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## Bound-bound pair production in relativistic collisions

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**Abstract.** Electron-positron pair production is considered in the relativistic collision of a nucleus and an anti-nucleus, in which both leptons are created in bound states of the corresponding nucleus-lepton system. Compared to free and bound-free pair production, this process is shown to display a qualitatively different dependency both on the impact energy and on the charges of the colliding particles. Interestingly, at high impact energies the cross section for this process is found to be larger (due to a bigger statistical weight) than that for the analogous atomic process of non-radiative electron capture, although the latter does not involve the creation of new particles.

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## 1. Introduction

One of the most fascinating predictions of quantum electrodynamics is the possibility of converting energy into matter. Starting with the paper by Sauter [1], electron–positron pair production from vacuum in the presence of external electromagnetic fields has been attracting the attention of different physical communities.

Pair production has been studied theoretically in the presence of electromagnetic fields of various configurations (e.g. in the combination of Coulomb and high-energy photon fields [2], in high-energy collisions of charged particles [3], in constant and uniform fields [4], in slowly varying super-strong Coulomb fields [5], in colliding laser fields [6] and in crystals [7]), and also in the presence of gravitational fields [8].

Pair production can occur with a noticeable probability (i) if the external field is strong enough to provide an energy of the order of the electron rest energy  $mc^2$  on a distance of the order of the electron Compton wave length  $\lambda_C = \hbar/mc$ ,<sup>2</sup> where  $\hbar$  is the Planck's constant; (ii) and/or if the field varies in time so rapidly that its typical frequencies multiplied by  $\hbar$  are larger than  $2mc^2$ .

Experimentally, pair production has been explored only in the case of rapidly varying electromagnetic fields (for instance, in relativistic heavy-ion collisions, photon–laser collisions [9], in the collision of an intense laser beam with a solid target [10]). Note also that the observation of pair production in the collision of two light beams is one of the main goals of future intense-laser facilities<sup>3</sup>.

Landau and Lifshitz [3] were the first to estimate the cross section for pair production in relativistic collisions of charged particles in which the created electron and positron freely move in space after the collision is over (see figure 1(a)). Such a process is termed free pair production and it was studied in much detail in a vast number of theoretical and experimental papers (see for recent reviews, e.g., [11] and also references therein).

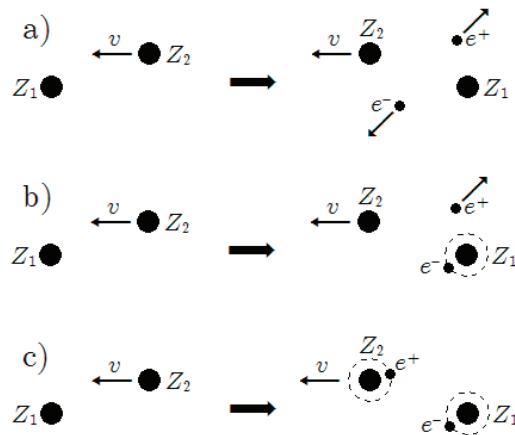
During the last two decades, another kind of pair production process occurring in relativistic nuclear collisions has attracted much attention (see, e.g., [12–17] and references therein), in which the electron is created in a bound state with one of the colliding nuclei (see figure 1(b)).

When the colliding nuclei possess charges of different signs, yet another pair production process becomes possible in which not only the electron but also the positron is created in a bound state (see figure 1(c)). Below we shall call this process bound–bound pair production. Compared to the free and bound-free cases, bound–bound pair production is expected to have a number of interesting features. In particular, it has an intrinsic non-perturbative dependence on charges of *both* colliding nuclei. This, as well as the fact that this process completes the picture of the basic (single-) pair production processes occurring in high-energy collisions of charged particles, makes its study of significant interest. Besides, bound–bound pair production, which results in creating bound states of antimatter (in particular, antihydrogen<sup>4</sup>), may also be relevant in connection with testing CPT invariance and the weak equivalence principle [18].

<sup>2</sup> A so strong field has an amplitude of the order of the so-called critical field  $E_{cr} = m^2 c^3 / \hbar e = 1.3 \times 10^{16} \text{ V cm}^{-1}$ , where  $e$  is the absolute value of the electron charge.

<sup>3</sup> See the European Light Infrastructure at <http://www.extreme-light-infrastructure.eu> and High Power Laser Energy Research at <http://www.hyperlaser.org>.

