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LETTER TO THE EDITOR

On the possibility of measuring the Earth's gravitomagnetic force in a new laboratory experiment

Lorenzo Iorio

Dipartimento di Fisica dell' Università di Bari, via Amendola 173, 70126, Bari, Italy

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Abstract

In this letter we propose, in a preliminary way, a new Earth-based laboratory experiment aimed at the detection of the gravitomagnetic field of the Earth. It consists of the measurement of the difference between the circular frequencies of two rotators moving along identical circular paths, but in opposite directions, on a horizontal friction-free plane in a vacuum chamber placed at the South Pole. The accuracy to our knowledge of the Earth's rotation from VLBI and the possibility of measuring the rotators' periods over many revolutions should allow for the feasibility of the proposed experiment.

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1. Introduction

In the weak-field and slow-motion approximation of general relativity, a test particle in the gravitational field of a slowly rotating body of mass M and angular momentum $\boldsymbol{J}$, assumed to be constant, is acted upon by a non-central acceleration of the form (Ciufolini and Wheeler 1995, Ruggiero and Tartaglia 2002)

$$\boldsymbol{a}_{\text{GM}} = \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}_{\text{g}}, \tag{1}$$

in which $\boldsymbol{v}$ is the velocity of the test particle, c is the speed of light in vacuum and $\boldsymbol{B}_{\text{g}}$ is the gravitomagnetic field given by

$$\boldsymbol{B}_{\text{g}} = \frac{2G}{c}\frac{[\boldsymbol{J} - 3(\boldsymbol{J}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}}]}{r^3}, \tag{2}$$

where $\hat{\boldsymbol{r}}$ is the unit position vector of the test particle and G is the Newtonian gravitational constant.

For a freely orbiting test particle equation (1) induces on its orbit the so-called Lense–Thirring drag of inertial frames (Lense and Thirring 1918, Ciufolini and Wheeler 1995). Such general relativistic spin–orbit effect has been experimentally checked for the first time by analysing the laser-ranged data to the LAGEOS and LAGEOS II artificial satellites in the gravitational field of the Earth with a claimed accuracy of the order of 20% (Ciufolini *et al* 1998). The use of the proposed LARES satellite would greatly increase the accuracy of such space-based measurements (Ciufolini 1986, 1998, Iorio *et al* 2002a). The famous GP-B mission (Everitt *et al* 2001), whose flight is scheduled for the beginning of 2003, is aimed at the detection, among other things, of a general relativistic spin–spin effect on the orientation of four spaceborne gyroscopes. The claimed accuracy is of the order of 1% or better.

With regard to the possibility of measuring the terrestrial gravitomagnetic field in an Earth-based laboratory experiment, many experiments have been proposed[1], but, up to now, none of them has been practically implemented due to relevant practical difficulties. Maybe the most famous of them involves the detection of the gravitomagnetic precession of the swinging plane of a Foucault pendulum at the South Pole (Braginsky *et al* 1984). Such a proposal would not be easy to implement practically due to many sources of error recently re-analysed in Pascual-Sánchez (2002). Also the proposal by Cerdonio *et al* (1988) should be mentioned. It is based on an off-line comparison between an astrometric measurement of the Earth's angular velocity and an inertial measurement of the angular velocity of the laboratory. Recently, in Tartaglia and Ruggiero (2002), a proposal for detecting the terrestrial gravitomagnetic field by means of electromagnetic waves in a Michelson–Morley-type experiment has been put forward. The effect, in terms of interferometric fringe shift, is really quite small; however, the advances in technology related to the gravitational wave detectors such as LIGO and VIRGO might allow for a detection of such an effect in future.

Very recently, the influence of a phenomenological gravitational spin–spin coupling on the free fall of mechanical gyroscopes in the terrestrial gravitational field which might violate the equivalence principle (Zhang *et al* 2001) has been investigated experimentally (Luo *et al* 2002, Zhou *et al* 2002) in some laboratory preliminary tests. According to them, it seems that there is no violation of the equivalence principle for extended rotating bodies at the level of 10^{-7}.

In this paper, we intend to present a possible new Earth-based laboratory experiment which exploits, in a certain sense, the concept of the gravitomagnetic clock effect of two counter-orbiting test particles along identical circular orbits (Iorio *et al* 2002b).

2. An Earth-based gravitomagnetic clock effect

According to the gravitational analogue of the Larmor theorem (Mashhoon 1993), we could obtain equation (1) by considering an accelerated frame rotating with angular velocity

$$\boldsymbol{\Omega}_{\mathrm{LT}} = \frac{\boldsymbol{B}_{\mathrm{g}}}{2c}. \tag{3}$$

Indeed, in it an inertial Coriolis acceleration

$$\boldsymbol{a}_{\mathrm{Cor}} = 2\boldsymbol{v} \times \boldsymbol{\Omega}_{\mathrm{LT}} \tag{4}$$

is experienced by the proof mass. It is the same acceleration which, among other things, induces the Lense–Thirring precession of the swinging plane of the Foucault pendulum in the experiment proposed by Braginsky and co-workers (Braginsky *et al* 1984) by means of the component of $\boldsymbol{\Omega}_{\mathrm{LT}}$ along the local vertical direction.

