On the role of the Avogadro constant in redefining SI units for mass and amount of substance

2007 Metrologia 44 82

(http://iopscience.iop.org/0026-1394/44/1/012)

INSTITUTE OF PHYSICS PUBLISHING METROLOGIA

Metrologia **44** (2007) 82–86 doi:10.1088/0026-1394/44/1/012

On the role of the Avogadro constant in redefining SI units for mass and amount of substance

B P Leonard

Department of Mechanical Engineering, The University of Akron, Akron, OH 44325, USA

Received 8 December 2006
Published 2 February 2007
Online at stacks.iop.org/Met/44/82

Abstract
There is a common misconception that the Avogadro constant is one of the fundamental constants of nature, in the same category as the speed of light, the Planck constant and the invariant masses of atomic-scale particles. Although the absolute mass of any specified atomic-scale entity is an invariant universal constant of nature, the Avogadro constant relating this to a macroscopic quantity is not. Rather, it is a man-made construct, designed by convention to define a convenient unit relating the atomic and macroscopic scales. The misportrayal seems to stem from the widespread use of the term 'fixed-Avogadro-constant' for describing a redefinition of the kilogram that is, in fact, based on a fixed atomic-scale particle mass. This paper endeavours to clarify the role of the Avogadro constant in current definitions of SI units for mass and amount of substance as well as recently proposed redefinitions of these units—in particular, those based on fixing the numerical values of the Planck and Avogadro constants, respectively. Precise definitions lead naturally to a rational, straightforward and intuitively obvious construction of appropriate (exactly defined) atomic-scale units for these quantities. And this, in turn, suggests a direct and easily comprehended two-part statement of the fixed-Planck-constant kilogram definition involving a well-understood and physically meaningful de Broglie–Compton frequency.

1. Introduction

The Avogadro constant, N_A, is usually described as a fundamental physical constant or an invariant of nature, in the same category as the vacuum light speed, the elementary charge and the Planck and Boltzmann constants, for example [1–4]. In contrast to these other fundamental physical constants, however, the Avogadro constant is not a constant of nature. Rather, it is a man-made construct, used as a convenient scaling factor designed for relating properties of atomic-scale entities and macroscopic quantities, its implicit numerical value in this role having been decided by fixing the numerical value of the amount-specific mass of carbon-12 at exactly $12\,\mathrm{g\,mol^{-1}}$. The basic concept defining the Avogadro constant is the number of (specified elementary) entities per unit of amount of substance. In this general sense, it is in the same category as, for example, the 'retail' constant: 12 per dozen, the 'wholesale' constant: 144 per gross and the 'paper-packaging' constant: about 500 per ream—where dozen, gross and ream describe various convenient commercial-amount units for measuring amounts of specified discrete items. The numerical value of N_A, of course, depends on the chemical-amount unit used: $N_A \approx 6.02 \times 10^{20}\,\mathrm{mmol^{-1}}$ or 2.73×10^{26} lb-mol^{-1}, for example.

If $m(\mathrm{X})$ is the absolute mass of an individual entity X, the amount-specific mass, $M(\mathrm{X})$, of an amount of these entities is given by

$$M(\mathrm{X}) = N_A m(\mathrm{X}), \tag{1}$$

showing the role of N_A in relating macroscopic and atomic-scale properties. In the following, the role of N_A in current definitions and proposed redefinitions of SI units for mass and amount of substance will be clarified explicitly. We will see that it is not N_A itself that is a universal constant of nature but, rather, $M(\mathrm{X})/N_A$—which, of course, is the absolute mass of the specified entity X. The Avogadro constant plays an important role in constructing atomic-scale units for mass and amount of substance, leading to a logical redefinition of the atomic-scale reference mass, the dalton—defined exactly in terms of the kilogram—that is more straightforward and much more easily comprehended than the strategy recently proposed by Mills *et al* [4] retaining the conventional inexact definition of the atomic-scale mass unit based on $m(^{12}\mathrm{C})$, thereby requiring a potentially confusing correction factor for relating amount-specific mass to the relative atomic-scale mass. The exact definition of the dalton, in turn, suggests an easily understood two-part statement of the fixed-Planck-constant definition of the kilogram involving a physically realizable de Broglie–Compton frequency corresponding to the exactly defined dalton.

