Considerations on future redefinitions of the kilogram, the mole and of other units

2007 Metrologia 44 1

(http://iopscience.iop.org/0026-1394/44/1/001)

INSTITUTE OF PHYSICS PUBLISHING METROLOGIA
Metrologia **44** (2007) 1–14 doi:10.1088/0026-1394/44/1/001

Considerations on future redefinitions of the kilogram, the mole and of other units

P Becker[1], **P De Bièvre**[2], **K Fujii**[3], **M Glaeser**[1,6], **B Inglis**[4], **H Luebbig**[1] **and G Mana**[5]

[1] Physikalisch-Technische Bundesanstalt (PTB), Bundesallee 100, 38116 Braunschweig, Germany
[2] Independent Consultant on Metrology in Chemistry (formerly with the Institute for Reference Materials and Measurements, IRMM, Geel, 1961–2002) B-2460 Kasterlee, Belgium
[3] National Metrology Institute of Japan (NMIJ), AIST, Tsukuba 305-8563, Japan
[4] National Measurement Institute, Australia (NMIA), PO Box 264, Lindfield NSW 2070, Australia
[5] Istituto Nazionale di Ricerca Metrologica (INRIM), str. delle cacce 91, 10135 Torino, Italy

E-mail: michael.glaeser@ptb.de

Received 15 September 2006
Published 11 December 2006
Online at stacks.iop.org/Met/44/1

Abstract

The definitions of some units of the Système International are likely to be revised as early as 2011 by basing them on fixed values of fundamental constants of nature, provided experimental realizations are demonstrated with sufficiently small uncertainties. As regards the kilogram, experiments aiming at linking it to the Planck constant and the atomic mass constant are under way in several laboratories. The other units likely to be redefined are the ampere, the kelvin and the mole. We discuss the advantages and disadvantages of different alternatives for revised definitions of the kilogram and the mole. From physical considerations, metrological consequences and ease of understanding, a definition of the kilogram based on the mass of a particle, such as an atom or the electron, is favoured. One of the proposed definitions fixes the value of the Planck constant through the Compton frequency of a material, though unphysical, particle. Finally, a redefinition of the mole, the counting unit of the amount-of-substance, is proposed which fixes the Avogadro constant as a dimensionless number.

1. Introduction

Among the seven base units of the Système International (SI), the definition of the kilogram is the only one that is still based on an artefact, the international prototype of the kilogram kept at the Bureau International des Poids et Mesures (BIPM) in Sèvres near Paris. The other units are defined by the values of specified quantities and the definitions of those that do not depend on the kilogram can be realized as primary measurement standards (see definition in [1]) at any place and at any time. In the definition of the metre, such a quantity value is the speed of light in vacuum, c_0, a fundamental constant of physics. Another constant is the magnetic constant (permeability of vacuum) μ_0, fixed in the definition of the ampere, which, together with the electric constant (permittivity of vacuum), $\varepsilon_0 = 1/(\mu_0 c_0^2)$, ensures the equality of the SI units of mechanical and electromagnetic force, energy and power. The quantity values fixed in the definitions of the remaining base units are atomic or material constants considered as being sufficiently invariant for all practical applications. With the increased requirements for smaller measurement uncertainties and better stability of the measurement units, efforts are made to also link the other base units of the SI to numerical values of fundamental constants of nature. About 30 years ago, two experiments aimed at linking the mass of the prototype of the kilogram to such a constant were begun. One is known as the watt balance experiment, the other as the Avogadro project. Other experiments followed, including those known as the voltage balance, the magnetic levitation and the ion

---

[6] Author to whom any correspondence should be addressed.

accumulation experiments [2–4]. The need for redefining the kilogram has been confirmed by the results of the third verification of the national copies of the kilogram against the international prototype in the period 1988–1992 [5, 6]. These results, together with those of the second verification (1946–1953), have shown that the mass values of the six official copies of the international prototype, the working prototypes of the BIPM and the national prototypes had increased, on average by about 50 µg in about 100 years relative to the international prototype. Taking into account that all these artefacts, including the international prototype, are made of the same material, manufactured in the same way and kept with great care over time, it is nevertheless probable that the mass of the international prototype has decreased, perhaps by as much as 50 µg. Furthermore, all these prototypes may have changed by an additional unknown amount, for which only an upper limit can be estimated [7].

