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The ancient art of laying rope

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Abstract – We describe a geometrical property of helical structures and show how it accounts for the early art of rope-making. Helices have a maximum number of rotations that can be added to them —and it is shown that this is a geometrical feature, not a material property. This geometrical insight explains why nearly identically appearing ropes can be made from very different materials and it is also the reason behind the unyielding nature of ropes. Maximally rotated strands behave as zero-twist structures. Hence, under strain they neither rotate in one direction nor in the other. The necessity for the rope to be stretched while being laid, known from Egyptian tomb scenes, follows straightforwardly, as does the function of the top, an old tool for laying ropes.

Introduction. – The crafting of rope, string and cordage has been an essential skill through the times going back to early prehistoric life. The image of rope is easily discernible and could perhaps be said to be iconic. It has also been important for early symbolic meaning and for creed. Examples are the spinning seidh [1], the shimenawa prayer rope [2], the shen ring, and the cartouche in hieroglyphs [3]. Scenes from Egyptian tombs display advanced rope-making [4,5].

Figure 1 shows a scene from the tomb of Akhethotep and Ptahhotep, and it appears to be depicted that the ropes are held under tensile stress while being laid. The hieroglyphics on the scene reads *"Rope-making for Shipbuilding"*. The round tool hanging from the rope is perhaps a stone helping the ropemakers to gauge that sufficient tensile stress is present: to maintain a nearly straight rope requires the presence of adequate tensile strength depending on the weight of the stone. In a scene from the tomb of Rekhmire a special belt is depicted which help the ropemakers to apply tensile stress by the use of their body weight [5]. Large quantities of ancient Egyptian rope have been found in a cave at the Red Sea coast [6,7], also found at the site are two limestones, with holes, now being discussed in the context of anchors [8,9]. We believe that another possible use of these stones, which only weigh a few kilograms in contrast to most stone anchors, might have been as weights used during the rope production akin to the tomb scene depicted in fig. 1.

Classical ropes appear with an easy discernible geometrical structure, even though they have been fabricated in different human cultures from a large variety of fibrous materials with diverse physical properties. Relatively recently, Zhang *et al.* [10] have demonstrated that yarn formation can be performed with the use of nano-sized strands. Why does the resulting geometry of rope appear so similar, as if it depends little on the material used, and why are ropes inextensible? Here we show, that these properties of rope are due to a universal behavior of helical structures which depends on geometry. It stems from the observation, derived below, that there is a maximum number of rotations that can be added to an N-helix, where $N \geqslant 2$. One consequence is that a tightly laid rope, where each of the strands are rotated to their maximum in one direction while being helically laid with the maximum number of rotations in the opposite direction will be interlocked, unable to unwind, and hence a functional rope.

Method. – Mathematical aspects of the helical geometry of yarns are described by Treloar [11] and by Fraser *et al.* [12], and a comprehensive review for wire rope is given by Costello [13]. The counter-twisting of the strands and the rope, respectively, has been discussed from a mathematical perspective [12], and rope-making in a historical context [14]. From the point of view of topology it is important to consider the amount of writhing, see Thompson and Campneys [15] and Stump *et al.* [16].

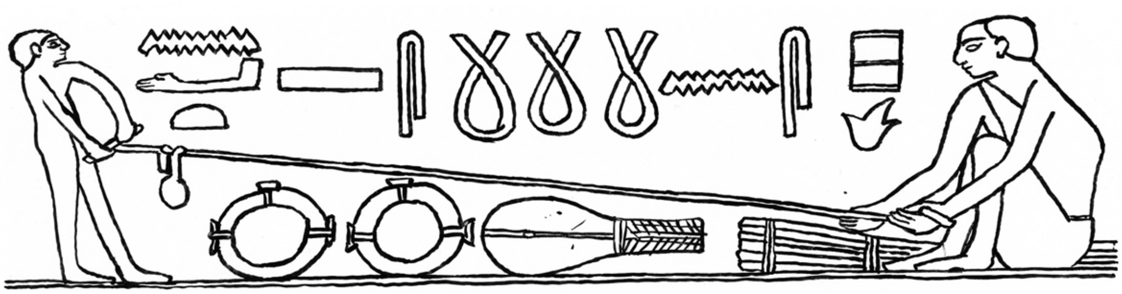

Fig. 1: Rope-making in ancient egypt. Tomb of Akhethotep and Ptahhotep, about 2300 BC. The round tool hanging from the rope close to the person to the left is perhaps a stone helping the ropemakers to gauge that sufficient tensile stress is present. Drawing by K. Olsen.

