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2009 Phys. Med. Biol. 54 N9

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## NOTE

## Single-Arc IMRT?

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Received 19 August 2008, in final form 16 November 2008

Published 10 December 2008

Online at [stacks.iop.org/PMB/54/N9](http://stacks.iop.org/PMB/54/N9)

### Abstract

The idea of delivering intensity-modulated radiation therapy (IMRT) with a multileaf collimator in a continuous dynamic mode during a single rotation of the gantry has recently gained momentum both in research and industry. In this note we investigate the potential of this Single-Arc IMRT technique at a conceptual level. We consider the original theoretical example case from Brahme *et al* that got the field of IMRT started. Using analytical methods, we derive deliverable intensity ‘landscapes’ for Single-Arc as well as standard IMRT and Tomotherapy. We find that Tomotherapy provides the greatest flexibility in shaping intensity landscapes and that it allows one to deliver IMRT in a way that comes close to the ideal case in the transverse plane. Single-Arc and standard IMRT make compromises in different areas. Only in relatively simple cases that do not require substantial intensity modulation will Single-Arc be dosimetrically comparable to Tomotherapy. Compared with standard IMRT, Single-Arc could be dosimetrically superior in certain cases if one is willing to accept the spreading of low dose values over large volumes of normal tissue. In terms of treatment planning, Single-Arc poses a more challenging optimization problem than Tomotherapy or standard IMRT. We conclude that Single-Arc holds potential as an efficient IMRT technique especially for relatively simple cases. In very complex cases, Single-Arc may unduly compromise the quality of the dose distribution, if one tries to keep the treatment time below 2 min or so. As with all IMRT techniques, it is important to explore the tradeoff between plan quality and the efficiency of its delivery carefully for each individual case.

<sup>3</sup> This work was initiated when the first author was the Haddow Visiting Professor at the Institute of Cancer Research, upon an invitation by the second author.

## 1. Introduction

The recent development of rotational delivery techniques ('RapidArc' from Varian<sup>4</sup> and VMAT (volumetric arc therapy) from Elekta<sup>5</sup>) has triggered a new wave of interest in intensity-modulated radiation therapy (IMRT). The announcement by the Tomotherapy corporation<sup>6</sup> that they will pay a high dollar-amount to anybody who can demonstrate that RapidArc is dosimetrically superior to Tomotherapy has further stirred up the discussion (McEntee 2008). The fact that the development of IMRT technology is now mainly driven by commercial companies does not help to objectify or clarify the issues. Some early comparison studies were also carried out by the developers of the new technology (Cozzi *et al* 2008, Palma *et al* 2008). While we will undoubtedly see many more papers comparing RapidArc and VMAT with various other IMRT techniques for specific clinical cases, we believe that some of the open questions can and should be addressed from first principles, which is the purpose of this note. The basic questions are:

- How can an IMRT-like dose distribution be delivered with a multileaf collimator during a single rotation of the gantry, where there is no intensity modulation from any gantry angle?
- Why and where is there a potential benefit in delivering IMRT dynamically while the gantry rotates, as opposed to the standard fixed field technique?
- Can a single arc technique with a multileaf collimator possibly compete with tomotherapy?
- Can these delivery questions be separated from aspects of treatment planning and optimization? In other words, when we see differences between different techniques, can those be attributed to the delivery techniques only or are they possibly rooted in limitations of the optimization systems?

While this note investigates the comparison of various IMRT techniques from a general and somewhat philosophical perspective, we will also look specifically at the example of the original problem formulated by Brahme *et al* (1982). This will be done mainly for illustration purposes. But the Brahme problem has the great advantage that a solution is known analytically, which serves as a benchmark solution.

## 2. Some definitions

Let us begin with some definitions. Both RapidArc and VMAT are similar to intensity-modulated arc therapy (IMAT), which was described for the first time by Yu (1995). They deliver radiation therapy with a multileaf collimator (MLC), which changes the shape of the treatment field dynamically while the gantry rotates around the patient. The unique feature of RapidArc is that it delivers the whole treatment with only one rotation of the gantry and is therefore potentially faster. This idea is actually not so new. To the best of our knowledge, it was first presented in public by Boyer (2001) and has subsequently been developed by several researchers, whose approaches differ in the way they calculate the trajectories of the MLC leaves, and in the orientation of the MLC (parallel to the gantry rotation axis (Cameron 2005), perpendicular to it (Ulrich *et al* 2007), and under 45° (Otto 2008). We will refer to all these techniques as 'Single-Arc' techniques. None of these rotational techniques has been used very much or at all in the clinic, mainly because until recently there were few research and no commercial planning solutions available. The clinical IMRT workhorses still are standard