<sup>4</sup> Note that antihydrogen has been produced via bound-free pair creation at CERN and later at FERMILAB. For more information, see e.g. [11].



**Figure 1.** A sketch of different pair production (sub-)processes: free (a), bound-free (b) and bound-bound (c) pair production.

To our knowledge, bound-bound pair production has not yet been considered in the literature and it is the goal of this article to investigate this process.

## 2. General considerations and results

Let us consider the collision of two nuclei with charges  $Z_1$  and  $Z_2$  (say  $Z_1 > 0$  and  $Z_2 < 0$ )<sup>5</sup>. Impact parameter values characteristic for this process are of the order of  $\lambda_C$  (see figure 3 below) and, thus, are much larger than the nuclear size. Therefore, one can treat the nuclei as point-like particles (our estimation using extended (‘real’) nuclei instead of point-like ones indeed shows very little change—well below 1%—in the cross section for bound-bound pair production). Since we are dealing with high impact energies, one can also assume that the initial velocities of these particles are not changed in the collision.

Our considerations will be based on the semi-classical approximation in which only the electron and the positron are treated using quantum theory, while the heavy charges  $Z_1$  and  $Z_2$  are regarded as classical particles. Although the formalism used below is Lorentz-covariant, for certainty we shall employ the rest frame of the charge  $Z_1$  as our reference frame. We take the position of this charge as the origin and assume that in this frame the charge  $Z_2$  moves along a straight-line classical trajectory  $\mathbf{R}(t) = \mathbf{b} + \mathbf{v}t$ , where  $\mathbf{b} = (b_x, b_y, 0)$  is the impact parameter,  $\mathbf{v} = (0, 0, v)$  is the collision velocity and  $t$  is the time.

Starting with the QED Lagrangian, neglecting the electron-positron interaction and radiative corrections, the ‘exact’ (prior form of the) transition amplitude for bound-bound pair production is given by

$$a(\mathbf{b}) = -i \int_{-\infty}^{+\infty} dt \int d^3\mathbf{r} \Psi_f^\dagger(\mathbf{r}, t) \hat{W}(\mathbf{r}, t) \psi_i(\mathbf{r}, t). \quad (1)$$

In this expression,  $\psi_i$  is the state of the negative-energy electron bound in the field of the charge  $Z_2$ ,  $\Psi_f$  is the exact state of the electron moving in the field of both charges  $Z_1$  and  $Z_2$ ,  $\mathbf{r} = (\mathbf{r}_\perp; z)$  is the lepton coordinate and  $\hat{W} = -Z_1/r$  is the interaction with the field of the charge  $Z_1$  not

<sup>5</sup> From now on atomic units are used unless otherwise indicated.

included in the state  $\psi_i$ . Since the form of the state  $\Psi_f$  is not known, we shall approximate  $\Psi_f$  by  $\psi_f$ , which is the state of the electron bound in the field of the charge  $Z_1$ , and arrive at the simplest form of the transition amplitude for bound-bound pair production.

Concerning the accuracy of such an approximation, one can note the following. A similar approximation is widely used in the studies of the analogous collision process of non-radiative electron capture. These studies (see, e.g., [16, 17]) have shown that the amplitude, obtained within this approximation, cannot yield very accurate results but it does enable one to get the correct order of magnitude for the cross section and the qualitatively correct dependencies on the collision velocity and nuclear charges. Therefore, one expects that the approximation, which we use, will correctly reproduce the main features of bound-bound pair production.

In the chosen reference frame the initial and final states read

$$\begin{aligned}\psi_i &= \sqrt{\frac{1+\gamma}{2}} \left( 1 + \frac{v}{c} \frac{\gamma}{1+\gamma} \alpha_z \right) \chi_i(\mathbf{s}) \exp(i\varepsilon_p \gamma (t - vz/c^2)), \\ \psi_f &= \varphi_f(\mathbf{r}) \exp(-i\varepsilon_e t).\end{aligned}\quad (2)$$

In (2),  $\chi_i$  is the initial negative-energy state,  $\varepsilon_p = mc^2 - I_p$  is the total energy of the positron where  $I_p$  is its binding energy; both these quantities are given in the rest frame of the charge  $Z_2$ . Further,  $\varphi_f$  is the bound state of the electron and  $\varepsilon_e = mc^2 - I_e$  is its total energy, with  $I_e$  being the binding energy.  $\gamma$  is the collisional Lorentz factor,  $\alpha_z$  is the Dirac matrix and  $\mathbf{s} = (\mathbf{r}_\perp - \mathbf{b}; \gamma(z - vt))$ .