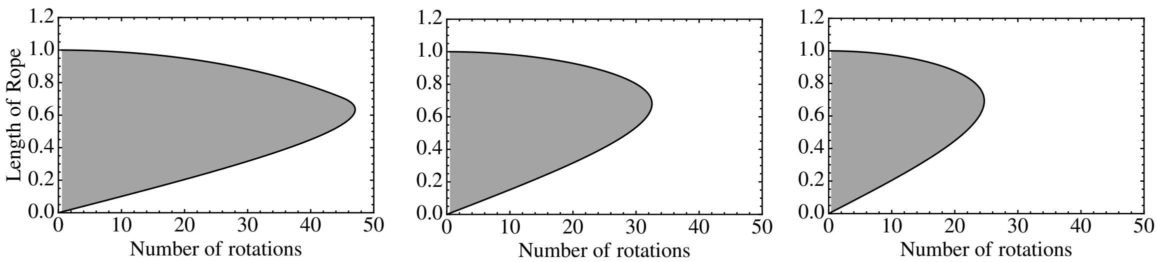

Fig. 2: Rope curve: the length of the formed rope as a function of the number of turns is given by the upper part of the solid line. The curve shown is for a two-, three- and four-stranded rope from left to right, respectively. The ropes are formed from strands each being 1 m long and having a strand diameter of 5 mm. The shape of these curves is universal for ropes with a given number of strands, while the specific number of rotations depend on the diameter and the length of the rope. At the maximally rotated point the triple-stranded rope is always 68% of the length of the individual strands. For a two- and four-stranded rope the numbers are 63% and 69%, respectively.

Thompson *et al.* have suggested that double helices will kinematically lock up at 45° [17], Gonzales and Maddocks have introduced the *global curvature* and investigated the significance hereof when understanding helical structure formation [18]. Przybył and Pierański have derived the conditions for self-contacts for single helices [19], and Neukirch and van der Heijden the condition for inter-strand contacts in an N-ply [20]. Przybył has considered the question of which tubular double helix is an ideal knot, *i.e.* uses the shortest length of tube per repeating helical unit [21]. We notice that this structure must be maximally rotated. If it were not maximally rotated another structure would have more repeats per unit length of tube. Recently, we have determined the close-packed helices from a calculation of the volume fraction for a helix as a function of the pitch angle. The molecular structures of α-helices and of DNA-A and -B are approximate examples of such optimally close-packed helices [22].

It is surprisingly simple to see that there is a geometrical limit to the number of rotations on a helix, and that helical structures can be maximally rotated. A helical curve in three-dimensional space is uniquely described by two independent variables; in differential geometry it is common to use curvature and torsion. For a description applicable to ropes formed from tubular strands, we will use (n_r, L_r) as parameters, where n_r is the number of turns the helical strands makes on the imaginary cylinder of the rope, and L_r is the length of the rope. Not all combinations of (n_r, L_r) are allowed and some are forbidden because of tubular interactions. The helical tubes are assumed to have hard walls and are therefore not allowed to intercept each other. The allowed values of (n_r, L_r) are gray shaded in fig. 2. The solid line is the boundary case where the helical tubes are in contact with each other, *i.e.* where the distance between two neighboring strands in the rope is equal to the diameter, D, of one strand. Hence the solid line in fig. 2 corresponds to all the packed structures. If a rope is laid under tensile stress one obtains the structures that correspond to the upper part of the solid curve. This follows from considering a point (n_r, L_r) within the shaded area of fig. 2 and applying a tensile stress such that n_r is held constant. In this case the point will move vertically until it reaches a point on the upper curve. The lower part corresponds to strands that are laid out flat on top of each other and then twisted around following the imaginary cylinder, which radius becomes smaller and smaller as one progresses along the curve towards the tip. At the tip of the curve the radius of the imaginary cylinder is larger than the radius of the strands, *i.e.* there remains a hollow central channel.

