Introducing relativistic mass: the `ultimate speed experiment' of William Bertozzi revisited

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Introducing relativistic mass: the 'ultimate speed experiment' of William Bertozzi revisited

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This paper presents a way of introducing students to the concept of relativistic mass via the rarely discussed experimental work of William Bertozzi.

The use of a constructivist approach to introduce concepts of special relativity has been discussed previously in this journal, for example by Ireson [1] and Palfreyman [2]. The reasoning behind the absence of Bertozzi's work is not the subject of this paper but it does appear a curious omission. Having reviewed the index of eight A-level and 12 undergraduate physics texts, the author found reference to the work of Bertozzi in one undergraduate text.

In Newtonian mechanics, the framework in which many physics students operate, there is no upper limit to velocity and mass is considered invariant. Hence, if a body is acted upon by a force, causing it to accelerate, then the work done is equal to the gain in kinetic energy as demonstrated by the increase in velocity.

French [3] uses the example of the acceleration due to gravity near the Earth's surface to demonstrate this effect. It can be shown that this acceleration, 9.81 $\mathrm{m\,s^{-2}}$, would result in a velocity of 3.1×10^8 $\mathrm{m\,s^{-1}}$ after one year for a body accelerated from rest. After two years this velocity would be 6.2×10^8 $\mathrm{m\,s^{-1}}$; students can easily reproduce these results.

In relativistic mechanics, the framework that the students are being guided towards, there is a limiting velocity of 3×10^8 $\mathrm{m\,s^{-1}}$ and mass is no longer considered to be invariant.

The work of Bertozzi [4] can be used to show students that their original Newtonian framework is inadequate to explain the observations, and by implication and reasoning this leads to a need for a new framework.

Bertozzi's 'ultimate speed experiment'

In his 1962 experiment Bertozzi accelerated electrons through a large potential difference before allowing them to travel through a 'drift space' of 8.4 m. Oscilloscope traces recorded the time of flight (see figure 1), which allows the average velocity to be determined by the dividing the distance travelled by the time taken. Although for energies greater than 1.5 MeV some acceleration took place within the Linac, the electrons travel almost the whole distance at constant velocity. By assuming that the velocity is constant it can be calculated from the kinetic energy acquired by the electrons using $\mathrm{KE} = \frac{1}{2}mv^2$.

A teaching approach

The application of the constructivist approach can be summarized in a simple form as:

1. raising the student's awareness of their current belief system
2. presenting the student with a situation that allows them to see that this framework is at fault
3. allowing the student to discover an idea that addresses the fault

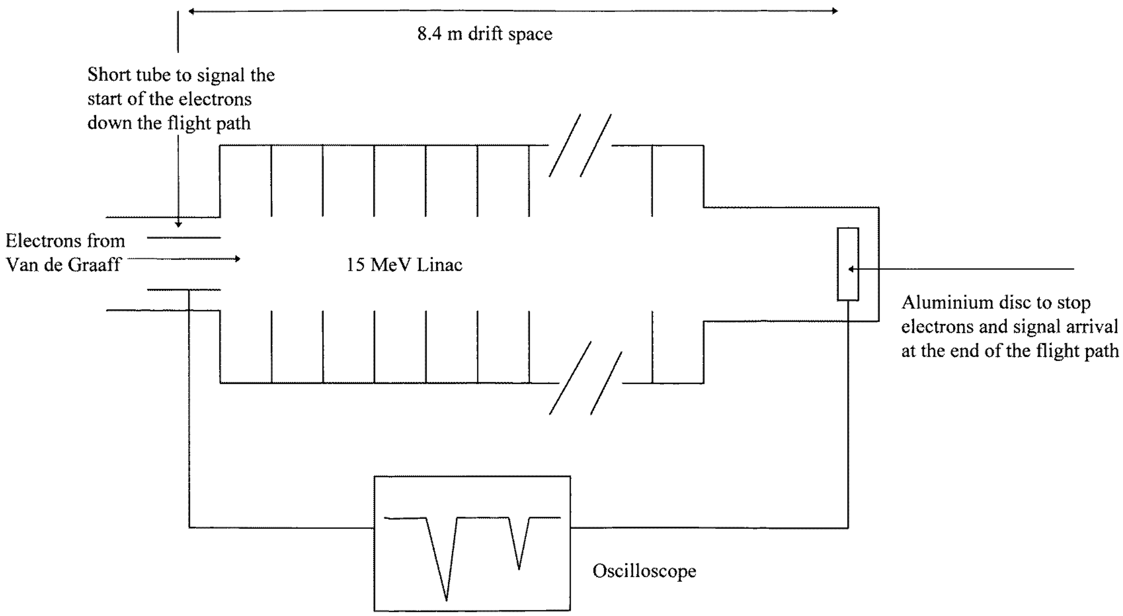

Figure 1. Simplified diagram of Bertozzi's apparatus.

