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Leptogenesis and tensor polarisation from a gravitational Chern-Simons term

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Abstract: Within an effective field theory derived from string theory, the universal axion has to be coupled to the the gravitational Chern-Simons (gCS) term. During any era when the axion field is varying, the vacuum fluctuation of the gravitational wave amplitude will then be circularly polarised, generating an expectation value for the gCS term. The polarisation may be observable through the Cosmic Microwave Background, and the vacuum expectation value of the gCS term may generate the baryon asymmetry of the Universe. We argue here that such effects cannot be computed without further input from string theory, since the 'vacuum' in question is unlikely to be the field-theoretic one.

Keywords: Chern-Simons Theories, Classical Theories of Gravity, Models of Quantum Gravity, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

It has been pointed out [1]–[6] that the action of the effective field theory should contain a gravitational Chern-Simons (gCS) term coupled to a scalar field such as the universal axion:

$$S_{\rm gCS} = \frac{M_{\rm P}^2}{4}\int d^4x(-g)^{1/2} f(\phi) R\tilde{R}\,, \tag{1.1}$$

$$R\tilde{R} \equiv \epsilon^{\alpha\beta\gamma\delta} R_{\alpha\beta\rho\sigma} R_{\gamma\delta}{}^{\rho\sigma}\,, \tag{1.2}$$

as a result of the Green-Schwarz mechanism [7]. Here $M_{\rm P} = 2.4\times10^{18}$ GeV is the reduced Planck mass, and f is an odd function.

In the reasonable approximation that f is linear it may be written as

$$f = \frac{\mathcal{N}}{\mu^2}\frac{\phi}{M_{\rm P}}\,, \tag{1.3}$$

where μ is the string scale (representing the ultra-violet cutoff for the effective field theory including gravity) and $\mathcal{N}$ is of order 1 at least for the case studied in [4][1].

If f varies during some era, the gCS term will polarise the vacuum fluctuation of the gravitational wave amplitude. On cosmological scales, this may be directly observable through the Cosmic Microwave Background [1, 2, 5]. On much smaller scales it may be observable indirectly, through the generation of baryon number [3, 4]. In this note we examine both effects.

[1] Our $\mathcal{N}$ is $(\mu/M_{\rm P})^2$ times that of reference [5].

2. Quantising the tensor perturbation

Quantisation of the gravitational wave amplitude in the presence of a gCS term has not been given before. We describe it here, adopting an approach which relies mainly on the field equation.

We write the line element for the expanding Universe, displaying only the tensor perturbation which corresponds to a gravitational wave amplitude:

$$ds^2 = a^2(\tau)\left(d\tau^2 - (\delta_{ij} + 2h_{ij}(\mathbf{x},\tau))\,dx^i dx^j\right). \tag{2.1}$$

Working to first order, the transverse and traceless gravitational wave amplitude h_{ij} may be written as

$$h_{ij}(\mathbf{x},\tau) = \frac{\sqrt{2}}{M_{Pl}} \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}} \sum_p \epsilon^p_{ij}(\mathbf{k}) h_p(\mathbf{k},\tau)\,, \tag{2.2}$$

where $p=$ R or L, and the polarisation tensors satisfy[2]

$$\begin{aligned}
k_i\epsilon_{ij}(p,\mathbf{k}) &= 0\,,\\
\epsilon^*_{ij}(p,\mathbf{k})\epsilon_{ij}(p',\mathbf{k}) &= 2\delta_{pp'}\,,\\
\epsilon^{ilm}\epsilon^*_{ij}(L,\mathbf{k})\epsilon_{jl}(R,\mathbf{k}) &= \epsilon^{ilm}\epsilon^*_{ij}(R,\mathbf{k})\epsilon_{jl}(L,\mathbf{k}) = 0\,,\\
\epsilon^{ilm}\epsilon^*_{ij}(L,\mathbf{k})\epsilon_{jl}(L,\mathbf{k}) &= -\epsilon^{ilm}\epsilon^*_{ij}(R,\mathbf{k})\epsilon_{jl}(R,\mathbf{k}) = 2i\frac{k_m}{|\mathbf{k}|}.
\end{aligned} \tag{2.3}$$

We first recall the familiar situation where there is no gCS term, presenting it in a way which will make it easy to include the gCS term. We need the field equation for the mode functions h_p, evaluated to first order so that it is linear in h_p. It is given by the Einstein action

$$S_{\rm E} = \frac{M_{\rm P}^2}{2}\int d^4x(-g)^{1/2}R\,, \tag{2.4}$$

and it may be obtained by either of two routes.