Now we discuss in more details how fig. 2 is obtained. For the calculation of the solid line it is useful to notice that the number of turns, n_r, can be expressed as

$$n_r = (L_s/2\pi a)\cos v_\perp. \quad (1)$$

Here L_s is the length of the strands, $v_\perp$ is the pitch angle of the strands defined relative to the equatorial plane, and a is the radius of an imaginary cylinder surface hosting the helical center line of the strands. The length of the fabricated rope, L_r, is

$$L_r = L_s \sin v_\perp. \quad (2)$$

These equations contain the radius, a, of the imaginary cylinder hosting the helical lines. The radius a, which is not a constant as it depends on $v_\perp$, can be determined from the requirement that the strands are in contact with each other. Recently, this requirement has been solved for tubular helices [19,20,22]. The N strands are described by their parametric equations,

$$\vec{r}_i = \left(a\cos t_i, a\sin t_i, ht_i + \frac{2\pi(i-1)h}{N}\right), \quad (3)$$

where $i \in \{1, 2, \ldots, N\}$ and $t_i \in \mathbb{R}$. The reduced pitch, h, is given as

$$h = \frac{L_r}{2\pi n_r}. \quad (4)$$

The requirement that the minimum distance between two strands is equal to the strand diameter is solved in two steps. First, the solutions are obtained for the transcendental equation,

$$\sin t + \left(\frac{L_r}{2\pi n_r a}\right)^2 \left(t + \frac{2\pi}{N}\right) = 0. \quad (5)$$

Here t is the parametric distance, $t = t_{i+1} - t_i$, between nearest points on neighboring helical curves describing the strands. Then, the radius a is found by the requirement that the nearest distance equals the strand diameter, D. *I.e.*, through the condition that

$$D^2 = a^2(\cos t - 1)^2 + a^2 \sin^2 t + \left(\frac{L_r}{2\pi n_r}\right)^2 \left(t + \frac{2\pi}{N}\right)^2. \quad (6)$$

In fig. 2 (where $N = 2, 3, 4$) a striking result is immediately visible, namely that there is a maximum number of turns that one can have on a rope of a given length!

The peculiar point on the curve in fig. 2 where the maximum possible number of rotations is obtained is at the turning point of $L_r(n_r)$ for the packed helices. At this point it is impossible for the strand to be further twisted.

These maximally rotated N-helices have the property that stretching will neither make the individual strands rotate in one direction, nor in the other. This follows from the tangent of the curve being vertical and therefore $\mathrm{d}n_r/\mathrm{d}L_r = 0$. The total twist, Θ, is the angular rotation of the strand around the imaginary cylinder, *i.e.* $\Theta = 2\pi n_r$, and thus we have, $\mathrm{d}\Theta/\mathrm{d}L_r = 0$ for this particular number of rotations. Henceforth, we therefore denote these helices as being zero-twist structures.

Table 1: Pitch angle, v_{ZT}, for the zero-twist structures given as a function of the number of strands. For the zero-twist structure there is no coupling from strain to rotation. Further, to these helical structures one cannot add additional rotations.

No. of strands	1	2	3	4	∞
v_{ZT}	–	39.4°	42.8°	43.8°	45°

Generally, we will denote any structure that has a vanishing strain-twist coupling as being a zero-twist structure. For $N \geqslant 2$ the geometrical zero-twist structures are given in table 1 in terms of their pitch angles. The case $N = 1$ is special as there is a divergence of the geometrical solution into two branches and therefore no universal geometrical point. If non-vanishing elastic constants are considered the divergence disappears and hence there is a material-dependent maximum number of turns that can be applied in the $N = 1$ case. Figure 3 displays the N-helix ($N = 2, 3, 4$) with a pitch angle of the zero-twist structures as a mathematical idealization.

An ideal tightly laid rope is a configuration where the strands are twisted to their zero-twist configuration in one direction while the rope is laid at its zero twist in the opposite direction. *E.g.*, the strands are at the tip of a rope curve pointing to the right, while the rope at the same time is at the tip of an other rope curve pointing to the left. Such a rope would therefore be infinitely rigid when taken as an ideal tubular idealization with hard walls, *i.e.* it would have no flexibility. In practice even a tightly laid rope will have some flexibility. One typical type of a classical rope has three strands where the individual strands each have two substrands. When flexibility is introduced by laying the rope close to, but not exactly at the zero-twist configuration, the rope will self-lock at the equilibrium given by

$$\frac{\mathrm{d}L_r}{\mathrm{d}n_r} = \frac{\partial L_r}{\partial n_r} + \frac{\partial L_r}{\partial L_s}\frac{\partial L_s}{\partial n_s}\frac{\partial n_s}{\partial n_r} = 0. \quad (7)$$

Here n_r is the number of turns the helical strands makes on the imaginary cylinder of the rope, and n_s quantify the counter-rotation from the pre-twisting of the strands. For the ideal tightly laid rope $n_r + n_s$ is equal to the sum of the maximum number of rotations of the rope and of the strands. In general, $n_r + n_s = \mathrm{const}$, where the size of the constant determines how hard the rope is laid. A triple stranded rope where n_r is 94% of its maximum will have a pitch angle of 52°.