2. Current definitions

The current definition of the SI unit for amount of substance fixes, by convention [5], the amount-specific mass of carbon-12 at exactly 12 g mol^{-1} (= 12 kg kmol^{-1}):

$$M(^{12}\mathrm{C}) = N_\mathrm{A} m(^{12}\mathrm{C}) = 12\,\mathrm{kg\,kmol^{-1}},\ \text{exactly}. \tag{2}$$

Note that since the atomic mass of carbon-12 is indeed an invariant of nature, this means that, given the current definition, the Avogadro constant also appears to be invariant, although by convention rather than by nature. However, with the current definition of the kilogram, as the mass of the international prototype of the kilogram (IPK), a physical artefact, the kilogram unit appearing on the right-hand side of equation (2) is not invariant with respect to a truly invariant mass (such as that of the carbon-12 atom itself, for example). We can write

$$\mathrm{kg} \equiv m_\mathrm{IPK}(t) = m_\mathrm{Ref}[1+\delta(t)], \tag{3}$$

where m_Ref is a reference value of the mass of the IPK and $\delta(t)$ is the relative deviation representing the drift in its mass over time due to contamination and handling—although, of course, $m_\mathrm{IPK}(t)$ is always equal to 1 kg, by definition.

It is instructive to relate $m_\mathrm{IPK}(t)$ to the invariant mass, $m(^{12}\mathrm{C})$:

$$\mathrm{kg} \equiv m_\mathrm{IPK}(t) = N_\mathrm{Ref}[1+\delta(t)]m(^{12}\mathrm{C}), \tag{4}$$

where N_Ref is a time-invariant dimensionless constant that must be determined by experiment (i.e. by measuring the mass of the carbon-12 atom in units of m_Ref). Then, from equations (2) and (4), we have, for the Avogadro constant,

$$N_\mathrm{A} = 12 N_\mathrm{Ref}[1+\delta(t)]\,\mathrm{kmol^{-1}}, \tag{5}$$

showing the corresponding drift in the nominally invariant magnitude of N_A due to its dependence on m_IPK. When the value of N_Ref is determined experimentally, its relative experimental deviation, $\varepsilon_\mathrm{expt}$, appears as follows:

$$N_\mathrm{Ref} = N_0(1+\varepsilon_\mathrm{expt}), \tag{6}$$

where N_0 is a known constant. Thus

$$N_\mathrm{A} = 12 N_0(1+\varepsilon_\mathrm{expt})[1+\delta(t)]\,\mathrm{kmol^{-1}}, \tag{7}$$

or, since $\varepsilon_\mathrm{expt}$ and $\delta(t)$ are extremely small, we need to keep only the first-order terms:

$$N_\mathrm{A} = 12 N_0[1+\varepsilon_\mathrm{expt}+\delta(t)]\,\mathrm{kmol^{-1}}, \tag{8}$$

showing explicitly the experimental and IPK-drift deviations, given the current definitions of SI units for mass and amount of substance and the coupling between them.

On rearranging equation (8), we have

$$\mathrm{kmol} = 12 N_0[1+\varepsilon_\mathrm{expt}+\delta(t)]/N_\mathrm{A}. \tag{9}$$

At this point it is essential to the following analysis to introduce an atomic-scale unit of amount of substance equal to that consisting of exactly one (specified elementary) entity. Call this unit 'entity' with a corresponding symbol 'ent.' One thing we can say for certain about the Avogadro constant (number of entities per unit of amount of substance) is that it is equal to *exactly* one per entity:

$$N_\mathrm{A} = 1\,\mathrm{ent^{-1}},\ \text{exactly}. \tag{10}$$

This is the only form in which the Avogadro constant might be called 'naturally invariant.' Substituting this into equation (9) gives