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IMRT (S-IMRT), which delivers fully intensity-modulated radiation fields with an MLC from a relatively small number (about 7) of fixed gantry angles, and Tomotherapy, which uses a special collimator that allows full intensity modulation during *dynamic* gantry rotation (i.e., with a large number of gantry angles), but it treats only one slice at a time. Both these latter classes of IMRT delivery are subdivided into two further implementation choices. S-IMRT either uses the dynamic MLC delivery (DMLC) (Stein *et al* (1994)) or the multiple-static-field MLC delivery (MSF-MLC) (Bortfeld *et al* (1994)). Tomotherapy either uses the NOMOS<sup>7</sup> MIMiC system (important between 1994 and a few years later but far less used today) or the Tomotherapy spiral system emanating from work initially at the University of Wisconsin (Mackie *et al* (1993)) and in clinical use and commercially available since 2002.

### 2.1. Is the single-Arc technique IMRT?

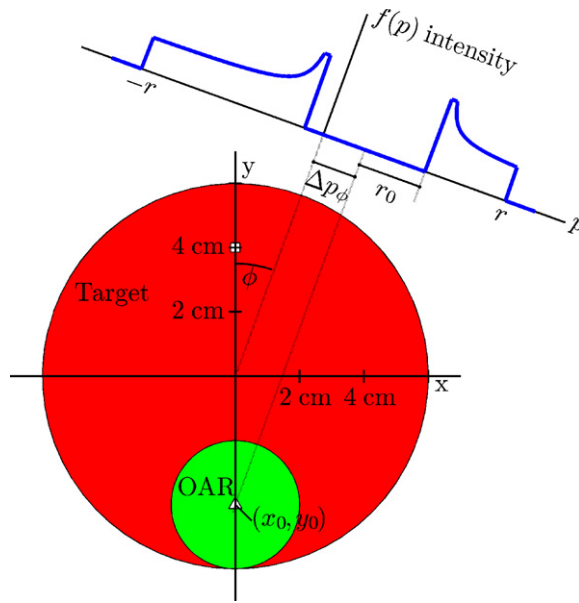
Because Single-Arc delivers an irregularly shaped but unmodulated treatment field from each gantry angle, it is actually not IMRT in the strict sense that requires each field *from a given direction* to be spatially modulated (even if that modulation arises from a series of temporal spatial changes in the fluence pattern, Webb and Lomax (2001)). In Single-Arc, the beam intensity may be modulated as a function of the gantry angle but that would not normally be called IMRT. Nevertheless, we believe that Single-Arc should be considered as a form of IMRT, as long as the MLC does more than to simply follow the shape of the target and the critical structures as the gantry rotates around the patient, i.e., more than 3D conformal radiation therapy. In Single-Arc, different sections of the target volume are being treated, albeit uniformly, from different gantry angles. This can be considered as a form of binary (on/off) intensity modulation. Incidentally, this technique has some similarity with the ‘patch field’ technique known from proton therapy, in which different parts of the target volume are covered uniformly with dose from different beam angles (Bussi ere and Adams 2003). The added flexibility, compared with the 3D conformal technique, can be exploited to achieve greater overall dose conformality. We may note that actually all IMRT techniques (other than compensator based IMRT, which we will not discuss here) create a spatial modulation of intensity (or fluence) by the summation of binary (on/off) spatial patterns of intensity delivered at different times. In the Tomotherapy methods the spatial pattern is a set of vane-defined shapes; in the other IMRT methods the spatial pattern at any one time is a MLC-shaped field.