The amplitude (1) is written in the impact-parameter space. However, it is more convenient to calculate the cross section using the transition amplitude written in the momentum space,

$$S(\mathbf{q}_\perp) = \frac{1}{2\pi} \int d^2\mathbf{b} a(\mathbf{b}) \exp(i\mathbf{q}_\perp \cdot \mathbf{b}). \quad (3)$$

Using equations (1)–(3), we obtain

$$S(\mathbf{q}_\perp) = i \frac{Z_1}{2\pi v \gamma} \sqrt{\frac{1+\gamma}{2}} \int d^3\mathbf{r} \varphi_f^\dagger(\mathbf{r}) \frac{1}{r} \exp(i\mathbf{q} \cdot \mathbf{r}) \left( 1 + \frac{v}{c} \frac{\gamma}{1+\gamma} \alpha_z \right) \int d^3\mathbf{s} \chi_i(\mathbf{s}) \exp(-i\mathbf{q}' \cdot \mathbf{s}). \quad (4)$$

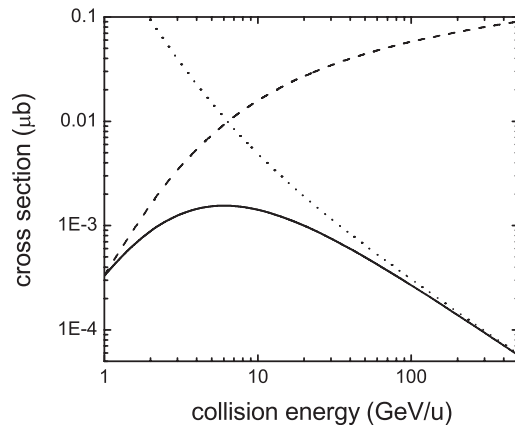
The quantities  $\mathbf{q}$  and  $\mathbf{q}'$  have the meaning of the momentum transfers as viewed in the rest frames of the charges  $Z_1$  and  $Z_2$ , respectively, and are given by

$$\begin{aligned}\mathbf{q} &= \left( \mathbf{q}_\perp, \frac{mc^2 - I_e + (mc^2 - I_p)/\gamma}{v} \right), \\ \mathbf{q}' &= \left( \mathbf{q}_\perp, \frac{mc^2 - I_p + (mc^2 - I_e)/\gamma}{v} \right).\end{aligned}\quad (5)$$

The total cross section for bound-bound pair production reads

$$\sigma = \int d^2\mathbf{q}_\perp |S(\mathbf{q}_\perp)|^2. \quad (6)$$

It follows from equations (4) and (5) that at asymptotically high collision energies the only dependence of the amplitude  $S$  on the collision energy is given by the factor  $1/\sqrt{\gamma}$ . Thus, at these energies the cross section is proportional to  $1/\gamma$ .



**Figure 2.** Cross sections for pair production and electron capture given as a function of the collision energy. Solid curve:  $p^- + U^{92+} \rightarrow \bar{H}(1s) + U^{91+}(1s)$ . Dashed curve:  $p^- + U^{92+} \rightarrow \bar{H}(1s) + e^- + U^{92+}$ . Dotted curve shows twice the cross section for the reaction  $H(1s) + U^{92+} \rightarrow p^+ + U^{91+}(1s)$ .

Further, the typical momentum transfers, involved in the process, are of the order of a few  $mc$  and, thus, their squares are substantially larger than the squares of the typical electron and positron momenta in the corresponding bound states (the order of magnitude of the latter momenta is  $Z_1$  and  $|Z_2|$ , respectively). Using this observation and the explicit form of the bound states, it is not difficult to show that the amplitude (4) is roughly proportional to  $Z_1^{5/2} |Z_2|^{5/2}$ .