$$\mathrm{kmol} = 12 N_0[1+\varepsilon_\mathrm{expt}+\delta(t)]\,\mathrm{ent}, \tag{11}$$

thereby defining the kilomole in a simple and direct (although not invariant) way in terms of a number of entities. Since the atomic-scale unit entity is defined exactly, we see explicitly that the (current) definition of amount of substance exhibits two types of deviation from an exact value: $\varepsilon_\mathrm{expt}$ stemming from determining the mass (in terms of a reference kilogram) of the carbon-12 atom by experiment and $\delta(t)$ due to the drift of the IPK. Given the goal of constructing *invariant* definitions of SI units, it is clear that units for mass and amount of substance need to be redefined.

3. Redefinitions

Redefining the kilogram in an invariant form has been a topic of concern for some time [1–4, 6–11]. The two most common proposals are the fixed-Planck-constant definition (fixing the numerical value of h when expressed in SI units) and the so-called 'fixed-Avogadro-constant' definition. The latter fixes the value of N_A—*and* the value of $M(^{12}\mathrm{C})$ (at exactly 12 kg kmol^{-1})—thereby fixing the value of $m(^{12}\mathrm{C})$ when expressed in units of kilogram. In other words, this is really a 'fixed-carbon-12-atomic-mass' definition. For example, if a fixed value of $m(^{12}\mathrm{C})$ is used to define the kilogram, we can write

$$\mathrm{kg} = N_\mathrm{C} m(^{12}\mathrm{C}),\ \text{exactly}, \tag{12}$$

where N_C is a specified invariant dimensionless constant (chosen to assure continuity with the IPK-based kilogram at the time of redefinition). Then, by keeping $M(^{12}\mathrm{C}) = 12\,\mathrm{kg\,kmol^{-1}}$, exactly, from equations (2), (10) and (12), we have a straightforward corresponding definition for the kilomole:

$$\mathrm{kmol} = 12 N_\mathrm{C}\,\mathrm{ent},\ \text{exactly}. \tag{13}$$

The kilomole definition is now exact and invariant, as well. The Avogadro constant is thus explicitly and exactly defined in a compatible way as seen by rearranging equation (13):

$$N_\mathrm{A} = 1\,\mathrm{ent^{-1}} = 12 N_\mathrm{C}\,\mathrm{kmol^{-1}},\ \text{exactly}, \tag{14}$$

and this fixed value of the Avogadro constant guarantees that $M(^{12}\text{C}) = 12\,\text{kg}\,\text{kmol}^{-1}$, exactly. Also, the atomic-scale reference mass unit, dalton, is defined exactly in terms of the kilogram: $\text{Da} = \text{kg}/(12N_\text{C})$. We should note, however, that, with these definitions, the numerical value of the Planck constant, h, expressed in SI units, cannot be fixed independently and must be determined by experiment [11].

With the fixed-h definition of the kilogram, there are two choices for redefining the unit for amount of substance. Keeping the current implicit definition, fixing $M(^{12}\text{C})$, still involves experimental uncertainty in determining $m(^{12}\text{C})$ and, thus, in the unit definition. The alternative is to fix N_A independently; however, in this case, the exactness constraint on $M(^{12}\text{C})$ must be relaxed [11]. In other words, the two alternatives are to have either an exact $M(^{12}\text{C})$ and an experimental uncertainty in the definition of the SI unit for amount of substance or an invariant definition and an experimental uncertainty in the value of $M(^{12}\text{C})$—as is the case for all other entities. The latter choice is clearly more in keeping with the overall goal of constructing *invariant* definitions of SI units, if possible.

For the fixed-h definition of the kilogram, since h has units of joule second, we can write

$$\text{kg} = N_h(h/c_0^2)\cdot(\text{s}^{-1}),\ \text{exactly}, \tag{15}$$

where N_h is a known dimensionless constant whose value is chosen to assure continuity with the current definition of the mass unit and c_0 is the fixed vacuum light speed. Independently, the kilomole can be written explicitly as a fixed number of entities:

$$\text{kmol} = N^*\,\text{ent},\ \ \text{exactly}, \tag{16}$$

where N^* is a known dimensionless constant related to the Avogadro constant:

$$N_\text{A} = 1\,\text{ent}^{-1} = N^*\,\text{kmol}^{-1},\ \ \text{exactly}. \tag{17}$$

In other words, N^* is the numerical value of the Avogadro constant when N_A is expressed as a number per kilomole.