Among the experiments aiming at redefining the kilogram, the smallest relative uncertainty, i.e. 5.2×10^{-8}, was obtained for the Planck constant in the NIST watt balance work in 2005 [8]. The Avogadro constant N_A was redetermined with a relative uncertainty of 3.1×10^{-7} in 2005 [9–11]. However, when the measurement results for h and N_A are compared using a relationship involving additional, well-known fundamental constants, it turns out that their experimental values, converted to the same quantity, have a relative difference of 10^{-6}, a value that is statistically incompatible with their quoted uncertainties.

The first NPL watt balance achieved a relative uncertainty of 2×10^{-7} on the Planck constant h in 1990 [12]. The Federal Office of Metrology (METAS) started to set up a watt balance in 1997, the Laboratoire Nationale d'Essais (LNE) in 2000 and the BIPM in 2002. The voltage balance experiments achieved relative uncertainties of about 3×10^{-7} [13, 14], but this work was terminated because smaller uncertainties could not be expected with reasonable effort. The magnetic levitation experiment of the National Metrology Institute of Japan (NMIJ/AIST) has also been discontinued after achieving a relative reproducibility of 10^{-6} [15]. The All Russia D I Mendeleyev Scientific and Research Institute for Metrology (VNIIM) and the Centre for Metrology and Accreditation (MIKES) have plans for a new magnetic levitation experiment [16]. The ion accumulation experiment of the Physikalisch-Technische Bundesanstalt (PTB) started in 1990. Even though the principle of ion accumulation has been demonstrated [4] and even though it might conceptually be considered as the 'obvious' experiment for the realization of a new kilogram definition as the mass of a number of atoms, it seems unlikely that it will reach the required relative uncertainty within the time frame anticipated (see below).

The redefinition of the units: kilogram, mole, ampere and kelvin was on the agenda of the meetings of the European Collaboration in Measurement Standards (EUROMET) technical committee for mass, of several consultative committees of the Comité International des Poids et Mesures (CIPM) and of the CIPM itself in 2005. In particular, a proposal was discussed [17] for redefining these units in 2007, when the next Conférence Générale des Poids et Mesures (CGPM) will meet. Statements by several national metrology institutes (NMIs) on this proposal have been sent to the relevant committees. The unanimous opinion was that a redefinition of the kilogram in 2007 is untimely, because of the above-mentioned discrepancy between the N_A value derived from h of the watt balance and the N_A value of the Si Avogadro work. The CIPM now envisages that redefinitions will be proposed to the 2011 CGPM, provided the measurement results of the ongoing projects are 'indeed acceptable' [18]. It must be mentioned that the Comité consultatif pour la masse et les grandeurs apparantées (CCM) formulated conditions under which the kilogram may be redefined and that one of them is that a relative uncertainty of 2×10^{-8} is demonstrated for the future realizations of the definition at the 1 kg level [19].

The following criteria for revised definitions of SI units seem to be widely accepted:

- continuity between the realizations of the old and the new definition,
- realization of the unit with a smaller (or at least the same) measurement uncertainty as before,
- better stability of the quantity concerned,
- consistency and coherence with the other SI base units,
- possibility of realization anywhere and at any time, at least in principle,
- based on commonly accepted laws in physics,
- conceptually clear and easy to understand.

We outline the metrological requirements for revised definitions of some SI units, based on the physical background of such definitions and on considerations about the general comprehensibility of such definitions. A recent publication proposes a redefinition of the kilogram by fixing the numerical value of the Planck constant and a redefinition of the mole by fixing the numerical value of the Avogadro constant [20]. Another option for the kilogram that fixes the Compton frequency of the kilogram is also discussed in [20]. Here, we present further options for revised definitions of the kilogram and the mole, and we discuss their various advantages and disadvantages provided future experimental results achieve sufficiently small uncertainties. A definition of the kilogram that is based on the mass of a particle, such as an atom or an electron, is favoured. An option that fixes the value of the Planck constant and relates it to the mass of a material particle is also presented for consideration. Finally, a revised definition of the mole, based on the Avogadro constant being redefined as a number, is proposed.

