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JHEP11(2009)077

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Doubling of background solution in 5D stabilized brane world model

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ABSTRACT: We discuss a model providing two different stationary background solutions with flat and dS_4 metric on the branes under the same values of the fundamental parameters. It is shown that only an additional fine-tuning of the brane scalar field potentials can provide a separation between two background solutions.

KEYWORDS: p-branes, Large Extra Dimensions, Cosmology of Theories beyond the SM

¹The work was supported by grant of Russian Ministry of Education and Science NS-1456.2008.2, FASI state contract 02.740.11.0244, grant for young scientists MK-5602.2008.2 of the President of Russian Federation, grant of the "Dynasty" Foundation, RFBR grant 08-02-92499-CNRS-a and scholarship for young teachers and scientists of M.V. Lomonosov Moscow State University.

Stabilized brane world models have been widely discussed in the last years. The most consistent model with flat metric on the branes was proposed in paper [1], where exact solutions to equations of motion for the background metric and the scalar field were found. The size of the extra dimension is defined by the boundary conditions for the scalar field on the branes.

Most brane world models assume the metric on the branes to be the flat Minkowski metric. At the same time it is evident that more realistic models should account for a cosmological evolution on the branes. This problem is widely discussed in scientific literature, see, for example [1, 2], reviews [3, 4] and references therein. Quite an interesting class of the brane world models is the one describing background solutions with dS_4 metric on the branes. Different models with the scalar field living in the bulk and dS_4 metric on the branes were discussed in [1, 5, 6].

In this paper we discuss a model which provides different exact solutions (with flat Minkowski or dS_4 background metric on the branes) for different values of the fine-tuned brane tensions in both cases (the same problem was discussed in [5], where an exact solution with zero Hubble parameter on the branes and an approximate solution with nonzero constant Hubble parameter on the branes were obtained for the same bulk scalar field potential). We also show that under an appropriate value of the parameter of the bulk scalar field potential the equations of motion lead to these different exact solutions even for the same values of the brane tensions in both cases.

To this end let us consider gravity in a five-dimensional space-time $E = M_4 \times S^1/Z_2$, interacting with two branes and with the scalar field ϕ . Let us denote coordinates in E by $\{x^M\} = \{t, x^i, y\}$, $M = 0, 1, 2, 3, 4$, where $x^0 \equiv t$; $\{x^i\}$, $i = 1, 2, 3$ are three-dimensional spatial coordinates and the coordinate $y \equiv x^4$, $-L \leq y \leq L$, corresponds to the extra dimension. The extra dimension forms the orbifold S^1/Z_2 , which is a circle of diameter $2L/\pi$ with the points y and $-y$ identified. Correspondingly, the metric g_{MN} and the scalar field ϕ satisfy the orbifold symmetry conditions

$$\begin{aligned} g_{\mu\nu}(x, -y) &= g_{\mu\nu}(x, y), & g_{\mu 4}(x, -y) &= -g_{\mu 4}(x, y), \\ g_{44}(x, -y) &= g_{44}(x, y), & \phi(x, -y) &= \phi(x, y), \end{aligned} \tag{1}$$

$\mu = 0, 1, 2, 3$. The branes are located at the fixed points of the orbifold $y = 0$ and $y = L$.

The action of the model has the form

$$\begin{aligned} S &= M^3 \int R \sqrt{-g} d^5x - \int \left(\frac{1}{2} \partial_M \phi \partial^M \phi + V(\phi) \right) \sqrt{-g} d^5x - \\ &\quad - \int (\lambda_1(\phi) \delta(y) + \lambda_2(\phi) \delta(y - L)) \sqrt{-\tilde{g}} d^5x, \end{aligned} \tag{2}$$

where M is the five-dimensional Planck mass, $\lambda_{1,2}(\phi)$ are the scalar field potentials on the branes and $\tilde{g}_{\mu\nu}$ is the induced metric on the branes.