## 3. Comparison between different IMRT techniques

### 3.1. Single-Arc IMRT versus standard IMRT

It may be worthwhile mentioning that, in a theoretical limit, there is actually no difference between Single-Arc IMRT and S-IMRT at all. First of all, they use the same hardware. They can also be ‘mapped’ onto each other: S-IMRT can, in principle, do everything that Single-Arc can do, and vice versa. To understand the first implication: by using many more gantry angles than the typical 7, S-IMRT can deliver dose distributions that are equivalent to those obtained by a dynamic rotation of the gantry. On the other hand, by slowing the gantry down to infinitesimally small rotation speeds, Single-Arc can deliver intensity-modulated fields from quasistatic gantry angles, like S-IMRT. In both cases, the price for this ‘mapping’ is increased treatment time. If we ignore the treatment time altogether, Single-Arc and standard IMRT can yield perfectly equivalent dose distributions. The important question about the *tradeoff*

<sup>7</sup> Now best nomos, Pittsburgh, PA, USA.



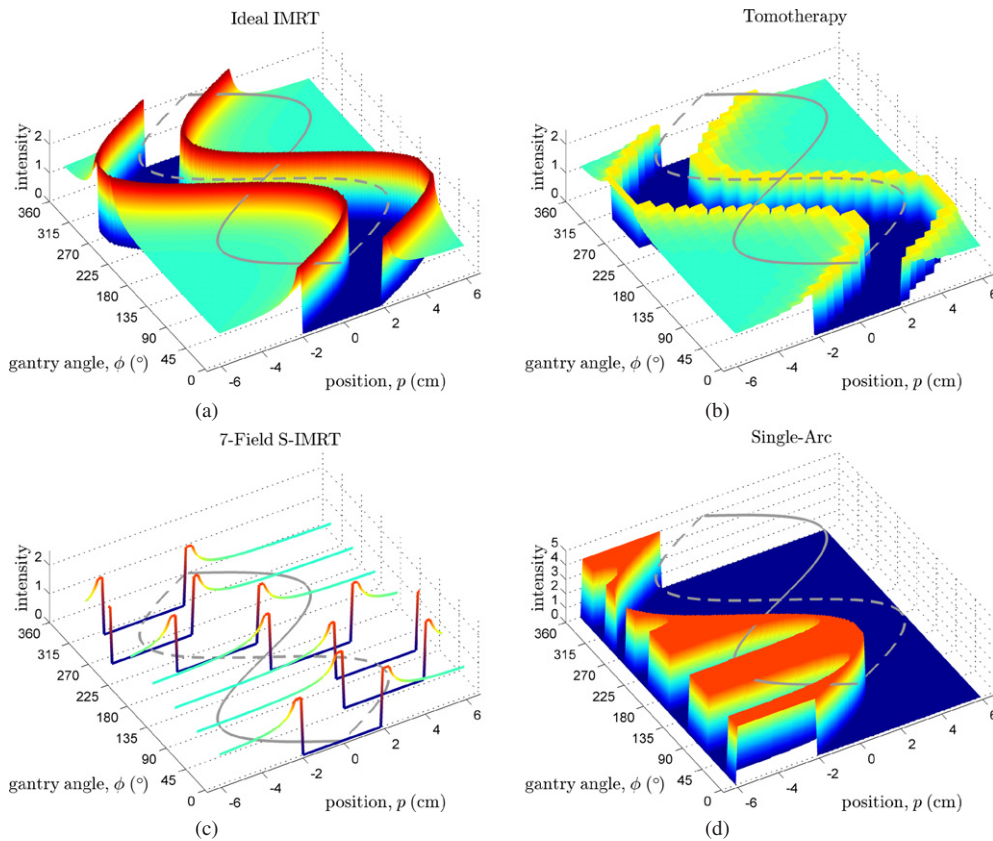
**Figure 1.** The archetypical IMRT example case where the target volume (red) wraps around an organ at risk (OAR) (Brahme *et al* 1982). This case requires substantial intensity modulation. Simply ‘blocking’ dose from reaching the OAR does not yield the desired uniform dose coverage of the target volume. One intensity profile from a gantry angle of  $\phi = 20^\circ$  is shown.

between the quality of the treatment plan and the treatment time, i.e., between quality and efficiency, will be discussed later.

