RESEARCH NOTE

Characteristics of propagation of ion waves in a collisionless plasma with relative drift velocity between ions and electrons

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THE characteristics of propagation and damping of small amplitude ion sound waves in a collisionless plasma have been intensively studied using the mathematical methods developed by LANDAU (1946) for electron plasma oscillations. Both the situations with and without a relative velocity between ions and electrons have been considered. It has been observed that with the relative drift a two-stream amplification of the wave can occur. Critical velocities for marginal stability and maximum growth rates have been computed as a function of the ratio between ion and electron temperature (JACKSON E. A., 1960; JACKSON J. D., 1960; FRIED and GOULD, 1961).

An experiment on ion wave propagation and damping, performed in an alkali plasma for the case where ion and electron temperature were equal, has shown good quantitative agreement with theory (WONG *et al.*, 1964). Recently, two different methods for varying the ratio between ion and electron temperatures in alkali plasmas have been developed (LEVINE *et al.*, to be published; BLAU *et al.*, 1966).

We suggest here that measurements of ion wave velocities and damping can be used in a quiescent plasma as a diagnostic means for obtaining temperatures and drift velocities of both particles with respect to the laboratory frame. For this purpose we calculate both the real and imaginary part of the wave velocity in the limit that the wavelength is much larger than the Debye length and that the ratio between electron and ion mass is very small. Runaway particles and collisions are neglected; Gaussian distribution functions are assumed.

The linear dispersion relation for plane longitudinal electrostatic oscillations in an infinite uniform plasma of negligible Debye length is

$$\omega_{pe}^2 \int_C \frac{\partial f_e/\partial v}{v - \omega/\kappa}\,dv + \omega_{pi}^2 \int_C \frac{\partial f_i/\partial v}{v - \omega/\kappa} = 0 \qquad (1)$$

where $\omega_{pe,i}^2 = 4\pi n_p e^2/m_{e,i}$; C is the Landau integration contour, f_e and f_i are the unperturbed distribution functions for ions and electrons and an electrostatic perturbation of the form $\exp(i\kappa x - i\omega t)$ is considered.

The assumed distribution functions are:

$$f_e = \frac{1}{\pi^{1/2} c_e} \exp\left[-(v - v_D)^2/c_e^2\right]$$
$$f_i = \frac{1}{\pi^{1/2} c_i} \exp\left(-v^2/c_i^2\right) \qquad (2)$$

where $c_{e,i} = (2T_{e,i}/m_{e,i})^{1/2}$ and v_D is the drift velocity of the electrons with respect to the ions.

The dispersion relation reads:

$$Z'\left(\frac{\omega}{\kappa c_e} - \frac{v_D}{c_e}\right) + \frac{T_e}{T_i} Z'\left(\frac{\omega}{\kappa c_i}\right) = 0 \qquad (3)$$

where $Z'(z)$ is the derivative with respect to the argument of the plasma dispersion function (FRIED and CONTE, 1961). In the limit $m_e \ll m_i$ we can consider $|\omega/\kappa| \sim c_i \ll c_e$ as verified *a posteriori*. Since the function $Z'(z)$ is regular for $z = 0$, we can neglect $\omega/\kappa c_e$ in the argument of the first term for all values of v_D/c_e.

In this approximation solutions of equation (3) were found by graphical methods with the aid of a table of the $Z'(z)$ function prepared for this calculation. In this table the resolution in the imaginary part of the argument is five times greater than in the Fried and Conte tables for both positive and negative values of the imaginary part of z.