We consider the standard form of the background metric, which is often used in brane world models (see, for example, [1])

$$ds^2 = \gamma_{MN} dx^M dx^N = e^{2A(y)} (-dt^2 + a^2(t) \eta_{ij} dx^i dx^j) + dy^2, \tag{3}$$

where $\eta_{ij} = \text{diag}(1, 1, 1)$ and

$$\phi(x, y) = \phi(y). \quad (4)$$

Below we will consider a maximally-symmetric metric on the branes with

$$a(t) = e^{Ht}, \quad (5)$$

where H is the four-dimensional Hubble parameter. In this case the equations of motion, following from (2), take the form [1]

$$\phi'' + 4A'\phi' = \frac{dV}{d\phi} + \frac{d\lambda_1}{d\phi}\delta(y) + \frac{d\lambda_2}{d\phi}\delta(y-L), \quad (6)$$

$$M^3 \left(A'^2 - H^2 e^{-2A} \right) = \frac{\phi'^2}{24} - \frac{V}{12}, \quad (7)$$

$$3M^3 \left(A'' + H^2 e^{-2A} \right) = -\frac{\phi'^2}{2} - \frac{1}{2}\lambda_1\delta(y) - \frac{1}{2}\lambda_2\delta(y-L). \quad (8)$$

We note that the equations for 00-component of the Einstein equations and ij -component of the Einstein equations have an equal form and lead to (8).

Equations (6), (7), (8) can be rewritten as equations on the interval $(0, L)$

$$\phi'' + 4A'\phi' = \frac{dV}{d\phi}, \quad (9)$$

$$M^3 \left(A'^2 - H^2 e^{-2A} \right) = \frac{\phi'^2}{24} - \frac{V}{12}, \quad (10)$$

$$3M^3 \left(A'' + H^2 e^{-2A} \right) = -\frac{\phi'^2}{2} \quad (11)$$

and standard boundary conditions on the branes

$$\phi'|_{y=+0} = \frac{1}{2} \frac{d\lambda_1}{d\phi}, \quad \phi'|_{y=L-0} = -\frac{1}{2} \frac{d\lambda_2}{d\phi}, \quad (12)$$

$$A'|_{y=+0} = -\frac{1}{12M^3}\lambda_1, \quad A'|_{y=L-0} = \frac{1}{12M^3}\lambda_2. \quad (13)$$

It is not difficult to show that only two equations of (9), (10), (11) are independent; one can show it by differentiating (10) and substituting the result into (11).

We consider the standard exponential bulk potential

$$V = -\beta^2 e^{-\gamma\phi} \quad (14)$$

and the following brane potentials

$$\lambda_1(\phi) = \epsilon_1 + F_1(x) \cdot (\phi - \phi_1), \quad (15)$$

$$\lambda_2(\phi) = \epsilon_2 + F_2(x) \cdot (\phi - \phi_2), \quad (16)$$

where $F_{1,2}(x)$ are auxiliary scalar fields, $\phi_{1,2}$ and $\epsilon_{1,2}$ are constants. Note that the fields $F_{1,2}(x)$ have no kinetic terms. One can recall that supersymmetry is based on the use of such auxiliary fields, which are necessary for reaching the closure of the supersymmetry

algebra [7]. A simple example with the fields of such type in classical field theory can be also found in [7]. The equations of motions for the fields $F_{1,2}(x)$ (which can be obtained by means of the standard variation procedure with respect to the fields $F_{1,2}(x)$) give

$$\phi|_{y=0} = \phi_1, \tag{17}$$

$$\phi|_{y=L} = \phi_2. \tag{18}$$

In fact the fields $F_{1,2}(x)$ play the role of Lagrange multipliers. We will see below that conditions (17) and (18) fix the size of the extra dimension. Such method of stabilization (with the help of auxiliary fields) seems to be quite simple and reduces the number of parameters to be fine-tuned. The physical consequences are equal to those of the stiff brane potentials used in [5].

Now let us consider two possible cases.

1. $H = 0$.