Table 1. Newtonian velocity prediction.

Kinetic energy (MeV)	Kinetic energy (10^{-14} J)	Velocity (10^8 m s^{-1})
0.5	8.0	2.96
1.0	16.0	4.19
1.5	24.0	5.13
4.5	72.0	8.89†
15.0	240	16.23†

† Some acceleration takes place within the Linac but the calculations are based on a uniform velocity (for 4.5 MeV only the first 1 m section of the Linac was used).

Table 2. Observed velocities.

Kinetic energy (MeV)	Time of flight (10^{-8} s)	Distance travelled (m)	Velocity (10^8 m s^{-1})
0.5	3.23	8.4	2.60
1.0	3.08	8.4	2.73
1.5	2.92	8.4	2.88
4.5	2.84	8.4	2.96
15.0	2.80	8.4	3.00

4. allowing the student to experience that the addition of the new idea generates a new framework which corrects the fault.

This simple outline can be used to generate a teaching scheme. After describing the experimental procedure of Bertozzi, students are asked to calculate the velocities and kinetic energy in joules, based on the kinetic energy in MeV, as shown in table 1.

This produces velocities in excess of 3×10^8 m s^{-1}, which may provide the 'fault in their framework' for some students whilst in others this 'fault' will come later.

Students are now asked to calculate the velocities from the observation of the time of flight, with the results shown in table 2.

This data set shows that the velocity approaches that of light, and whatever the student's current framework a conflict will now exist between the two data sets.

The two data sets can be graphed, on the same axes, and the discussion focused on the velocity at which departure from the Newtonian view becomes apparent and the indication of the limiting velocity (see figure 2). The application of $\mathrm{KE} = \frac{1}{2}mv^2$ can also be shown to be at fault since an increase in kinetic energy from 0.5 MeV to 15 MeV is a factor of 30 and hence the velocity increase should be by a factor of $\sqrt{30}$ or 5.5. The

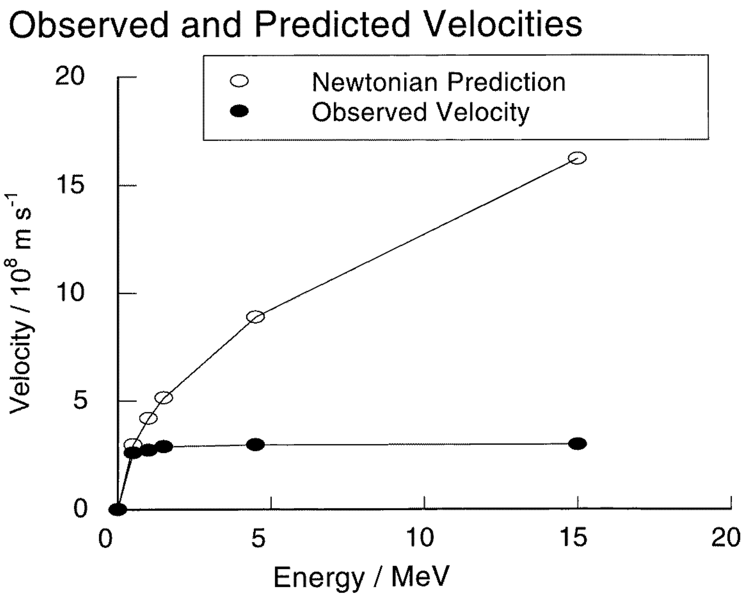

Figure 2. Graph to show the departure from Newtonian mechanics.

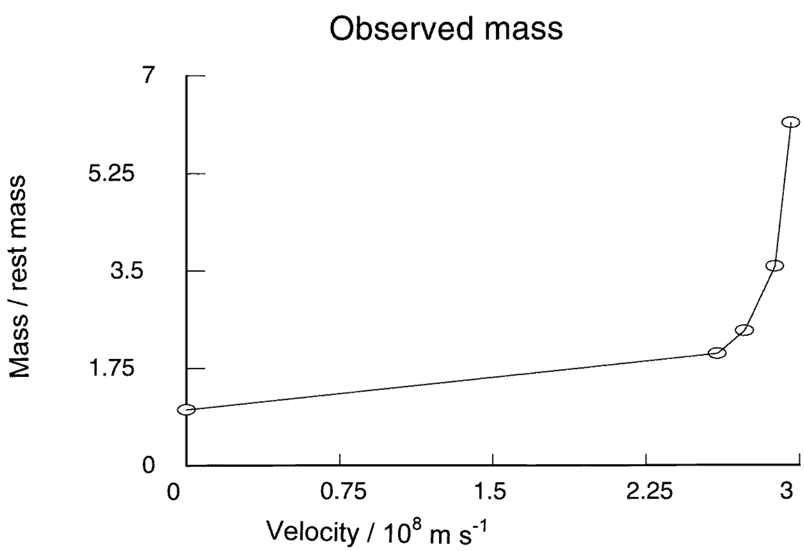

Figure 3. Graph to show the increase of mass with velocity.

observed increase the velocity is from 2.60×10^8 m s^{-1} to 3.00×10^8 m s^{-1} or a factor of 1.15.