Discussion. – A consequence of the presented analysis is that it is a geometrical property of the rope, and not a material property, that is responsible for the nature of the equilibrium given by eq. (7). This geometrical equilibrium of rope is what accounts for the unyielding nature of ropes (their ability to maintain a constant length). The Mersa/Wadi Gawasis expedition has lead to rich findings

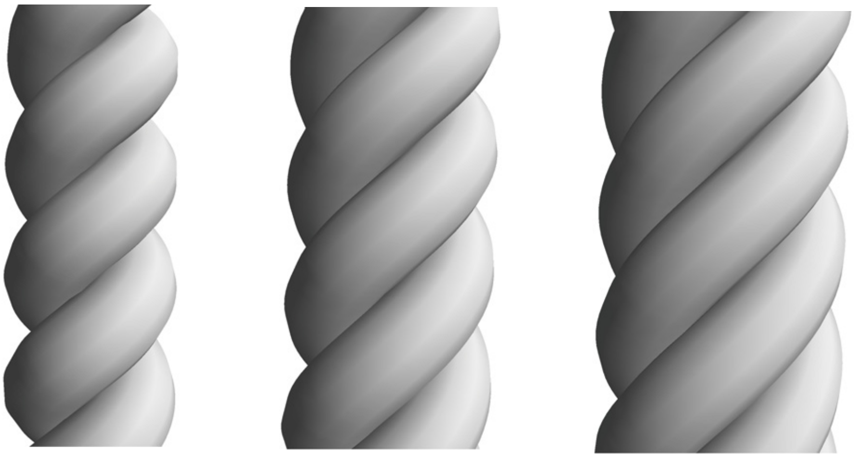

Fig. 3: An ideal representation of a two-, three- and four-stranded rope laid with a pitch angle corresponding to the maximally rotated zero-twist structures with the respective pitch angles of 39.4°, 42.8° and 43.8° relative to the equatorial plane. With these pitch angles, the strands will neither rotate in one or the other direction under vertical strain.

of ancient strings, cordage, and ropes, both as fragments, and as bundles and coils of well preserved ropes about 30 m long [6,7]. For the three rope fragments ($N = 3$) depicted in ref. [7] (fig. 22, item 3, 4, and 5) we measure the pitch angle to be 50°, 45°, and 55°, respectively. On the long ropes (fig. 23, ref. [7]) we measure 51°. For the more recent nano-yarn ($N = 2$) discussed by Zhang *et al.* [10] we measure the pitch angle to be 50°. This means that all of the ropes are laid within few percent of the zero-twist structure. The Egyptian fragments even included one that was very tightly laid.

Another consequence of the presented analysis is that when laying the rope it is necessary to add the strands in a way that will allow for strands under tensile stress to meet each other on the upper branch of the rope curve at the point dictated by eq. (7), *i.e.* for pitch angles down to the one of the zero-twist configuration. This is the purpose of the so-called top (a cone with grooves in) used at traditional ropewalks, and of similar tools. The top brings the strands together from a radius which is purposely too large corresponding to points within the shaded area of fig. 2.

Simple steel wires laid with a pitch angle significantly higher than what has been discussed above cannot be used in cranes with a single fall of the rope. The reason for this is that the load is not rotationally fixed when hanging and as a consequence the steel wire would unwind. This has lead to the manufacturing of counter-rotated multi-layered steel wires, which are denoted rotationally restricted [23]. It is interesting to note that the classical ropes discussed above are relatively well-performing rotationally restricted ropes, this being a consequence of the equilibrium described by eq. (7). The need for this type of engineering therefore did not arise before the advent of the steel wire. Steel wires are modeled using mechanical concepts [24].

In summary, we have discussed the significance of the zero-twist geometry for rope-making. In hard-wall models the physical forces that one strand is causing on another strand are indirectly implied, *i.e.* repulsions are assumed to be infinitely large when the hard-wall criterion is violated. This idealization of the forces is what allows for one to take a geometrical approach to understanding the helical structure of rope.

At last, one can speculate why the described geometrical nature of the art of rope-making has been overlooked. One explanation could be that by laying the rope through the usual procedures, it will —for the reasons described above— automatically be a functional rope. And, therefore the intrinsic geometry behind the art of laying rope is not something one have to know or be aware of, just the instructions which have been passed down through generations.

* * *

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