the transverse field $\Gamma(t)$ is our control field. We assume that the system can be easily prepared in the ground state at a large value of the control field $\tilde{\Gamma} = 10$, in which all the spins are polarized along the positive z -direction. The aim of the control is to enhance the concurrence between the first and the N th spin of the chain, possibly stabilizing the state. The concurrence

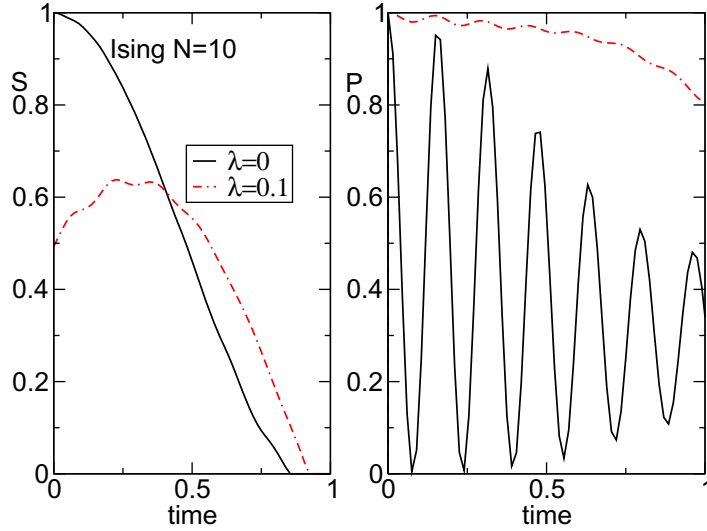


Figure 9. Concurrence $S(t)$ between extremal spins (left panel) and survival probability $P(t)$ (right panel) as a function of time (time units C^{-1}) in an Ising chain with $N = 10$ spins for two different λ values: $\lambda = 0$ (black continuous line) and $\lambda = 0.1$ (red dot-dashed line).

between two spins is defined as $S = \max\{0, e_1 - e_2 - e_3 - e_4\}$, where the e_i s are the eigenvalues in decreasing order of the Hermitian matrix $R = \sqrt{\sqrt{\tilde{\rho}}\tilde{\rho}\sqrt{\tilde{\rho}}}$, ρ is the reduced density matrix of the two extremal spins and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$ is the spin-flipped state [18].

At a large value of the transverse field, the eigenstates of the Hamiltonian are the classical states represented by all the possible up-down combinations of N spins, and states with the same numbers of flipped spins, although in different positions, are degenerate. A naive approach to build stable entangled states would then require a search for possibly entangled states in each degenerate subspace at a given energy. Such a search, however, represents a highly non-trivial task, due to the strong constraint imposed by requiring non-vanishing concurrence: again a suitable recipe for such a search should be provided and is non-trivial to find. In contrast, our protocol proposes an answer to the task without requiring any diagonalization, while automatically performing the search, therefore offering a clear advantage.

We perform a CRAB optimization in the time interval $[-T, 0]$ minimizing the function $\mathcal{F}(\lambda)_{|\psi(0)\rangle} = -S + \lambda\Delta\tilde{E}$, in which now S is the concurrence; then at the time $t = 0$ the control is switched off, the value of the field is kept constant ($\Gamma(t) = \tilde{\Gamma}$ for $t > 0$), and we observe the evolution of the optimized state. In figure 9, we show the behavior of the concurrence $S(t)$ and of the survival probability $P(t) = (\text{Tr}\sqrt{\sqrt{\rho(t)}\rho(0)\sqrt{\rho(t)}})^2$, excluding ($\lambda = 0$, black continuous line) and including ($\lambda = 0.1$, red dot-dashed line) the energy fluctuation term in the optimization procedure. As shown in the picture, although, as expected, the concurrence is smaller when $\lambda \neq 0$, the survival probability is stabilized by a factor bigger than 50 in time with respect to the $\lambda = 0$ case.

6. Conclusion

Exploiting optimal control we proposed a method to steer a system into *a priori* unknown eigenstates satisfying the desired properties. We demonstrated, on a particular system, that

this protocol can be effectively used to build long-lived entangled states with many-body systems, indicating a possible implementation of an ESU scalable with the system size. The method presented is compatible with different models (e.g. LMG and Ising) and measures of entanglement (e.g. von Neumann entropy and concurrence) and can be extended to any other property one is interested in, such as for example the squeezing of the target state [12]. It can be applied to different systems with *a priori* unknown properties: optimal control will select the states (if any) satisfying the desired property and robust to system perturbations. We stress that an adiabatic strategy is absolutely ineffective for this purpose, as transitions between different eigenstates are forbidden. Applying this protocol to the full open-dynamics description of the system, e.g. via a CRAB optimization of the Lindblad dynamics as done in [19], will result in an optimal search for a decoherence-free subspace (DFS) with desired properties [20]. If no DFS exists, the optimization would lead the system to an eigenstate of the superoperator with the longest lifetime and the desired properties [12]. Although the state so prepared may be unstable over long times, it represents the best and most robust state attainable, and additional (weak) control might be used to preserve its stability. Finally, working with excited states would reduce finite-temperature effects, relaxing low-temperature working-point conditions, simplifying the experimental requirements to build a reliable ESU.

Acknowledgments

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