Students can be presented with $m = m_0(1 - v^2/c^2)^{-1/2}$ and asked to calculate the observed mass of the electron at the velocities calculated from the time-of-flight data. Graphing these data (figure 3) allows discussion of the range of velocity for which mass can be considered invariant.

An alternative would be to allow the more able student to proceed along the lines of Bertozzi's original analysis by calculating the Newtonian and relativistic predictions of $(v/c)^2$ by the application of

$$(v/c)^2 = 2\mathrm{KE}/m_e c^2$$
$$(v/c)^2 = 1 - [m_e c^2/(m_e c^2 + \mathrm{KE})]^2$$

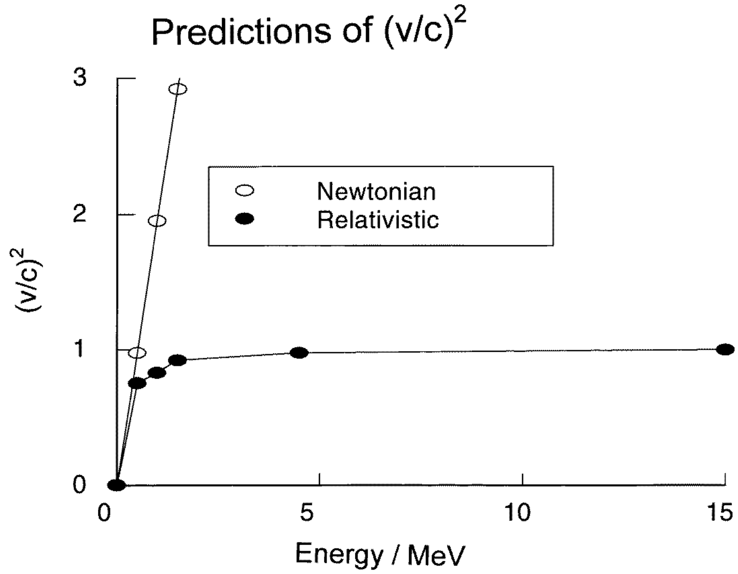

Figure 4. The prediction of $(v/c)^2$ from Newtonian and relativistic mechanics.

Table 3. Newtonian and relativistic predictions of $(v/c)^2$.

Energy (MeV)	Newtonian $(v/c)^2$	Relativistic $(v/c)^2$
0.5	0.974	0.752
1.0	1.95	0.828
1.5	2.92	0.922
4.5	8.78	0.974
15.0	29.3	1.000

where KE is the kinetic energy and m_e the rest mass of the electron. These results are shown in table 3 and figure 4.

Student feedback

This approach has been used with three cohorts of students and it has been shown to be useful across a wide range of ability and interest. The two most common questions during the discussion were:

1. How do we know that the electron gained the kinetic energy given by $\mathrm{KE} = eV$?
2. If a change in kinetic energy gives rise to a change in mass, is the same true for a change in potential or internal energy?

The first of these questions can be answered from Bertozzi's work since calorimetric measurements of the electron energy at the end of the flight path showed the energy to be as calculated.

The second is conceptually more difficult but can lead into discussions of $\Delta E = \Delta mc^2$, which students may have met in the context of nuclear decay. The example of melting ice, given by Beiser [5], can also be used, in which a certain quantity of ice at 0 °C melts into water at 0 °C, and in so doing gains 1 kg of mass. What was its initial mass? An addition to this would be the discussion point of why we can generally disregard this change.

Biographical note

William Bertozzi is currently Professor of Physics at Massachusetts Institute of Technology, where he remains active in both teaching and research. His research interests include electron scattering, nuclear reactions and the design and construction of spectrometers. He also serves of the Council for Primary and Secondary Education, which links the resources of MIT to students and educators in schools. The work described here was carried out during the production of a film entitled the '*The Ultimate Speed, An Exploration With High Energy*

Electrons' made at the Education Development Centre, Newton, Massachusetts during 1962. Whilst much experimental work had been carried out to affirm relativity, Bertozzi and his colleagues believed that previous work involved too many other concepts for the students to be able to readily see the effects of relativity.

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