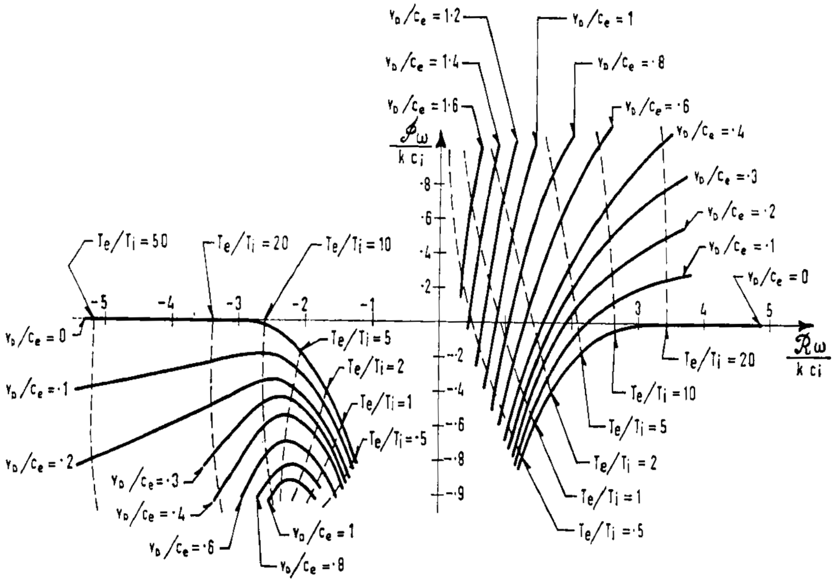

FIG. 1.—The real and imaginary part of $\omega/\kappa c_i$ for different values of T_e/T_i and v_D/c_e.

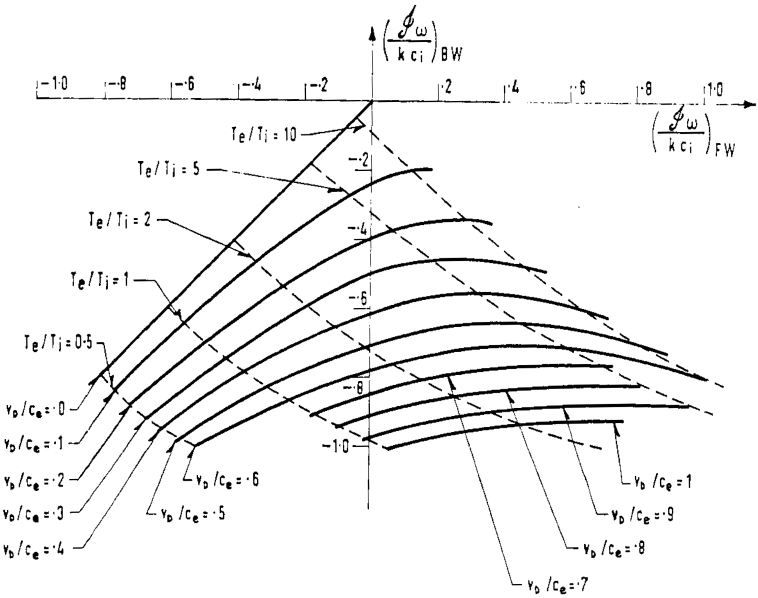

FIG. 2.—Imaginary part of phase velocity in the direction of the electron drift $(\mathscr{I}\omega/\kappa c_i)_{\rm FW}$ and in the opposite direction $(\mathscr{I}\omega/\kappa c_i)_{\rm BW}$ for different values of T_e/T_i and v_D/c_e.

As it is well known (FRIED and GOULD, 1961) equation (3) has an infinite number of solutions, corresponding to different modes. The real and the imaginary part of the wave velocity of the least damped mode in each direction are plotted in Fig. 1 as a function of T_e/T_i and v_D/c_e. Note that the wave travelling in the direction opposite to the electron drift direction is always damped, while the wave travelling in the direction of the electron drift direction can be amplified.

We wish to remark that by launching ion waves in the plasma and by measuring the four quantities

$$\mathscr{R}\left(\frac{\omega}{\kappa}\right)_{\mathrm{FW}}, \mathscr{I}\left(\frac{\omega}{\kappa}\right)_{\mathrm{FW}}, \mathscr{R}\left(\frac{\omega}{\kappa}\right)_{\mathrm{BW}}, \mathscr{I}\left(\frac{\omega}{\kappa}\right)_{\mathrm{BW}},$$

i.e. the phase velocities and damping rates in the laboratory frame of reference in the forward and backward direction (the direction of the electron drift velocity with respect to the ions and the opposite one) one can deduce the electron and ion temperatures and the drift velocity of both particles with respect to the laboratory frame. If one or more of these quantities are known already, one can deduce the others and obtain also one or more consistency relations. If, for instance, the ion temperature is known by means of other diagnostic tools, a measurement of

$$\mathscr{I}\left(\frac{\omega}{\kappa c_i}\right)_{\mathrm{FW}} \quad \text{and} \quad \mathscr{I}\left(\frac{\omega}{\kappa c_i}\right)_{\mathrm{BW}}$$

gives immediately T_e/T_i and v_D/c_e (see Fig. 2), where v_D is the drift velocity of the electrons with respect to the ions. The drift velocity of the ions with respect to the laboratory and an internal consistency relation can be obtained by use of the graph shown in Fig. 1.

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