1. Start with the full field equation $R_{\alpha\beta}=0$ and take its first-order perturbation [8].
2. Take the second-order perturbation of the full action and from it derive the first-order field equation [9].

Either way, it is convenient to consider a re-defined mode function

$$\mu_p \equiv z_p h_p\,, \tag{2.5}$$

with[3] $z_p \equiv a$. Then the field equation is

$$\mu_p'' + \left(k^2 - \frac{z_p''}{z_p}\right)\mu_p = 0\,, \tag{2.6}$$

where a prime will denote $d/d\tau$.

[2]These expressions follow from the properties of the rotational transformations.

[3]We use the notation z_p because this quantity will depend on the polarisation on p when we include the gCS term.

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To quantise one needs an action. The second-order action is given by $S_{\rm E} = \int d\tau L_{\rm E}$, with lagrangian

$$L_{\rm E} = \frac{1}{2}\int d^3k \left[\mu_p'^{\,2} + \left(k^2 + \frac{z_p''}{z_p}\right)\mu_p^2\right] . \tag{2.7}$$

This action is determined (up to irrelevant surface terms) by the field equation except for its normalisation. To determine that normalisation it is enough to know the action in the subhorizon regime $k \gg aH$. In this regime the term z_p''/z_p becomes negligible and, the action as well as the field equation is the same as for a massless scalar field with Fourier components μ_p. We see that the first approach, supplemented by an evaluation of the second-order action in the sub-horizon regime, can provide the full second-order action.[4]

Promoting the gravitational wave amplitude to an operator $\hat{h}_{ij}$, we work in the Heisenberg picture so that the operator satisfies the classical equation of motion. Expanding in Fourier modes we write

$$\hat{h}_{ij}(\mathbf{x},\tau) = \frac{\sqrt{2}}{M_{Pl}}\int \frac{d^3k}{(2\pi)^{3/2}}\sum_p \left[e^{i\mathbf{k}\cdot\mathbf{x}} h_p(k,\tau)\epsilon_{ij}(p,\mathbf{k})\hat{a}_p(\mathbf{k}) + \text{h.c.}\right] . \tag{2.8}$$

The mode function $h_p(k,\tau)$ is related to the operator $\hat{h}_p(\mathbf{k},\tau)$ appearing in the expansion of equation (2.2) by

$$\hat{h}_p(\mathbf{k},\tau) = h_p(k,\tau)\hat{a}_p(\mathbf{k}) + h_p^*(-k,\tau)\hat{a}_p^\dagger(-\mathbf{k}) . \tag{2.9}$$

Without loss of generality we impose the commutation relation

$$[\hat{a}_p(\mathbf{k}), \hat{a}_{p'}^\dagger(\mathbf{k}')] = \delta^3(\mathbf{k}-\mathbf{k}')\delta_{pp'} . \tag{2.10}$$

Then canonical quantisation corresponds to choosing the following Wronskian for the mode function;

$$\mu_p'\mu_p^* - \mu_p\mu_p^{*\prime} = -i . \tag{2.11}$$

We also may define a time-independent vacuum state by

$$\hat{a}_p(\mathbf{k})|0\rangle = 0 . \tag{2.12}$$

Finally, we demand that well before horizon entry the vacuum corresponds to the state with no gravitinos, which corresponds to choosing the mode function

$$\mu_p = \frac{1}{\sqrt{2k}}e^{-ik\tau} . \tag{2.13}$$

The mode function at later times is calculated by solving equation (2.6), but we shall not need it.

[4] It is even superfluous to write down the action except in the sub-horizon regime since canonical normalisation once established is preserved by the equation of motion. Similar considerations apply to the case of the curvature perturbation [10]. In both cases, the second approach has become the standard one, though the simpler first approach was the original one.

With these preliminaries it is easy to include the effect of the gCS term. Without loss of generality we can continue to define the vacuum by equation (2.12), but now the mode function is different. Either from the field equation $R_{\alpha\beta} = \tilde{T}_{\alpha\beta}$, where the right hand side is the effective energy-momentum tensor provided by the gCS term [2], or by deriving the contribution of the gCS term to the second-order action [5], one can show that the mode functions μ_p defined by equation (2.5) satisfy the field equation (2.6), with the modified factors

$$z_p = \sqrt{a^2 \pm k f'}\,, \tag{2.14}$$

where the plus sign is for $p = \mathrm{L}$ and the minus sign is for $p = \mathrm{R}$.