In order to assure continuity with current definitions, we note that although $M(^{12}\text{C})$ cannot be fixed independently once h and N_A are fixed, it must satisfy the compatibility equation [11]:

$$hN_\text{A} = K_C M(^{12}\text{C}) = \text{fixed value}, \tag{18}$$

where the compatibility constant (with dimensions of length squared per unit time) is given by

$$K_C = (c_0/2)[m_\text{e}/m(^{12}\text{C})]\,\alpha^2/R_\infty, \tag{19}$$

involving, in addition to c_0, which is exact, the electron/carbon-12 mass ratio, $m_\text{e}/m(^{12}\text{C})$, the fine-structure constant, α, and the Rydberg constant, R_∞. The numerical value of K_C must be determined by experiment and it will have a relative deviation $\delta(K_C)$:

$$K_C = K_{C0}[1+\delta(K_C)]. \tag{20}$$

In choosing the fixed values of h and N_A, we require, to assure continuity,

$$hN_\text{A} = 12M_\text{u}K_{C0}, \tag{21}$$

where $M_\text{u} = 1\,\text{kg}\,\text{kmol}^{-1}$, exactly. Then, from equations (18) and (20), $M(^{12}\text{C})$ will have an experimental deviation given by

$$M(^{12}\text{C}) = 12M_\text{u}(K_{C0}/K_C) = 12M_\text{u}/[1+\delta(K_C)] = [1-\delta(K_C)]12M_\text{u}, \tag{22}$$

to first order.

Using the following values and relative standard uncertainties taken from [2]: $c_0 = 299\,792\,458\,\text{m}\,\text{s}^{-1}$ (exact), $R_\infty = 10\,973\,731.568\,525\,\text{m}^{-1}$, $u_\text{r}(R_\infty) = 6.6\times10^{-12}$, $m_\text{e}/m(^{12}\text{C}) = M_\text{e}/(12M_\text{u}) = 4.571\,499\,2454\times10^{-5}$, $u_\text{r}(M_\text{e}) = 4.4\times10^{-10}$, and [12]: $\alpha = 7.297\,352\,5359\times10^{-3}$, $u_\text{r}(\alpha) = 7\times10^{-10}$, we can evaluate the compatibility constant to the indicated precision:

$$K_C = 3.325\,260\,5670(61)\times10^{-8}\,\text{m}^2\,\text{s}^{-1}, \tag{23}$$

with

$$u_\text{r}(K_C) = u_\text{r}(R_\infty) + u_\text{r}(M_\text{e}) + 2u_\text{r}(\alpha) = 18.5\times10^{-10}. \tag{24}$$

The fixed values of the Planck and Avogadro constants should satisfy

$$hN_\text{A} = 12M_\text{u}K_{C0} = 3.990\,312\,6804\times10^{-7}\,\text{J}\,\text{s}\,\text{kmol}^{-1}. \tag{25}$$

By setting h equal to $6.626\,069\,282\times10^{-34}\,\text{J}\,\text{s}$ and N_A equal to $6.022\,1415\times10^{26}\,\text{kmol}^{-1}$, we find that, to the indicated precision:

$$M(^{12}\text{C}) = 12.000\,000\,000(22)\,\text{kg}\,\text{kmol}^{-1}, \tag{26}$$

with $u_\text{r}[M(^{12}\text{C})] = u_\text{r}(K_C)$.