To illustrate the comparison between Single-Arc and S-IMRT for a more realistic scenario with practical delivery times, let us recall the original example from Brahme *et al* (1982), which arguably got the whole field of IMRT started (Webb 2003). Its geometry is shown in figure 1. This is supposed to represent a transverse slice of the tumor target volume, which has a circular cross section with 6 cm radius (shown in red) and wraps around a circular organ at risk (OAR, shown in green) with a radius  $r_0$  of 2 cm. It is the archetypical example of a concave target geometry in which intensity modulation is needed to achieve both a uniformly high dose in the target and a uniformly low dose in the OAR. In a slight modification of the problem by Brahme *et al* (1982), and to make this example more practically relevant, the OAR is not located at the center of the target, but at its periphery. One advantage of this simple geometry is that optimal intensity profiles can be calculated analytically, if the approximations of Brahme *et al* (1982) are made. The details of the calculations can be found in the appendix. Those intensity profiles therefore do not depend on the specifics of the implementation of an optimization algorithm. Figure 1 shows an intensity profile from a gantry angle of  $\phi = 20^\circ$ , with zero intensity in the projection of the OAR, and non-uniform intensity elsewhere.

We can now plot the intensity profiles for all gantry angles,  $\phi$ , in a sinogram-like surface plot (landscape), as shown in figure 2(a). The meandering ‘river’ represents zero intensity, to protect the OAR. The intensity peaks are needed to avoid dose-coldspots in the target volume close to the OAR. In principle these intensity peaks must be of infinite height, but here, in figure 2(a), we cut them off at a width of  $w = 2$  mm. More narrow peaks are not practically achievable due to scattered radiation.

In figure 2(c) and (d) we show the corresponding sinogram plots for an instantiation of S-IMRT and Single-Arc, respectively. They are obviously much different from each other and



**Figure 2.** ‘Landscapes’ of different IMRT delivery techniques for the case from figure 1. Each case shows a surface representation of the intensity as a function of lateral position and gantry angle. The ideal IMRT profile, albeit with capped intensities, is shown in part a. The blue intensity profile from figure 1 can be found at  $\phi = 20^\circ$ . In part b we see what is achievable with Tomotherapy. The intensity crest next to the ‘river’ valley is less pronounced due to the limited resolution of 6 mm, but overall the shape is similar to the ideal profile. Part c shows the intensity map for a typical S-IMRT with 7 fixed gantry angles. Finally, part d shows the Single-Arc technique. In all cases the intensity values were normalized to the intensity that would be required if all beams would treat the entire target volume uniformly. The dashed and solid gray lines represent the integration paths that yield the dose at the white triangle ( $y = -4$  cm) and at the white square (at  $y = +4$  cm) in figure 1.

from the ideal IMRT ‘landscape’ of figure 2(a). In S-IMRT we can practically only place a finite number of fields, which is seven in this case. But, each field can be highly intensity modulated. In the Single-Arc implementation of figure 2(d), we assumed that the MLC leaf motion is perpendicular to the gantry rotation axis, i.e., in the transverse plane. Leaves move in and out four times during one gantry rotation (crossing the field eight times), oscillating between a full coverage of one side of the target volume, and a minimum field size of  $w = 3$  mm. The details of the calculation can be found in the appendix. Note that the right leaf of the MLC is fixed at the tangent to the OAR, and defines the right border of the red fluence region in figure 2(d), starting at  $p = -2$  cm for  $\phi = 0^\circ$ . The left leaf, defining the left border, starts at the tangent to the PTV ( $p = -6$  cm), stays there for a while, and then moves to the right as the gantry moves from  $0^\circ$  in positive  $\phi$  direction. As it does so, its velocity is continually

decreased until the leaf stops at about  $\phi = 45^\circ$ ; it then moves back to the tangent of the PTV, and so forth. Note that only one side of the target volume is covered (one side of the ‘river’ in the sinogram) but with higher intensity. This is sufficient in this specific example due to its symmetry. Overall, one can view Single-Arc and S-IMRT as two techniques that make compromises in different areas: S-IMRT uses a very coarse sampling of the gantry angle  $\phi$  but with full intensity modulation from each angle, whereas Single-Arc utilizes all angles but without intensity modulation at a given angle  $\phi$ .

For cases with multiple organs at risk that ‘indent’ the target on different sides, no analytical solution is known. Probably there is simply no solution that provides a perfectly uniform dose in a concave target volume with two (or more) OARs on two sides, and spares those OARs perfectly. But, this is a hypothesis that is purely based on our intuition<sup>8</sup>. Interestingly, in cases with two (or more) OARs, the landscape would have two ‘rivers’, and there could be ‘isles’ between them. Those isles would be particularly difficult to treat with Single-Arc.