Solution to equations of motion (9), (10) and (11) in the interval $(0, L)$ has the form

$$A = \frac{2}{3\gamma^2 M^3} \ln(ky + C_1) + C, \tag{19}$$

$$\phi = \frac{2}{\gamma} \ln(ky + C_1), \tag{20}$$

where C, C_1 are constants and

$$k^2 = \frac{3\beta^2 \gamma^4 M^3}{16 - 6\gamma^2 M^3}. \tag{21}$$

We also suppose that $\gamma < 2\sqrt{\frac{2}{3M^3}}$.

Equation (17) defines the constant C_1

$$C_1 = e^{\frac{\gamma\phi_1}{2}},$$

whereas the size of the extra dimension is defined by equation (18)

$$L = \frac{e^{\frac{\gamma\phi_2}{2}} - e^{\frac{\gamma\phi_1}{2}}}{k}.$$

The constant C should be defined by the requirement to have Galilean coordinates on the brane. It means that the values of C are different for different branes: $C = -\frac{\phi_1}{3\gamma M^3}$ for the brane at $y = 0$ ($A(0) = 0$) and $C = -\frac{\phi_2}{3\gamma M^3}$ for the brane at $y = L$ ($A(L) = 0$), see detailed discussion about Galilean coordinates on the branes in [8–10]. In other words, its value is defined by the physical four-dimensional scale we suppose to use.

We see that all the parameters of the background solution appear to be defined. Boundary conditions (12) result in

$$F_1 = \frac{4k}{\gamma} e^{-\frac{\gamma\phi_1}{2}}, \tag{22}$$

$$F_2 = -\frac{4k}{\gamma} e^{-\frac{\gamma\phi_2}{2}}. \tag{23}$$

Boundary conditions (13) suggest that the brane tensions $\epsilon_{1,2}$ should be fine-tuned

$$\epsilon_1 = -\frac{8k}{\gamma^2} e^{-\frac{\gamma\phi_1}{2}} = -12M^3 \frac{\beta\gamma}{\sqrt{6}} \frac{e^{-\frac{\gamma\phi_1}{2}}}{\sqrt{2M^3\gamma^2 - \frac{3}{4}\gamma^4 M^6}}, \quad (24)$$

$$\epsilon_2 = \frac{8k}{\gamma^2} e^{-\frac{\gamma\phi_2}{2}} = 12M^3 \frac{\beta\gamma}{\sqrt{6}} \frac{e^{-\frac{\gamma\phi_2}{2}}}{\sqrt{2M^3\gamma^2 - \frac{3}{4}\gamma^4 M^6}}. \quad (25)$$

Such a fine-tuning is inherent to almost all five-dimensional brane world models with compact extra dimension.

2. $H \neq 0$.

In this case the solution to equations of motion (9), (10) and (11) in the interval $(0, L)$ has the form

$$A = \ln(ky + C_1) + C, \quad (26)$$

$$\phi = \frac{2}{\gamma} \ln(ky + C_1), \quad (27)$$

with

$$k^2 = \frac{\beta^2\gamma^2}{6} \quad (28)$$

and

$$H^2 e^{-2C} = \frac{k^2}{3\gamma^2 M^3} (3\gamma^2 M^3 - 2). \quad (29)$$

We also suppose that $\gamma > \sqrt{\frac{2}{3M^3}}$. Equation (17) defines the constant C_1

$$C_1 = e^{\frac{\gamma\phi_1}{2}}.$$

The size of the extra dimension is

$$L = \frac{e^{\frac{\gamma\phi_2}{2}} - e^{\frac{\gamma\phi_1}{2}}}{k}.$$

Boundary conditions (12) result in

$$F_1 = \frac{4k}{\gamma} e^{-\frac{\gamma\phi_1}{2}}, \quad (30)$$

$$F_2 = -\frac{4k}{\gamma} e^{-\frac{\gamma\phi_2}{2}}. \quad (31)$$

We see that the form of (30), (31) is the same as that of (22), (23). It follows from the fact that the form of the solutions for the scalar field is the same for both cases (see equations (20) and (27)).