4. Atomic-scale units

As experimental techniques for determining the value of K_C (primarily with regard to the mass ratio and fine-structure constant) improve, we need to distinguish between the known relative deviation in the value of $M(^{12}\text{C})$, as it increases in precision, and its (decreasing) relative uncertainty. From equation (22), we can write

$$M(^{12}\text{C}) = (1+\kappa_\text{known}+\kappa_\text{expt})12M_\text{u}, \tag{27}$$

where κ_known represents the known relative deviation and κ_expt is the corresponding experimental deviation, with $\kappa_\text{known} = 0$ and $|\kappa_\text{expt}| = \text{O}[u_\text{r}(K_C)_{t=0}]$ at the time of redefinition. Since we need to relate this to the atomic scale, the appropriate atomic-scale units therefore have to be defined with care and clarity.

Mills *et al* [4] define an atomic-scale mass constant in the conventional way:

$$m_\text{u} = m(^{12}\text{C})/12 = \text{u}, \tag{28}$$

where we should note that u is an inexact mass unit (in terms of the fixed-h kilogram), with $u_\text{r}(\text{u}) = u_\text{r}(K_C)$. Using equation (2), equation (27) can be rewritten as

$$N_\text{A}m_\text{u} = (1+\kappa_\text{known}+\kappa_\text{expt})M_\text{u}. \tag{29}$$

Following Mills *et al*, if we also define the relative atomic-scale mass of any entity in the conventional way,

$$A_\text{r}(\text{X}) = m(\text{X})/m_\text{u}, \tag{30}$$

noting that $A_\text{r}(^{12}\text{C}) = 12$, exactly, then the basic relationship, $M(\text{X}) = N_\text{A}m(\text{X})$, becomes

$$M(\text{X}) = N_\text{A}A_\text{r}(\text{X})m_\text{u} = (1+\kappa_\text{known}+\kappa_\text{expt})A_\text{r}(\text{X})M_\text{u}, \tag{31}$$

as compared with the well-known formula $M(\mathrm{X}) = A_\mathrm{r}(\mathrm{X})M_\mathrm{u}$ using current definitions. Mills *et al* recommend using equation (31) for relating $M(\mathrm{X})$ and $A_\mathrm{r}(\mathrm{X})$. Their κ is equal to $(\kappa_\mathrm{known} + \kappa_\mathrm{expt})$ used here. They state that $(1+\kappa)$ 'will initially be equal to one…', whereas, although $\kappa_\mathrm{known}(0) = 0$, the initial experimental deviation is $\kappa_\mathrm{expt}(0) \neq 0$. For 'real world' calculations, they recommend using the standard formula:

$$M(\mathrm{X}) \approx A_\mathrm{r}(\mathrm{X})M_\mathrm{u}, \tag{32}$$

since the relative uncertainties involved in most calculations are likely to be much larger than $|(\kappa_\mathrm{known} + \kappa_\mathrm{expt})|$. But fixing N_A exactly while retaining the conventional definitions of u and $A_\mathrm{r}(\mathrm{X})$ by using $m(^{12}\mathrm{C})/12$ as the (inexact) reference atomic-scale mass and introducing a multiplicative 'correction factor' into equation (31) and related equations (and then setting it equal to unity for practical calculations) is a very cumbersome and potentially confusing—and ultimately unnecessary—complication.

A much more straightforward strategy is to define the atomic-scale mass unit, the dalton, exactly in terms of the kilogram:

$$\mathrm{Da} = (1/N^*)\,\mathrm{kg},\ \text{exactly}, \tag{33}$$

where N^* is the numerical value of the Avogadro constant appearing in the fixed-N_A definition of the kilomole, equation (16), which itself can be rewritten as

$$\mathrm{ent} = (1/N^*)\,\mathrm{kmol},\ \text{exactly}, \tag{34}$$

paralleling the definition of the dalton. Absolute entity masses can be catalogued in units of dalton, directly. Then we have in a straightforward manner, stemming from the basic relationship, equation (2),

$$M(\mathrm{X}) = N_\mathrm{A} m(\mathrm{X}) = \{m(\mathrm{X})\}_\mathrm{Da}\,\mathrm{Da\,ent}^{-1}. \tag{35}$$