### 3.2. From intensity landscapes to dose distributions

One question arises: how can S-IMRT and Single-Arc with so vastly different intensity patterns produce dose distributions in the patient, which are at least somewhat comparable? To understand this, one needs to know that the dose at any volume element (voxel) in the patient is obtained by integrating the sinogram landscapes over sinusoidal trajectories (Cormack 1987). For the two voxels marked with a white triangle at  $y = -4$  cm (in the center of the OAR) and with a white square at  $y = +4$  cm in figure 1, these sinusoidal trajectories are shown as the dashed and solid gray lines, respectively, in figure 2. The dashed line follows the river valley and its integral therefore yields a very low (theoretically zero) dose value. The solid line ‘sees’ variable intensities and should yield the desired target dose.

This integration over the sinusoidal path washes out differences between the different intensity landscapes of figure 2 to a certain degree. However, this is only true for voxels within or close to the target. For voxels at large distances from the target, the amplitude of the sinusoidal integration paths is also greater, and they may miss the S-IMRT intensity profiles of figure 2(c) altogether, resulting in zero, or very low, dose values. This leads to the typical streaky dose pattern of S-IMRT, and a smoother dose distribution for Single-Arc. The choice between S-IMRT and Single Arc is therefore, to a large degree, one between distributing low dose values uniformly within large volumes of normal tissues away from the target volume, and delivering more dose to some normal tissues, and sparing others completely. Which of those options is preferable from a clinical standpoint is one of the oldest open questions in radiation oncology.

Even though differences in the dose distributions are generally smaller than the differences in the intensities, there are significant differences in the dose distributions produced by the different IMRT techniques illustrated in figure 2. Only the theoretical solution by Brahme leads, within the approximations made, to a perfectly uniform dose in the target and perfect sparing of the OAR. Capping the intensities as in figure 2(a) with a minimum peak width of  $w$  leads to an underdosage, in particular in the vicinity of the OAR. Disturbances also arise at distances greater than  $w$ , however becoming insignificant with increasing distance. The S-IMRT solution produces, by its discrete nature, dose steps across the target, which decrease

<sup>8</sup> As was pointed out to us by a referee, the hypothesis can be easily proven in cases where the multiple OARs are completely inside the target volume, see also Cormack (1987) and Webb (1993). The situation is not so clear in more realistic geometries with less extreme concavities, as encountered in head and neck tumors with brainstem and parotid as OARs.

with increasing number of beam angles. Furthermore, in the vertices of a regular 14-sided (twice the number of beams) polygon with the circular OAR inscribed, no primary dose would be deposited. The Single-Arc technique leads to dose variations of similar magnitude, but smoother variations.

### 3.3. *The tradeoff between quality and efficiency*

In a complex case like the one discussed above, a certain amount of intensity modulation is required to achieve satisfactory dose distributions. This, in turn, means that the leaves of the MLC have to move back and forth several times across the target volume during the treatment, which takes the same amount of time in S-IMRT and Single-Arc. The main difference is that, in Single Arc, the MLC leaf motion occurs while the gantry rotates around the patient. In S-IMRT, on the other hand, the rotation of the gantry takes up additional time, about 1 min. Based on this somewhat oversimplified viewpoint, Single-Arc should be about 1 min faster than S-IMRT. This difference is not negligible but it is certainly not a huge effect.

Restricting the delivery time to, say, 2 min, as in RapidArc, may be acceptable for simple cases that do not really need much intensity modulation, or need none at all. But, then S-IMRT should also be very fast. A problem is that IMRT optimization algorithms, especially the standard two-step approach (intensity optimization, then leaf sequencing) used for the planning of S-IMRT, sometimes yield complex intensity patterns even for very simple cases. Mathematically this is a consequence of the fact that the solution space is often quite flat (mathematical degeneracy), which means that different intensity maps yield similar dose distributions, and that there is no reward for picking solutions that are easy to deliver. Therefore, initial comparisons between Single-Arc and S-IMRT have shown much bigger differences than the 1 min stated above.

Restricting the overall delivery time to 2 min *for all cases* may unduly compromise the plan quality for complex geometries. Instead, vendors of treatment-planning systems should be encouraged to implement methods that let users explore the tradeoff between quality and efficiency, which is already possible in research systems (Coselmon *et al* 2005, Craft *et al* 2007). Of course, the tradeoff discussion has to be based on technical constraints of the delivery system, such as finite MLC leaf speed and acceleration, gantry rotation speed, and dose rate.