2. Who is affected by definitions of units and why?

Scientists and engineers need accurate metrological references for their measurement results. They wish to have measurement units with a high degree of invariance and universality. Also, very small uncertainties in the values of the fundamental constants are required in various fields of research. An ideal situation would ensue if these units were based on fundamental constants having numerical values fixed by definition. An example is the metre which is based on a fixed value, c_0, of the speed of light in vacuum. Other useful constants are the charge of the electron, the mass of an atom such as ^{12}C and the Planck constant. Measurements are made everywhere in society and play an important role in our daily life, in science, technology and engineering, in trade and commerce, in transport, medicine and travel. The definitions and the stability of the common measurement units are therefore of paramount public interest.

Mass measurement is among the most important and widespread of all areas of metrology. It is widely used in the important fields of trade, food technology and chemistry. The unit of mass underpins other SI base units such as the ampere, the mole and the candela and also provides the basis for many derived units such as force, torque, pressure, density and flow. Today, mass comparisons can be performed in national metrology institutes with relative uncertainties down to 10^{-10} at the 1 kg level. Verification and calibration of weighing instruments require mass standards with small uncertainties because customers need weighing instruments that will weigh over several decades. Class I weighing instruments, for example, must be verified with weights of a relative expanded uncertainty of 5×10^{-7}. Metrological traceability requires a calibration chain from commercial weights over, at least, four links up to the international prototype of the kilogram. In practice, the uncertainty increases at each link by a factor of, at least, 3 from the prototype down to commercial weights. National metrology institutes of states with an important industrial capacity need mass standards with a relative uncertainty of about 2 parts in 10^8, a value that has been confirmed in a recommendation of the CCM at its 2005 meeting.

Chemistry, biotechnology, medicine and environmental monitoring are fields where the amount-of-substance is measured and expressed in the unit mole. The uncertainty of such measurement results depends on measurements of other quantities such as mass, electric current, pressure or just on counting entities as in mass spectroscopy [21,22]. The concept of the quantity of amount-of-substance and the unit mole is often subject to various misunderstandings. A clarification of these concepts by a new definition of the mole is therefore useful at a time when other SI units are redefined.

The mass of elementary particles is of interest for high-energy physics. Examples of this interest are questions such as: 'Are neutrinos massive particles?' or 'What is the mass value of the Higgs boson?' and 'What are the mass origin and scale of the three generations of fundamental particles?'. A recent review paper expressed this interest by stating: 'The burning problems of today's particle physics are: mass, flavour, and unification" [23], Okun writes: 'Mass is one of the most fundamental concepts of physics' [24] and Wilczek (Nobel Prize 2004) asks 'What is the origin of mass?' [25]. Mass plays a key role also in astronomy and cosmology. The search for dark matter and dark energy is a high priority, because, with the present cosmological models, the knowledge of the universe is at odds [26].

In nuclear and sub-nuclear processes, mass and energy are converted one into the other. Therefore, particle masses (but also photon frequencies) are expressed in terms of the non-SI unit electron volt, eV. The conversion from atomic mass units to electron volts, i.e. to an absolute mass scale, requires the factor $\{m_{\mathrm{u}}c_0^2/e\}$, expressed in the unit eV u^{-1}, where m_{u} is the atomic mass constant, e is the elementary charge, the braces indicate that only the numerical value of the enclosed expression must be considered and u is the (unified) atomic mass unit. The uncertainties of the mass values of the electron, the proton and the neutron, expressed in atomic mass units, are given in CODATA 2002 [27]; atomic masses are known to within a relative uncertainty of 10^{-11} (^{4}He and ^{16}O) [28]. However, the uncertainty of the conversion factor (which is widely used, for example, in the tables of relative atomic masses of nuclides and of their mass excesses [28], as well as in the tables of molar masses [29]) is much larger and leads to corresponding larger uncertainties of the absolute masses; absolute mass excesses are known only to within a relative uncertainty of about 10^{-8} [28]. Another conversion, that of photon energies into frequency units, requires the h/e conversion factor. This factor presently has a moderately large uncertainty due to the discrepancy between the Avogadro measurements and the watt balance measurements. This factor will have either a very small uncertainty if m_{u} and e are fixed or no uncertainty if h and e are fixed.

3. Mass and fundamental constants of nature

3.1. The mass concept in classical and modern physics

In classical mechanics, mass, an empirical parameter to be determined experimentally, measures the bodies' inertia (i.e. their resistance to change in the rest or motion state) and gravitational charge. It was Newton's achievement to associate inertia and gravitation with the amount of matter, characterized by density and volume. More precisely, in the framework of an atomic theory of matter, mass is proportional to the amount-of-substance—as expressed by $M = N_{\mathrm{A}}m_{\mathrm{a}}$, where M and m_{a} are the molar mass and the atomic mass of a particle, respectively. But what is mass in the case of an elementary, structureless and point-like particle?