The total second-order action (coming from $S_{\mathrm{E}} + S_{\mathrm{gCS}}$) has the form of equation (2.7), up to irrelevant surface terms. Indeed, by analogy with the previous discussion, the action is defined up to a constant factor by the field equation, and the factor is defined by matching to an already-known limit, namely the limit $f' \to 0$ in which the gCS term is negligible.

The above argument, leading to the second-order action in the presence of a gCS term has not been given before. An expression for this action, derived from a lengthy calculation, has been given in [5], but it is not manifestly equivalent (up to surface terms) to equation (2.7). On the basis of the above argument we know that it must be equivalent, but there is no need to show this explicitly.

Since the action is the same as before, the commutation relation in equation (2.10) and the Wronskian condition in equation (2.11) are also the same and we can still define the vacuum by equation (2.12). The crucial difference, as we are now going to discuss, is that there is unlikely to be any initial sub-horizon regime where the gCS term is negligible (unless it is negligible at all times in which case there is nothing to discuss). In the absence of such a regime, the initial condition for the mode functions is at present unknown which means that one cannot calculate anything.

3. The generation of circularly polarised gravitational waves

During inflation, comoving scales comparable with the present Hubble distance are supposed to leave the horizon.[5] The vacuum fluctuation of the gravitational wave amplitude on these scales is then converted to a classical perturbation, which after horizon entry corresponds to gravitational waves whose effect on the Cosmic Microwave Background anisotropy may be observable. The idea [1, 2, 5] is that the gCS term will give the waves some circular polarisation whose effect may be detectable.

The amplitude of the gravitational waves is proportional to the inflationary Hubble parameter H_*, and the present bound $r \lesssim 10^{-1}$ on their spectrum relative to that of the curvature perturbation requires $H_* \lesssim 10^{-5} M_{\mathrm{P}}$. On the other hand, they will never be detectable [11] unless $r \gtrsim 10^{-4}$, corresponding to $H_* \gtrsim 10^{-6} M_{\mathrm{P}}$. We focus on this high range for H_*, while recognising that most inflation models give a smaller value [10] corresponding to unobservable gravitational waves.

[5] Recall that wavenumbers $k < aH$ are said to be outside the horizon, and bigger wavenumbers are said to be inside the horizon.

There are a three things to take into account, which have perhaps not been sufficiently emphasised before.

1. The effective field theory will presumably become valid only when the energy density $3M_{\rm P}^2H^2$ falls below μ^4.

2. The discussion for a given scale k should begin only when the wavenumber is below the string scale, $k/a < \mu$.

3. In order to obtain a prediction using known methods, we need an initial era when the gCS term has negligible time-dependence, so that the initial condition in equation (2.13) can be imposed on the mode function.

The problem, as we now explain, is that the second and third requirements are practically incompatible if the polarisation is to have an observable effect.

Taking f to have the linear form as in equation (1.3), z_p is then given by

$$(z_p/a)^2 - 1 = \pm\mathcal{N}\left(\frac{k}{a\mu}\right)\left(\frac{M_{\rm P}H}{\mu^2}\right)\left(\frac{\mu}{M_{\rm P}}\right)\left(\frac{\dot\phi}{M_{\rm P}H}\right) . \tag{3.1}$$

The term on the right comes from the gCS term, and to satisfy our third requirement we would need an initial era when this term is negligible.

The first, second, and third terms in brackets are obviously less than 1. Provided that ϕ is canonically normalised the final term is also less than 1, because the energy density then has a contribution $\frac{1}{2}\dot\phi^2$ which must be less than the total energy density. (For an estimate it is reasonable to suppose that one can take ϕ to be approximately canonically normalised while it is varying. Doing that might be artificial from the string viewpoint, altering the expectation $\mathcal{N} \sim 1$, but according to [4] one still expects $\mathcal{N} \lesssim 100$ or so.) If ϕ is the inflaton, or a component of the inflaton in a multi-field model, the final term is very much less than 1. In addition it is slowly varying so that it may be regarded as a constant when evaluating z_p''.