In other words, the amount-specific mass of a given entity in exactly defined atomic-scale units, $\mathrm{Da\,ent}^{-1}$, very simply has exactly the same numerical value as the absolute entity mass expressed in Da. But, from equations (33) and (34),

$$\mathrm{Da\,ent}^{-1} = \mathrm{kg\,kmol}^{-1} = \mathrm{g\,mol}^{-1}, \tag{36}$$

so that

$$M(\mathrm{X}) = \{m(\mathrm{X})\}_\mathrm{Da}\,\mathrm{kg\,kmol}^{-1} = \{m(\mathrm{X})\}_\mathrm{Da}\,\mathrm{g\,mol}^{-1}. \tag{37}$$

If desired, we could introduce the relative atomic-scale mass, $A_\mathrm{r}(\mathrm{X})$, defined as

$$A_\mathrm{r}(\mathrm{X}) = m(\mathrm{X})/m_\mathrm{u}, \tag{38}$$

where $m_\mathrm{u} = 1\,\mathrm{Da}$, exactly, is the atomic-scale mass unit—not equal to $m(^{12}\mathrm{C})/12$ in the case of the fixed-h kilogram—in which case we have

$$M(\mathrm{X}) = A_\mathrm{r}(\mathrm{X})M_\mathrm{u},\ \text{exactly}, \tag{39}$$

where $M_\mathrm{u} = 1\,\mathrm{Da\,ent}^{-1} = 1\,\mathrm{kg\,kmol}^{-1} = 1\,\mathrm{g\,mol}^{-1}$, etc. Of course, $A_\mathrm{r}(^{12}\mathrm{C})$ will not be equal to 12 exactly for the fixed-h kilogram definition, and the relative atomic-scale masses of some other entities might need to be updated as more precision is gained in the value of K_C. However, this would be a relatively simple short-term adjustment, easily incorporated into the periodic review of atomic-scale properties. This seems to be the most logical—and easily comprehended—approach to dealing with atomic-scale properties and relating them to macroscopic quantities.

5. Statement of the fixed-h kilogram definition

The use of the exactly defined dalton naturally leads to a two-part statement of the fixed-h definition of the kilogram that avoids the introduction of unphysical frequencies or wavelengths. As pointed out in [4], the fixed-h kilogram definition is based on $h\nu = mc_0^2$. If $m = 1\,\mathrm{kg}$, the corresponding de Broglie–Compton frequency is unphysically large ($>10^{50}$ Hz). Instead, we define the kilogram as equal to N^* dalton, exactly:

$$\mathrm{kg} = N^*\,\mathrm{Da},\ \text{exactly}, \tag{40}$$

where the dalton is, in turn, defined as

$$\mathrm{Da} = \nu^*(h/c_0^2),\ \text{exactly}, \tag{41}$$

where ν^* is the de Broglie–Compton frequency of a mass of exactly one dalton, given by $\nu^* = (1/N^*)\{c_0\}^{*2}/\{h\}^*\,\mathrm{Hz}$, exactly, with N^* being the numerical value of N_A when expressed in a number per kilomole and the notation $\{q\}^*$ denoting the fixed numerical value of the argument expressed in SI base units. Substituting $\{c_0\}^* = 299\,792\,458$ and $\{h\}^* = 6.626\,069\,282 \times 10^{-34}$, we find that the pseudo-frequency $\nu^* = 2.252\,342\,7378\ldots \times 10^{23}$ Hz, comparable to the corresponding frequency of the proton mass ($\nu_\mathrm{p} = 2.268\,731\,8151\ldots \times 10^{23}$ Hz), as would be expected. This two-part statement of the fixed-h kilogram definition is thus very easy to understand and directly introduces the exactly defined atomic-scale mass unit while avoiding the use of any unphysical quantities.