For a composite particle (for instance, a nucleus or atom), when Newton's law is derived from the low-velocity limit of special relativity, the proportionality factor between force and acceleration is E_0/c_0^2, where E_0 is the particle's internal energy. Therefore, special relativity entails the well-known relation

$$E_0 = m_0 c_0^2 \tag{1}$$

between (rest) mass m_0 and (internal) energy. The c_0 invariability suggests that mass is a nickname of the E_0/c_0^2 ratio and indicates that mass and energy are different names of the same quantity, an obvious statement when a system of units where $c_0 = 1$ is adopted. Similarly to classical mechanics, the question 'What is the internal energy of point-like and structureless particles like electrons?' is unanswered. Therefore, for elementary particles, the creation energy and, therefore, mass must be determined experimentally.

General relativity extends this idea and also identifies the gravitational charge with energy. It establishes a link between the energy distribution and the space-time metric. Newton's mass emerges when considering the low-field and low-velocity limits of Einstein's gravitation. In this limit, a point-like body appears to be a space–time singularity and its mass is given by

$$m_0 = \lambda_{\mathrm{G}} c^2/(2G), \tag{2}$$

where λ_{G} is the Schwarzschild radius and G is the gravitational constant. Extended bodies emerge similarly, but now the relevant concepts are density and space–time curvature.

A consequence of relativity is that, in the general case, $M = N_{\mathrm{A}}m_{\mathrm{a}}$ is no longer correct, mass depending on how particles are packed. The unification of mass and energy is evident in nuclear and sub-nuclear processes, when the exchanged energy is comparable to E_0. Material particles can be converted into energy, for example in electron–positron annihilation, and, conversely, energy (electromagnetic or kinetic) can be converted into particles. However, at low velocities mass and energy are separately conserved and split into different concepts. From this viewpoint, in Newtonian physics, the inertial and gravitational properties of a body are measured by a single independent quantity which is called mass and is additive and invariable; therefore, strictly speaking, mass is a classical quantity.

Relativistic quantum mechanics establishes a link between the frequency and energy of photons, in the case of the electromagnetic field, or of point-like and structureless particles, in the case of matter fields. As soon as the relativistic invariance is imposed, this link extends to their mass. For example, the non-relativistic limit of a Klein–Gordon field whose quanta are particles with rest mass m_0 is

$$\psi(x,t) = \varphi(x,t)\exp(2\pi \mathrm{i}\nu_{\mathrm{C}}t), \tag{3}$$

where $\varphi(x,t)$ is the Schrödinger wavefunction, x and t the space and time coordinates and the Compton frequency

$$\nu_{\mathrm{C}} = E_0/h = m_0c_0^2/h \tag{4}$$

is proportional to the particle's creation energy. According to (2) and (4), gravitation and relativistic quantum mechanics set two independent links between mass and space or time; if, by setting a system of units, we fix the speed of light and we enforce both, as in the case of the Planck units, then not only the mass unit, but also the space and time ones will follow.

Baryons, atoms and molecules are composite particles; they have a spectrum of internal energies and, consequently, a spectrum of mass values. In contrast, because leptons, in particular the electron, have no structure, their rest masses are uniquely defined. As regards macroscopic bodies, when mass is calculated from the number of atoms or molecules, the binding energy has to be considered. Therefore, if in a new definition of the kilogram a reference is made to the mass of a particle or of an atom, then the specifications such as 'free', 'in its ground state' and 'at rest' could be necessary in order to remove the ambiguities.

3.2. Fundamental constants in nature

Fundamental constants are key, universal and invariable properties of classes of physical systems, for example, the electron's charge and mass. Some are concept synthesizers [30] which unify seemingly separate fields of physics, for example, the speed of light in a vacuum in $E = mc_0^2$ (mass and energy equivalence) and the Planck, Boltzmann and Avogadro constants in $E = h\nu$ (wave–matter duality), $E = k_{\mathrm{B}}T$ (equivalence between temperature T and energy, with k_{B} the Boltzmann constant) and $M = N_{\mathrm{A}}m_{\mathrm{a}}$ (atomic structure of matter). They show that we are giving different names to quantities that are basically the same, but measured with different instruments. When the concept unification is deeply embedded in the common sense of reality, a rescaling occurs and the relevant fundamental constant declines to a mere conversion factor between measurement units; from this viewpoint, c_0^2 is the scaling factor between mass and energy, h between frequency and energy, k_{B} between temperature and energy and N_{A} between atomic and macroscopic masses. Whereas h and e are related to the atomic world, N_{A} (the number of entities, atoms or molecules in a given amount of a solid, liquid or gas) establishes the link between the micro- and macro-physics [21, 31].