Taking all this into account, we determine that the asymptotic form of the cross section for the bound-bound pair production is given by

$$\sigma \sim \frac{Z_1^5 |Z_2^5|}{\gamma}. \quad (7)$$

This dependence is significantly different from the corresponding ones in the case of free and bound-free pair productions, which read  $\sigma_f \sim Z_1^2 Z_2^2 \log^3(\gamma)$  and  $\sigma_{bf} \sim Z_1^5 Z_2^2 \log(\gamma)$ , respectively (see, e.g., [11, 16]). Note that in  $\sigma_{bf}$ ,  $Z_1$  is the charge of the nucleus carrying away the created electron.

In figure 2, we show the cross section for the reaction  $p^- + U^{92+} \rightarrow \bar{H}(1s) + U^{91+}(1s)$  (solid curve). The dependence of the cross section on the impact energy is not monotonous. At the relatively low collision energies, the cross section increases with the energy reaching a maximum at about  $5\text{--}7 \text{ GeV u}^{-1}$ . With a further energy increase, the cross section starts to decrease with an increasing slope and reaches its asymptotic energy dependence  $\sim 1/\gamma$  already within the energy interval displayed in the figure.

The cross section for bound-bound pair production can be compared with that for bound-free pair creation. We have calculated the cross section for the latter process (shown in figure 2 by the dashed curve), treating it as a transition between the negative- and positive-energy Coulomb states centred on the antiproton, which is induced by the interaction with the charge  $Z_1$  taken into account in the lowest order perturbation theory.

At relatively low impact energies, both cross sections increase and are rather close in magnitude to each other. However, at larger impact energies the cross sections start to demonstrate qualitatively different behaviours and the difference between them increases very rapidly.

Such an interrelation between these cross sections can be understood by noting the following. At very low collision energies the spectrum of the electromagnetic field generated by the colliding particles does not have enough high-frequency components to create an electron–positron pair. As a result, the cross sections for both pair production processes are very small. An increase in the impact energy leads to an increase in the high-frequency components of the field and both cross sections grow rather rapidly. However, when the impact energy increases further, the conditions for the bound–bound pair production begin to deteriorate. Indeed, the electron and positron are created on different nuclei and, therefore, the difference between their momenta increases with the impact energy. This effectively reduces the overlap between the states  $\psi_i$  and  $\psi_f$ , making bound–bound pair production more difficult.

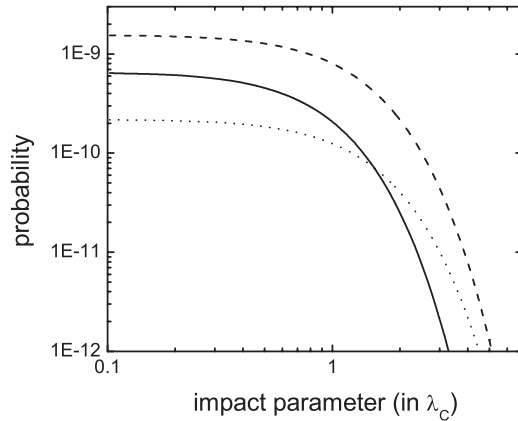
This, of course, does not occur in bound-free pair production since both leptons are created on/around the same nucleus. In this case, when the impact energy grows, the range of the impact parameters efficiently contributing to the process grows as well ( $\sim \gamma$ ), leading to the logarithmic increase in the cross section.

Bound–bound pair production can be viewed as a collision-induced transition between states of the electron with negative and positive total energies bound in the field of the charge  $Z_2 < 0$  and charge  $Z_1 > 0$ , respectively. This is reminiscent of the atomic collision process of non-radiative electron capture (for a review, see, e.g., [16, 17]), in which an electron initially bound in the atom undergoes a transition into a bound state in the ion:  $(Z_a + e^-) + Z_i \rightarrow Z_a + (Z_i + e^-)$ , where  $Z_a$  and  $Z_i$  are the charges of the atomic and ionic nuclei. Indeed, within the simplest description of non-radiative capture, its amplitude is given by equation (1), in which  $\psi_i$  and  $\psi_f$  are now the states of the electron bound in the atom and ion, respectively, and, therefore, it is of interest to compare the cross sections for these two processes.

Such a comparison is presented in figure 2, where the dotted curve shows twice the cross section for the reaction  $H(1s) + U^{92+} \rightarrow p^+ + U^{91+}(1s)$  calculated using the simplest description mentioned above. At relatively low and intermediate collision energies, where the electron capture is much more probable than the bound–bound pair production, the two cross sections show a qualitatively different behaviour. However, at higher impact energies the cross sections approach each other, cross and, when the energy increases further, demonstrate exactly the same energy dependence, with the bound–bound pair production cross section being a factor of 2 larger.