6. Summary

Unlike fundamental constants of nature (such as c_0, h and e), the Avogadro constant is a man-made scaling factor relating atomic-scale and macroscopic quantities, with its value in this role currently implicitly defined by the convention of setting the amount-specific mass of carbon-12 equal to $12\,\mathrm{g\,mol}^{-1}$, exactly. The misportrayal of N_A as a fundamental constant of nature arises from the so-called 'fixed-Avogadro-constant' definition of the kilogram, which always requires, in addition, that $M^{12}(\mathrm{C}) = 12\,\mathrm{g\,mol}^{-1}$, exactly—a man-made condition imposed by convention. In fact, this is a 'fixed-carbon-12-atomic-mass' definition. It is $m(^{12}\mathrm{C}) = M(^{12}\mathrm{C})/N_\mathrm{A}$ that is the invariant constant of nature, not N_A itself. Once the definition of the unit for amount of substance is written in terms of an exact constant, as, for example, $\mathrm{kmol} = N^*/N_\mathrm{A}$, the only thing one can say definitely about the Avogadro constant in terms of an exactly defined unit for amount of substance is that it is exactly $1\,\mathrm{ent}^{-1}$, giving $\mathrm{kmol} = N^*\,\mathrm{ent}$ as the invariant definition for the kilomole, thereby fixing $N_\mathrm{A} = N^*\,\mathrm{kmol}^{-1}$, exactly.

Redefinitions of SI units for mass and amount of substance designed by fixing the numerical values of h and N_A independently require relaxation of the exactness condition on $M(^{12}\mathrm{C})$. Expected progress in reducing uncertainties in $m_\mathrm{e}/m(^{12}\mathrm{C})$ and α in the near future will allow straightforward and easily comprehended definitions of atomic-scale units: the exactly defined dalton, Da, and entity, ent, categorized as units in use with SI. Absolute entity masses can be catalogued directly in terms of dalton. Then the amount-specific mass of a specified entity can be expressed in Da ent^{-1}, kg kmol^{-1}, g mol^{-1} or mg mmol^{-1} (all of which are equal), with a numerical value exactly equal to that of the absolute entity mass expressed in dalton. If desired, one could introduce the relative atomic-scale mass, equation (38), and use the familiar formula, $M(\mathrm{X}) = A_\mathrm{r}(\mathrm{X})M_\mathrm{u}$, satisfied exactly—without any correction factor.

Finally, the use of the exactly defined dalton, equation (33), suggests a two-part statement of the fixed-h definition of the kilogram which is very easily understood and does not rely on the introduction of physically impossible quantities.

References

[1] Taylor B N 1994 Determining the Avogadro constant from electrical measurements *Metrologia* **31** 181–94

[2] Mohr P J and Taylor B N 2005 CODATA recommended values of the fundamental constants 2002 *Rev. Mod. Phys.* **77** 1–107

[3] Mills I M, Mohr P J, Quinn T J, Taylor B N and Williams E R 2005 Redefinition of the kilogram: a decision whose time has come *Metrologia* **42** 71–80

[4] Mills I M, Mohr P J, Quinn T J, Taylor B N and Williams E R 2006 Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation 1 (CI–2005) *Metrologia* **43** 227–46

[5] Terrien J 1972 News from the Bureau International des Poids et Mesures *Metrologia* **8** 32–6

[6] Taylor B N 1991 The possible role of the fundamental constants in redefining the kilogram *IEEE Trans. Instrum. Meas.* **40** 86–91

[7] Taylor B N and Mohr P J 1999 On the redefinition of the kilogram *Metrologia* **36** 63–4

[8] Steiner R L, Williams E R, Newell D B and Liu R 2005 Towards an electronic kilogram: an improved measurement of the Planck constant and electron mass *Metrologia* **42** 431–41

[9] Flowers J L and Petley B W 2005 The kilogram redefinition—an interim solution *Metrologia* **42** L31–4

[10] Picard A 2006 Mass determinations of a 1 kg silicon sphere for the Avogadro project *Metrologia* **43** 46–52

[11] Leonard B P 2006 Note on invariant redefinitions of SI base units for both mass and amount of substance *Metrologia* **43** L3–5

[12] Gabrielse G, Hanneke D, Kinoshita T, Nio M and Odom B 2006 New determination of the fine structure constant from the electron g value and QED *Phys. Rev. Lett.* **97** 030802