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[1] For a review, see chapter 6 of Ciufolini and Wheeler (1995) and Ruggiero and Tartaglia (2002) and references therein. See also Braginsky *et al* (1977).

Let us choose a horizontal plane at, say, the South Pole[2]: here the Earth's angular velocity vector $\boldsymbol{\omega}_{\oplus}$ and $\boldsymbol{\Omega}_{\mathrm{LT}}$ are perpendicular to it and have opposite directions. Let us choose, as unit vector for the z-axis, the unit vector $\hat{\boldsymbol{\Omega}}_{\mathrm{LT}}$ so that

$$\boldsymbol{\Omega}_{\mathrm{LT}} = \frac{2GJ}{c^2 R_{\mathrm{p}}^3}\hat{\boldsymbol{z}}, \tag{5}$$

$$\boldsymbol{\omega}_{\oplus} = -\omega_{\oplus}\hat{\boldsymbol{z}}, \tag{6}$$

$$\boldsymbol{g} = -g\hat{\boldsymbol{z}}, \tag{7}$$

where $\boldsymbol{J}$ is the proper angular momentum of the Earth[3], R_{p} is the Earth polar radius and $\boldsymbol{g}$ is the gravitoelectric acceleration to which, at the poles, the centrifugal acceleration does not contribute.

A particle which moves with velocity $\boldsymbol{v}$ in such a polar horizontal plane is acted upon by the Coriolis inertial force induced by the noninertiality of the terrestrial reference frame[4] and also by the gravitational force of equation (4). Such forces have the same line of action and opposite directions: in a horizontal plane at the South Pole the resultant acceleration is $2v\tilde{\Omega} \equiv 2v(\Omega_{\mathrm{LT}} - \omega_{\oplus})$ and it lies in the plane orthogonally to $\boldsymbol{v}$.

Let us consider an experimental apparatus consisting of a friction-free horizontal plane placed in a vacuum chamber. Upon such a desk a small tungsten mass m, tied to a sapphire fibre of length l, tension $\boldsymbol{T}$ and fixed at the other extremity, is put in a circular uniform motion. Indeed, the forces which act on m are the tension of the wire, the Coriolis inertial force and the Lense–Thirring gravitational force which are all directed radially; the weight force $\boldsymbol{W} = m\boldsymbol{g}$ is balanced by the normal reaction $\boldsymbol{N}$ of the plane and there is neither the atmospheric drag nor the friction of the plane. Let us assume the anticlockwise rotation as the positive direction of motion for m. At equilibrium, the equation of motion is

$$m\omega_+^2 l = T - 2m\omega_+ l\tilde{\Omega}, \tag{8}$$

where ω_+ is the angular velocity of the mass m when it rotates anticlockwise and l is the radius of the circle described by m. If the Earth did not rotate, the angular velocity of the particle would be

$$\omega_0 = \sqrt{\frac{T}{ml}}. \tag{9}$$

For, say, $m = 100$ g, $T = mg = 9.798 \times 10^4$ dyne and $l = 100$ cm, $\omega_0 = 3.13$ rad s^{-1} and $P_0 = 2$ s. The gravitomagnetic and the Coriolis forces slightly change such a circular frequency. Since $\omega_0 \gg \tilde{\Omega}$, from equation (8) it follows for both the anticlockwise and clockwise directions of rotation

$$\omega_{\pm} = \omega_0 \mp \tilde{\Omega}, \tag{10}$$

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[2] Of course, it is easier to prepare an experimental setup in the Antarctic continent rather than in the Arctic floe. Thanks to the small size of the proposed apparatus, it should not be too difficult to find an Antarctic region free enough from seismic noise and other geological disturbances. Last but not the least, several scientific stations already exist in the Antarctic continent.

[3] In fact, while $\boldsymbol{J}$ remains constant, the Earth's angular velocity vector $\boldsymbol{\omega}_{\oplus}$ does not (Bertotti and Farinella 1990). Indeed, among other things, it moves around $\boldsymbol{J}$ with an approximate period of 14 months (the Chandler wobble) due to the oblateness of the Earth (motion with respect to the terrestrial reference frame). Moreover, there are also the secular precession of the equinoxes induced by the external lunisolar torque on the equatorial bulge with a period of 26 000 years and other faster variations (motion with respect to the celestial reference frame). The angular velocity of the Earth $\omega_{\oplus}$ suffers a secular deceleration due to the lunar torque so that the length of day increases in a year by about 2×10^{-5} s. In addition, we have other changes in $\omega_{\oplus}$ over shorter time scales due to moving masses within and on the Earth, in particular the oceans and the atmosphere. However, over the characteristic time scales of the experiment all such variations of $\boldsymbol{\omega}_{\oplus}$ can be neglected.