Boundary conditions (13) suggest that the brane tensions $\epsilon_{1,2}$ should be also fine-tuned (compare with (24), (25))

$$\epsilon_1 = -12M^3 \frac{\beta\gamma}{\sqrt{6}} e^{-\frac{\gamma\phi_1}{2}}, \quad (32)$$

$$\epsilon_2 = 12M^3 \frac{\beta\gamma}{\sqrt{6}} e^{-\frac{\gamma\phi_2}{2}}. \quad (33)$$

Thus, we have shown that for $\sqrt{\frac{2}{3M^3}} < \gamma < 2\sqrt{\frac{2}{3M^3}}$ there exist two different solutions for the system with the bulk potential (14). The only difference is in the fine-tuned values of the brane tensions, see (24), (25) and (32), (33). Very similar situation of two different background solutions with a fixed bulk scalar field potential was discussed in [5]. We note that the background solutions presented above are exact.

A very peculiar case is $\gamma = \sqrt{\frac{2}{M^3}}$. For $\gamma = \sqrt{\frac{2}{M^3}}$ we get fine-tuned tensions

$$\epsilon_1 = -4\beta\sqrt{3M^3} e^{-\frac{\phi_1}{\sqrt{2M^3}}}, \quad (34)$$

$$\epsilon_2 = 4\beta\sqrt{3M^3} e^{-\frac{\phi_2}{\sqrt{2M^3}}} \quad (35)$$

for both cases ($H = 0$ and $H \neq 0$)! It means that there are two different stabilized solutions corresponding to the bulk scalar field potential (14) and brane tensions (34), (35). The first one is

$$A = \frac{1}{3} \ln \left(\sqrt{\frac{3}{M^3}} \beta |y| + e^{\frac{\phi_1}{\sqrt{2M^3}}} \right) + C, \quad \phi = \sqrt{2M^3} \ln \left(\sqrt{\frac{3}{M^3}} \beta |y| + e^{\frac{\phi_1}{\sqrt{2M^3}}} \right), \quad (36)$$

$$H = 0, \quad L = \sqrt{\frac{M^3}{3}} \left(\frac{e^{\frac{\phi_2}{\sqrt{2M^3}}} - e^{\frac{\phi_1}{\sqrt{2M^3}}}}{\beta} \right), \quad (37)$$

$$F_1 = \sqrt{24}\beta e^{-\frac{\phi_1}{\sqrt{2M^3}}}, \quad F_2 = -\sqrt{24}\beta e^{-\frac{\phi_2}{\sqrt{2M^3}}}; \quad (38)$$

whereas the second solution is

$$A = \ln \left(\sqrt{\frac{1}{3M^3}} \beta |y| + e^{\frac{\phi_1}{\sqrt{2M^3}}} \right) + C, \quad \phi = \sqrt{2M^3} \ln \left(\sqrt{\frac{1}{3M^3}} \beta |y| + e^{\frac{\phi_1}{\sqrt{2M^3}}} \right), \quad (39)$$

$$H = e^C \frac{\sqrt{2}\beta}{3\sqrt{M^3}}, \quad L = \sqrt{3M^3} \left(\frac{e^{\frac{\phi_2}{\sqrt{2M^3}}} - e^{\frac{\phi_1}{\sqrt{2M^3}}}}{\beta} \right), \quad (40)$$

$$F_1 = \sqrt{\frac{8}{3}}\beta e^{-\frac{\phi_1}{\sqrt{2M^3}}}, \quad F_2 = -\sqrt{\frac{8}{3}}\beta e^{-\frac{\phi_2}{\sqrt{2M^3}}}. \quad (41)$$

We see that there is a doubling of the background solution for appropriate values of the model parameters. Both solutions correspond to fixed sizes of the extra dimension, maximally-symmetric spaces on the branes (maximally symmetric spaces are Minkowski, dS and AdS, see [11]) and both correspond to stationary cosmological solutions. Thus it seems that there are no phenomenological criteria (such as a symmetry criterion) to choose between the solutions.