We see that the effect of the gCS term will be significant only if the smallness of the product of the four terms in brackets is compensated by a sufficiently large value of $\mathcal{N}$. Assuming that this happens, we will now explain why the requirements 2 and 3 are likely to be incompatible.

The scale k in which we are interested (corresponding to the size of the presently observable Universe) leaves the horizon during inflation at the epoch $k = aH_*$. This scale emerges from the string scale (at the epoch $k = a\mu$) only $N_{\rm str} \equiv \ln(\mu/H_*) < \ln(M_{\rm P}/H_*)$ e-folds before it leaves the horizon, unless inflation begins a fewer number of e-folds before that epoch. But to get an observable tensor perturbation one needs $H_* \gtrsim 10^{-6}M_{\rm P}$, making $N_{\rm str} < 14$. It seems unlikely that inflation will start such a small number of e-folds before the observable Universe leaves the horizon. Discounting that possibility, we see that *on the scale of interest, the gCS term can be significant only during the first few Hubble times after it emerges from the string scale.* This is why requirements 2 and 3 are practically incompatible; they require that the motion of ϕ switches on suddenly during inflation, within just the few Hubble times before the observable Universe leaves the horizon.

Having explained why the gCS term will not be initially negligible (unless it is always so) let us consider the opposite possibility that it initially dominates making $|z_p/a|^2 \gg 1$. Then evolution will become singular [5] when $(z_p/a)^2$ passes through zero for one of the polarisation states. At this point, at least the linear calculation becomes unphysical.

It may be that a non-linear calculation would make sense, allowing $|z_p/a|^2 \gg 1$ as an initial condition. It has been noticed [4] that the initial value of z_p''/z_p is then practically zero, if $\dot{\phi}$ is practically constant corresponding to ϕ rolling very slowly. In that case equation (2.13) is a correctly-normalised solution of the mode function equation [5], but it seems hard to justify the use of this solution as an initial condition. The physical mode function h_p decreases like $a^{-1/2}$, as opposed to the a^{-1} behaviour that would correspond to a redshifting gravitino momentum, which is hardly surprising since it is the gCS term rather than Einstein gravity which is dominating the dynamics of h_p. As a result it is hard to see how the vacuum state in equation (2.12) can be regarded as a no-particle state representing minimal energy density, which is the usual justification for using such a state. However we can still say that $\hat{a}$ annihilates the vacuum, as in equation (2.12), at the expense of compensating this effect with an adequate, but at the moment unknown, definition of the mode function. We conclude that even if the gCS term dominates initially, one still does not know the initial condition for the mode function.

4. Leptogenesis

It is known [12] that the gravitational anomaly violates lepton number conservation through a term

$$\partial_\mu J^\mu_L \supset \frac{1}{16\pi^2} R\tilde{R} \,. \tag{4.1}$$

If the gCS term is nonzero during some era $t_1 < t < t_2$, lepton number $n_L \equiv J^0_\ell$ is generated:

$$n_L = \frac{1}{16\pi^2} \int_{t_1}^{t_2} R\tilde{R}\, dt \,. \tag{4.2}$$

Alexander, Peskin, and Sheikh-Jabbari [3] have pointed out that this could be the origin of the observed baryon number, with lepton number generating baryon number at the electroweak transition.

The idea is to generate the gCS term from the vacuum fluctuation that we have been discussing. Up to second order, the contribution of h_{ij} to $R\tilde{R}$ is[6]

$$R\tilde{R} = -\frac{8}{a^4}\,\epsilon^{ijk}\left(\frac{\partial^2}{\partial_l \partial\tau} h_{jm} \frac{\partial^2}{\partial_m \partial_i} h_{kl} - \frac{\partial^2}{\partial_l \partial\tau} h_{jm} \frac{\partial^2}{\partial_l \partial_i} h_{km} + \frac{\partial^2}{\partial\tau^2} h_{jl} \frac{\partial^2}{\partial_i \partial\tau} h_{lk}\right). \tag{4.3}$$

Inserting equation (2.8) the vacuum expectation value, given here for the first time, is[7]

$$\begin{aligned}\langle 0 \mid \widehat{R\tilde{R}} \mid 0\rangle = \frac{16}{a^4 M_{\rm P}^2} \int \frac{d^3k}{(2\pi)^3} \big[&- k^2 h_L^*(k,\tau) h_L'(k,\tau) + k^2 h_R^*(k,\tau) h_R'(k,\tau) + \\ &+ h_L'^*(k,\tau) h_L''(k,\tau) - h_R'^*(k,\tau) h_R''(k,\tau)\big] \,. \end{aligned} \tag{4.4}$$

[6]This expression reproduces the Eq. (10) in reference [3] for the case where gravity waves move in the z direction.