Another dimension in the tradeoff is leakage radiation, which may lead to radiation induced cancers (Hall and Wu 2003). However, since the total treatment time is correlated with the amount of leakage radiation, we will not discuss this issue separately.

### 3.4. *Comparison with tomotherapy*

In spiral Tomotherapy the gantry rotates at a constant speed and the goal is to deliver a modulated beam planned at 51 fixed directions. The leaves are operated from 51 arc segments. The high intensity beamlets are delivered by opening the leaf at the beginning of the arc segment and closing at the end of the arc segment. The crossing time is about 20 ms and the response of the leaves is taken into account in the control system. A low intensity beamlet is delivered by opening the leaf just before the center of the arc segment and closing it just after the center of the arc. However, the minimum opening time is about 5% of the possible opening time and the maximum opening time is about 95% of the possible opening time. Also since there are 64 such vanes the spatial increment of placing the field components is discrete with a resolution of about 6 mm.

This shows that, in Tomotherapy as well, beam delivery is limited in several ways. For example, the finite size of the vanes limits the height of the intensity peaks and leads to a discrete fluence pattern as shown in figure 2(b). Nevertheless, it is clear from figure 2 that Tomotherapy offers much greater flexibility than either S-IMRT or Single-Arc. It is practically only limited by the 6 mm resolution of its collimator, which leads to the difference between figures 2(a) and (b). Tomotherapy should therefore almost always yield better dose distributions *in the transverse plane*.

But what about the resolution in the longitudinal direction? This is determined by the longitudinal field width, which is user adjustable by setting the collimator width to any value between 0 and 5 cm. So, while in principle high spatial resolutions are achievable in all directions, in practice one has to make a compromise between quality and efficiency again. With very narrow field widths of a few mm, a treatment would take way too long to deliver. In practice the field width is often set to values as large as 2.5 cm, which means that the longitudinal resolution is worse than in volumetric IMRT.

#### 4. Some optimization issues

The purpose of this note is to discuss the potential of various IMRT delivery techniques at a higher conceptual or philosophical level. When comparing different IMRT techniques for concrete clinical cases, it is difficult or impossible to judge the potential of the delivery technique separately from the underlying plan optimization method. The optimization of volumetric rotational IMRT techniques, Single-Arc and IMAT, is a difficult non-convex optimization problem. This may be the reason why IMAT, though announced 13 years ago, suffered for nearly a decade from lack of planning-computer developments, later first solved by Earl *et al* (2003). Today there exist various planning methods for Single-Arc and IMAT, but they have not been in widespread clinical use yet. Due to its mathematical nature, Single-Arc plan optimization can get stuck in a local minimum far away from the true global optimum (Ulrich *et al* 2007). This may go undetected, unless one runs several plan optimizations with different parameters. The published Single-Arc optimization methods use simulated annealing or occasional large steps through the search space to escape local minima (Cameron 2005, Ulrich *et al* 2007, Otto 2008). They produce very promising plans, but one needs to keep in mind that they were all tested in a research environment, which allows one to do a lot of parameter tweaking. Whether these systems will reliably yield good plans in a clinical environment, where there is no time for extensive parameter tweaking, remains to be seen. In comparison, the optimization of S-IMRT (with the two-step approach and fixed beam angles) and Tomotherapy is a much easier convex problem. These optimization systems may still not find the true optimum, but there is no risk that they will end up far way from the optimum. However, these statements are only valid if one assumes that the beam angles in S-IMRT are fixed. The optimization of beam angles in S-IMRT is also a very difficult non-convex optimization problem, which is as yet unsolved, or at least no fully satisfying solution is available. In fact, it is a major advantage of rotational techniques, including Single-Arc, that one does not need to worry about the selection of beam angles.

#### 5. Summary and conclusions

Single Arc IMRT is a relatively recent addition to the IMRT family and has been touted commercially with great fanfare. Indeed the Single-Arc technique has a number of attractive features. Being a rotational technique, unlike S-IMRT, it does not require the selection of

a number of fixed beam orientations and allows greater flexibility because it lets radiation come from basically all beam orientations. Also, because it delivers the entire treatment in one rotation of the gantry, it is potentially more efficient, both in terms of treatment time and monitor units, than Tomotherapy and S-IMRT.