Of course, such doubling is the consequence of our choice of the stabilizing brane potentials (15), (16). One can chose a more familiar form of the potentials (see, for example, [1]):

$$\lambda_1(\phi) = -4\beta\sqrt{3M^3} e^{-\frac{\phi_1}{\sqrt{2M^3}}} + Q_1 \cdot (\phi - \phi_1) + q_1^2 (\phi - \phi_1)^2, \quad (42)$$

$$\lambda_2(\phi) = 4\beta\sqrt{3M^3} e^{-\frac{\phi_2}{\sqrt{2M^3}}} - Q_2 \cdot (\phi - \phi_2) + q_2^2 (\phi - \phi_2)^2, \quad (43)$$

where Q_1, Q_2, q_1, q_2 are constants. If $Q_1 = \sqrt{24} \beta e^{-\frac{\phi_1}{\sqrt{2M^3}}}$, $Q_2 = \sqrt{24} \beta e^{-\frac{\phi_2}{\sqrt{2M^3}}}$, then one should consider the first background solution with $H = 0$, if $Q_1 = \sqrt{\frac{8}{3}} \beta e^{-\frac{\phi_1}{\sqrt{2M^3}}}$, $Q_2 = \sqrt{\frac{8}{3}} \beta e^{-\frac{\phi_2}{\sqrt{2M^3}}}$, then one should consider the second background solution with $H \neq 0$. But the latter brane potentials appear to be more fine-tuned than potentials (15), (16).

Now let us discuss the obtained results and compare them with those obtained earlier. In [1] the problem of uniqueness of solutions to (6)–(8) was discussed. Although the main solutions in [1] were obtained with the help of the superpotential method (see also [12]), some statements for the general case were also made. It was argued that there should exist a solution to (6)–(8) without fine-tuning of the parameters and this solution is (locally) unique. This seems to be not correct in general. Indeed, even with quite relaxed brane potentials (15), (16), which contain additional degrees of freedom described by the scalar fields $F_{1,2}(x)$, there should be fine-tuning of the brane tensions (24), (25) or (34), (35) in order to get the solutions. For the case $\gamma = \sqrt{\frac{2}{M^3}}$ the solutions appear to be not unique globally, and only additional fine-tuning can separate them (although the uniqueness of the solution was discussed in [1] for the case $H \neq 0$, formally the case $H = 0$ should be included as a possible solution to this system of equations). The subtle point is that our relaxed brane potentials allow the existence of two different global solutions. If we take highly fine-tuned brane potentials (42), (43), which depend only on ϕ (analogous potentials were considered in [1]), the solutions seem to be even globally unique for the given values of Q_1, Q_2 .

Nevertheless, it was noted in [1] that at least for $H = 0$ there can be a discrete set of solutions to the equations of motion with fixed values of ϕ_1 and ϕ_2 . As we have seen above in a special case the solutions can even belong to different classes — one with $H = 0$ and another with $H \neq 0$.

In this connection it should be mentioned that, as it was noted in the beginning of the paper, in [5] an analogous situation with two solutions corresponding to $H = 0$ and $H \neq 0$ for a given bulk scalar field potential was considered. To find the solutions the superpotential method was used, at the same time the brane potentials were chosen to be stiff, which led to the same physical consequences as our choice (15), (16): the only boundary conditions for the scalar field in both cases are (17), (18). In this sense, besides the choice of the bulk scalar field potential, our model and the model of [5] are very similar. The first exact solution in [5] with $H = 0$ was the one previously obtained in [1], whereas the second approximate solution corresponding to $H \neq 0$ was found perturbatively. It has been shown in [5] that the second solution corresponds to the case where there is no fine-tuning of the tension on the second brane (but with the fine-tuning on the first brane retained). Our results show that such situation can be realized even if we retain fine-tuning of the second brane tension.

Finally we would like to note that although the doubling of background solution happens only for a particular choice of the parameter γ in (14) and in the special case of brane potentials (15), (16), more realistic brane world models could also lead to different background solutions for equal values of fundamental parameters. It is necessary to bear this in mind while examining brane world models.

Acknowledgments

The author is grateful to I.P. Volobuev for valuable discussions.

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