[4] Of course, the centrifugal force is absent at the Earth's poles.

so that we could adopt as the observable

$$\Delta\omega \equiv \omega_- - \omega_+ = 2\tilde{\Omega} \equiv 2(\Omega_{\rm LT} - \omega_\oplus). \tag{11}$$

Of course, the physical properties of the sapphire fibre and tungsten mass should not change from a set of rotations in a direction to another set of rotations in the opposite direction, so as to allow an exact cancellation of ω_0 in equation (11).

Since on the Earth's surface at the poles $\Omega_{\rm LT} = 3.4 \times 10^{-14}$ rad s^{-1}, would the experimental sensitivity of the sketched apparatus allow us to measure such a tiny effect? If we measure the frequency shift $\Delta\omega$ from the rotational periods of the mass m, we have

$$\delta\Omega_{\rm LT} = \frac{\delta(\Delta\omega)^{\rm exp} + \delta\omega_\oplus}{2} \tag{12}$$

with

$$\delta(\Delta\omega)^{\rm exp} = \delta\omega_-^{\rm exp} - \delta\omega_+^{\rm exp} = 2\pi\left[\left(\frac{\delta P_-}{P_-^2}\right)^{\rm exp} - \left(\frac{\delta P_+}{P_+^2}\right)^{\rm exp}\right]. \tag{13}$$

The Earth's angular velocity $\omega_\oplus$ is very well known in a kinematically, dynamically independent way from the very long baseline interferometry (VLBI) technique with an accuracy of the order of[5] $\delta\omega_\oplus \sim 10^{-18}$ rad s^{-1}. In fact, $\omega_\oplus$ is not exactly uniform and experiences rather irregular changes which are monitored in terms of length-of-day (LOD) by the Bureau Internationale des Poids et Mesures-Time Section (BIMP) on a continuous basis[6]. Such changes are of the order of $\Delta\omega_\oplus \sim 0.25$ milliarcseconds per year (mas yr^{-1}) $= 3.8 \times 10^{-17}$ rad s^{-1}, so that they are negligible. A possible source of error might come from our uncertainty in the position of the proposed polar setup with respect to the Earth's crust, i.e. from the polar motion of the instantaneous axis of rotation of the Earth $\hat{\omega}_\oplus$ in terms of the small angles x and y. It turns out that this phenomenon has three components: a free oscillation with a (measured) period of 435 days (Chandler wobble) and an amplitude of less than 1 arcsecond (asec), an annual oscillation forced by the seasonal displacement of the air and water masses of the order of 10^{-1} asecs and an irregular drift of the order of some arcseconds. There are also some diurnal and semi-diurnal tidally induced oscillations with an amplitude less than 1 mas. As a consequence, the position of the pole is unknown at a level of some metres. The small size of the apparatus should overcome such a problem. Moreover, it can be easily seen that the impact of such offsets of $\hat{\omega}_\oplus$ on the Coriolis force is $2v\omega_\oplus\cos\delta$ with δ of the order of some arcseconds or less, so that it is negligible.

With regard to the experimental measurement of the periods $P_\pm$, it should be possible to strongly constrain equation (13) by choosing the parameters of the apparatus suitably so as to increase the periods and/or by measuring them after many revolutions. However, the important point is that only their difference is important, and it should be possible to reduce such a difference to the accuracy level required.

3. Discussion and conclusions

In this paper, we have proposed a new experiment for measuring the gravitomagnetic Lense–Thirring effect in an Earth-based laboratory setup.

The key point consists of the measurement of the difference between the rotational frequencies of two test bodies which rotate uniformly along the same circular paths, but in opposite directions, in a horizontal, friction-free plane in a vacuum chamber at the South Pole.

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[5] See http://www.iers.org/iers/products/eop/long.html and http://hpiers.obspm.fr/eop-pc/.

[6] For these topics see http://einstein.gge.unb.ca/tutorial/ and http://hpiers.obspm.fr/eop-pc/. See also (Bertotti and Farinella 1990).

The value of the Earth's daily rotation rate has to be subtracted from such a quantity, but the great accuracy, to our knowledge, of it from VLBI would allow us to single out the tiny relativistic effect. Over many revolutions it should be possible to experimentally measure the difference between the rotational frequencies from the periods to a sufficiently high level of accuracy to allow for an extraction of the investigated gravitomagnetic effect.

Of course, many practical difficulties would make the proposed measurement very hard to implement. For example, it turns out that the friction force of the plane should be less than 2×10^{-9} dyne. Moreover, in order to reach the quoted accuracy in measuring $\omega_\oplus$ with VLBI, several years of continuous observation would be required.

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