On the other hand, expectations should not be too high. In terms of efficiency, when comparing Single-Arc and S-IMRT for complex cases, the leaves of the MLC have to move back and forth across the target volume many times during a treatment to do the appropriate dose shaping, and this takes time for both techniques. Therefore, Single-Arc cannot be so much faster than S-IMRT. To first order, the time saving is simply the time it takes to rotate the gantry around the patient, which is approximately 1 min. Keeping the total treatment time below 2 min or so may indeed work for some cases, especially those that do not really require IMRT at all, but it can unduly compromise the quality of the dose distribution in very complex cases.

In our opinion it is crucial that treatment-planning systems allow the treatment planner and physician to make an informed decision about the tradeoff between the quality of a treatment plan and the time it takes to deliver the plan. If, for example, a much better plan can be delivered by extending the treatment time by 1 min, this would practically always be worthwhile.

By and large, the choice between Single-Arc and S-IMRT is about the rock and the hard place of spreading low dose values uniformly over large volumes of normal tissues, and delivering larger dose values to smaller volumes of normal tissue.

Tomotherapy has, by design, a much greater flexibility of shaping dose distributions in transverse slices than both S-IMRT and Single-Arc: it allows full intensity modulation per beam direction (like S-IMRT but not Single-Arc), and beams can come from practically all (co-planar) directions (like Single-Arc but not S-IMRT). Limitations in Tomotherapy come from the finite width of the vanes and the choice of the slice thickness, which again requires a tradeoff to be made between quality and efficiency. We believe we have stated in this note what can be said about this topic from a philosophical standpoint. Further statements can be made only by looking at individual clinical cases. This requires planning comparisons for the three techniques discussed, subject to known constraints. We have mentioned some difficulties associated with such comparisons, especially with respect to the underlying optimization algorithms. We and others are currently working on those issues. Some comparison studies of this type have actually been recently published or are in print, e.g., Cao *et al* (2007), Fogliata *et al* (2008). In the study by Cao *et al* (2007), multiple arc IMAT is compared with Tomotherapy. They conclude that, in the most complex cases, Tomotherapy has a dosimetric advantage over IMAT, which should be bigger when compared with Single-Arc. This is in agreement with our findings. However, the authors acknowledge that the differences may be in part due to their specific implementation of IMAT optimization by use of an ‘arc-sequencer’. In the study by Fogliata *et al* (2008), Single-Arc is compared with Tomotherapy and S-IMRT. They find that all three techniques yield somewhat comparable results in the cases studied, but they also acknowledge several shortcomings of their study, and conclude that further investigations with more complex cases should be performed.

### Acknowledgments

We wish to thank Dr David Craft (Massachusetts General Hospital) and three unknown referees for their constructive comments that have made this note better. This study was supported in parts by grants CA118200 and CA103904 from the National Institute of Health of the United States of America.

## Appendix A

In this appendix we derive the intensity landscapes shown in figure 2 for the ideal case and the three different IMRT techniques (Tomotherapy, S-IMRT, and Single-Arc) in analytical form.

### A.1. Intensity profiles

For the illustrations in this note we used a geometrical two-dimensional case with a circular target volume with radius  $r = 6$  cm with its center at the isocenter  $(0, 0)$ , and a critical structure with a radius  $r_0 = 2$  cm and center at  $(x_0, y_0) = (0, -4)$  cm, see figure 1. The optimal intensity profile as a function of the lateral position  $p$  for a beam with the gantry angle  $\phi$  can be derived from Brahme *et al* (1982) and is given by

$$f(p) = \begin{cases} \frac{|p - \Delta p_\phi|}{\sqrt{(p - \Delta p_\phi)^2 - r_0^2}} & \text{if } |p - \Delta p_\phi| > r_0 \text{ and } |p| \leq r, \\ 0 & \text{otherwise,} \end{cases} \quad (\text{A.1})$$

where

$$\Delta p_\phi = x_0 \cos \phi - y_0 \sin \phi \quad (\text{A.2})$$

is a shift incurred due to the noncentrally located critical structure. Here we have used the Brahme–Roos–Lax approximation (Brahme *et al* 1982), which neglects scatter and disregards beam divergence, and we have also ignored beam attenuation, which actually changes the intensity profile only very slightly. A proportionality factor of  $D_0/2\pi$ , where  $D_0$  is the prescribed dose level, has been disregarded to simplify the notation.