The factor of 2 is of statistical origin, reflecting the difference between the averaging over the spin projections in the initial state in the case of electron capture and the corresponding summation in the case of pair production. Apart from this, the cross sections in the limit  $\gamma \gg 1$  are identical. This circumstance can be understood by taking into account the symmetry between these two processes and observing that at  $\gamma \gg 1$  the absolute values of the momentum transfers in both processes become essentially the same. Thus, at asymptotically high impact energies it would be easier to capture electron and positron from vacuum into the corresponding bound states in the collision with an antiproton than to pick up the already existing electron from an atomic hydrogen.

Additional insight into the physics of bound–bound pair production can be obtained by considering the probability  $P(b)$  for this process as a function of the impact parameter  $b$  ( $b = |\mathbf{b}|$ ). The probability, which is related to the cross section via  $\sigma = 2\pi \int_0^\infty db b P(b)$ , is shown in figure 3 for the reaction  $p^- + U^{92+} \rightarrow \bar{H}(1s) + U^{91+}(1s)$  for three different impact energies, 1, 6 and 100 GeV  $u^{-1}$ . In all these cases the impact parameters characteristic of the process are of the order of  $\lambda_C$ , but the range of  $b$  significantly contributing to the cross section



**Figure 3.** Probability for the reaction  $p^- + U^{92+} \rightarrow \bar{H}(1s) + U^{91+}(1s)$  given as a function of the impact parameter  $b$ . Solid, dashed and dotted curves display the probability for the impact energy of 1, 6 and 100  $\text{GeV u}^{-1}$ , respectively.

slightly broadens with the increase in the impact energy. Both these points can be understood by considering equations (5), which show that at  $v \simeq c$  the longitudinal components of the momentum transfers are of the order of  $mc$  and decrease when the impact energy increases, and also by taking into account that the transverse and longitudinal components of the momentum transfer in this process are (typically) of the same order of magnitude.

### 3. Conclusion

In conclusion, we have considered bound-bound  $e^+e^-$  pair production in which both these particles are created in bound states. Compared to free and bound-free pair production, it represents a qualitatively new sub-process whose cross section has different dependencies on the impact energy and charges of the colliding nuclei. Besides, its consideration also enables one to establish an interesting correspondence between the pair production and the more usual atomic collision process, in which the already existing electrons undergo transitions between colliding centres but no new particles is created.

In non-relativistic quantum theory, only the positive energy states exist and within the non-relativistic consideration of ion-atom (ion-ion) collisions only three basic atomic processes appear: excitation, ionization and electron capture. The relativistic theory adds up the negative energy states into consideration. This results in the existence of pair production and the corresponding extension of the group of the basic atomic collision processes to six. Thus, bound-bound pair production not only fills in the ‘vacancy’ in the set of the (single-) pair production processes but can also be viewed as completing the whole picture of the basic (effectively single-lepton) atomic processes possible in ion-atom (ion-ion) collisions.

We have focused our attention on bound-bound pair production in collisions involving a highly charged nucleus and an antiproton. However, this process also occurs if the antiproton is replaced in the collision by another particle with negative charge, for example by an electron or a muon  $\mu^-$ . In particular, since the mass of  $\mu^-$  is much larger than that of an electron/positron, our results obtained for collisions involving antiprotons are directly applicable to collisions with muons. In the case of electron-nucleus collision, where our results are, of course, not applicable, a different treatment is required.



In the present paper, bound–bound pair production is considered using the simplest possible approximation. As already mentioned, such an approach gives the correct quantitative dependencies of the cross section on the collision energy and the charges of the colliding particles, but it cannot yield very accurate results for the values of the cross section. More accurate approaches, used to calculate electron capture, are well documented in the literature (see [16, 17]). In particular, in the case of very asymmetric collisions, like e.g. between antiprotons and uranium ions, better results for the cross section could be obtained by employing an impulse-like approximation.

The cross section for bound–bound pair production is so small that, at present, experimental detection of this process does not seem to be possible. In the future, the experimental exploration of this process will be feasible provided that high-luminosity beams of heavy nuclei and antiprotons (or muons) are available. The possibility to detect bound–bound pair production using the future facilities at GSI (Darmstadt, Germany) is currently under discussion.

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