[7]This expression is complex, corresponding to the fact that the naively-defined operator $\widehat{R\tilde{R}}$ is not hermitian. Presumably we should take the real part, corresponding to the hermitian part of the naively-defined operator.

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The integration over k is cut off at the value $k \simeq a\mu$, and the presence of spacetime derivatives presumably will mean that values close to the cutoff will dominate the integral. If that is the case we need to know each mode function just after it emerges from the string scale. Its subsequent evolution is irrelevant, assuming that it is not so drastic as to prevent the value $k \simeq a\mu$ from dominating the integral.

Although it is only of historical interest, we end with the following extended comment. The earlier discussions [3, 4] ignored the problem that the initial condition is unknown, and also dropped terms containing more than two spacetime derivatives from the evolution equation. The appropriately normalised solutions after the gCS term switches on are then

$$h_p(k,\tau) = \frac{1}{a} e^{-ik\tau} e^{\pm k\Theta(\tau-\tau_0)/2}\,, \tag{4.5}$$

where

$$\Theta \equiv \frac{4}{a^2}\left(f'' + aHf'\right). \tag{4.6}$$

Inserting this into equation (4.4) gives[8]

$$\langle 0 \mid \widehat{R\tilde{R}} \mid 0 \rangle = \frac{4H^2\Theta}{\pi^2 M_P^2}\mu^4\,. \tag{4.7}$$

This expression differs from the one obtained in [3]. The derivations cannot be directly compared because we have used the Fourier decomposition whereas those of [3] used a Green function method. As one easily checks, we would recover the results of [3] if we multiplied equation (4.5) by a factor ik/H, but this would not correspond to canonical normalisation nor would it satisfy the requirement that the factors k and a should enter into physical quantities only through the physical wavenumber k/a.

5. Conclusion

The initial condition should presumably be provided by string theory. Since the presence of a gCS term is indeed a requirement in string theory, it seems urgent to either find the initial condition or to become convinced that the universal axion is frozen until the energy density falls well below the string scale rendering the gCS term negligible.

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[8] In an earlier work [13] we assumed that the $+$, $\times$ states decoupled (instead of the L and R states), arriving at the incorrect result that $R\tilde{R} = 0$. We thank the authors of [3] for pointing out this mistake.

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Erratum

- Eq. (1.1) should be replaced by

$$S_{\rm gCS} = \frac{M_{\rm P}^2}{4} \int d^4x f(\phi) R\tilde{R} \,,$$

- The fourth expression in eq. (2.3) should be replaced by

$$\epsilon^{ilm}\epsilon^{*}_{ij}(L,\mathbf{k})\epsilon_{jl}(L,\mathbf{k}) = -\epsilon^{ilm}\epsilon^{*}_{ij}(R,\mathbf{k})\epsilon_{jl}(R,\mathbf{k}) = -2i\frac{k_m}{|\mathbf{k}|}\,.$$

- Eq. (2.8) should be replaced by

$$\hat{h}_{ij}(\mathbf{x},\tau) = \frac{\sqrt{2}}{M_{pl}} \int \frac{d^3k}{(2\pi)^{3/2}} \sum_p \left[e^{i\mathbf{k}\cdot\mathbf{x}} h_p(k,\tau)\epsilon_{ij}(p,\mathbf{k})\hat{a}_p(\mathbf{k}) \right].$$

- In the second line of the second paragraph in page 6, the cited reference [4] should be actually the reference [5].
- Eq. (4.4) should be replaced by

$$\langle 0 \mid \widehat{R\tilde{R}} \mid 0\rangle = \frac{16}{a^4 M_{\rm P}^2} \int \frac{d^3k}{(2\pi)^3} \Big[k^2 h^{*}_L(k,\tau)h'_L(k,\tau) - k^2 h^{*}_R(k,\tau)h'_R(k,\tau) - \\ - h'^{*}_L(k,\tau)h''_L(k,\tau) + h'^{*}_R(k,\tau)h''_R(k,\tau) \Big]\,.$$