The intensity profile of equation (A.1) cannot be delivered due to its singularities at  $p = \Delta p_\phi \pm r_0$ . Here we have circumvented this issue simply by cutting intensity peaks off at a level where the width of the peaks is  $w$ . This yields the intensity profile

$$\hat{f}(p) = \begin{cases} \frac{r_0 + w}{\sqrt{(r_0 + w)^2 - r_0^2}} & \text{if } r_0 \leq |p - \Delta p_\phi| \leq r_0 + w, \\ \frac{|p - \Delta p_\phi|}{\sqrt{(p - \Delta p_\phi)^2 - r_0^2}} & \text{if } |p - \Delta p_\phi| > r_0 + w \text{ and } |p| \leq r, \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A.3})$$

For the numerical simulations we have chosen a value of  $w = 2$  mm for the ideal case,  $w = 6$  mm for Tomotherapy, and  $w = 3$  mm for S-IMRT and Single-Arc. The latter has been motivated by our previous study (Bortfeld *et al* 2000).

### A.2. Leaf trajectories

*A.2.1. Static gantry position.* Let us focus on the left side of the intensity profile where  $-r \leq p < \Delta p_\phi - r_0$ . This is all we will need for our simple illustrative example because of its symmetry. We can define the leaf trajectories that will deliver this profile using the standard ‘sliding window’ recipe. We first invert equation (A.1) in the relevant range ( $f$  non-negative), which yields:

$$p(f) = \begin{cases} -r & \text{if } f \leq f_{\min}(\phi), \\ \Delta p_\phi - r_0 \frac{f}{\sqrt{f^2 - 1}} & \text{otherwise,} \end{cases} \quad (\text{A.4})$$

where

$$f_{\min}(\phi) = \frac{r + \Delta p_{\phi}}{\sqrt{(r + \Delta p_{\phi})^2 - r_0^2}}. \quad (\text{A.5})$$

Now we write the intensity (fluence)  $f$  as a function of time  $t$  in the form  $f = f_{\max}t/t_{\max}$ , where  $f_{\max} = \frac{r_0+w}{\sqrt{(r_0+w)^2 - r_0^2}}$ , and  $t_{\max}$  is the time it takes to reach the intensity (fluence) level  $f_{\max}$ . Then we can use equation (A.4) to determine the leaf trajectory  $A_{\phi}(t)$  of the left A-leaf of the multileaf collimator (MLC):

$$A_{\phi}(t) = \begin{cases} -r & \text{if } t \leq t_{\max} \frac{f_{\min}(\phi)}{f_{\max}}, \\ \Delta p_{\phi} - r_0 \frac{t}{\sqrt{t^2 - t_{\max}^2 \left(1 - \frac{r_0^2}{(r_0+w)^2}\right)}} & \text{otherwise.} \end{cases} \quad (\text{A.6})$$

The right B-leaf is stationary at

$$B_{\phi}(t) = \Delta p_{\phi} - r_0. \quad (\text{A.7})$$

If we deliver this sequence  $A_{\phi}(t), B_{\phi}(t)$  with an ideal MLC (no leakage etc.) for  $t = 0, \dots, t_{\max}$ , we will obtain the left part of the fluence profile from equation (A.3).

**A.2.2. Moving gantry (arc therapy).** If we want to move the leaves while the gantry rotates for (single) arc therapy, an obvious way to do that is simply by use of equations (A.6) and (A.7), but taking the increasing  $\phi$  into account as the gantry rotates. This yields trajectories  $\tilde{A}(\phi) = A_{\phi}(t_{\max}(\phi - \phi_0)/\Delta\phi)$  and  $\tilde{B}(\phi) = B_{\phi}(t_{\max}(\phi - \phi_0)/\Delta\phi)$ . Here  $\phi_0$  is some starting angle and  $\Delta\phi$  is the angular interval during which the leaves move across from left to right, and the time goes from 0 to  $t_{\max}$ . These are the trajectories used in our illustration of the single arc technique, see figure 2(d). There we have alternated between leaves moving in and out in subsequent  $\Delta\phi$  intervals, and have used  $\Delta\phi = 45^\circ$ .

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