3.3. A fundamental constant for a new definition of the kilogram

In the discussions about a new definition of the kilogram, the question of whether the mass of an atom or an elementary particle is more fundamental than the Planck constant has been raised [32, 33]. Since there is a spectrum of particles, each one having a different mass and none more fundamental than the others, giving a fixed value to the Planck constant is surely less arbitrary than giving a fixed value to the mass of an atom. Accordingly, it has been proposed [32–34] to base the kilogram definition on the Compton frequency using the relation (4), and the watt balance was suggested as its primary *mise en pratique*.

However, some difficulties must be considered. In the first place, (4) holds in the framework of the relativistic quantum-mechanical description of point-like and structureless particles, but a kilogram is the mass of a macroscopic body. How the classical world emerges from the quantum-mechanical one is far from being clarified and phenomena entailing a direct relation such as (4) between h and a macroscopic mass do not exist. Besides, the Compton frequency of a 1 kg body, about 10^{50} Hz, is higher than the Planck frequency, about 10^{43} Hz; a remedy is to base the kilogram definition on an ensemble of photons each having an accessible frequency and whose total energy sums up to a value that corresponds to a mass of 1 kg.

Second, the measurement equation (4) implies both the relations $E = h\nu$ and $E = mc_0^2$, which have not yet been directly tested with the same 2×10^{-8} relative accuracy required in order to enforce a new kilogram definition. While the ratio $mc_0^2/(hc/\lambda)$ has been tested to within 4.4×10^{-7} relative uncertainty [35], the first equation will require a significantly improved repetition of Millikan's 1916 verification of Einstein's model of the photoelectric effect [36].

In the third place, both relativistic quantum mechanics and Einstein's gravitation set links between mass and time. Since we do not yet have a satisfactory quantum theory of gravitation, the mass and time unifications implied by these two theories lead to different mass concepts. Although a realization of the mass unit via equation (3) is still beyond any technical possibilities—the present relative accuracy of G measurements is limited to about 10^{-4}, from a conceptual viewpoint, a kilogram definition based on the Planck constant is necessarily biased towards quantum mechanics.

Fourth, a satisfactory realization of the kilogram via the measurement equation (4) should access directly the Compton frequency, but the watt balance [12] establishes only an indirect link. It virtually compares mechanical and electrical powers by combining measurements in two different operation modes (gravitational and moving-coil) of the balance. In the gravitational mode, the force on a mass m in a uniform gravitational field g is balanced by the electromechanical force on a coil with current I in a magnetic field with a vertical magnetic flux gradient $\partial\Phi/\partial x$; in the moving-coil mode, a voltage U is induced by the coil moving with a velocity v in the same magnetic field. By combining the relevant equations of the two modes the value of the magnetic flux gradient is eliminated, and by arithmetical operations a mechanical power can be equated with an electrical power. I and U are measured in terms of the representations of the volt and the ohm based on macroscopic quantum effects [37]—a procedure that introduces h by means of the photon-assisted Josephson effect and the quantum Hall effect of integer order. The final measurement equation is

$$4UI = h\nu_{\mathrm{m}}\nu_{\mathrm{g}} = 4mgv, \tag{5}$$

where ν_{m} and ν_{g} are, respectively, the microwave frequencies irradiating a Josephson junction in the moving and gravitational operating modes of the balance (even though the exact relations of Josephson voltage and quantized Hall resistance to $h/(2e)$ and h/e^2, respectively, have been verified experimentally with high reproducibility and accuracy, their theoretical validity is still in question [38, 39]). This measurement equation resembles (4), but the measurement process does not actually involve a Compton frequency.

Finally, mass is a classical concept and, whereas a definition based on the mass of an atom is quite obvious, a definition relating it to the Planck constant will depart from classical physics. From a classical viewpoint, h and ν_{C} are not quantities of the same kind as mass; therefore, they are not obvious references for mass measurements. If the value of h is fixed, according to the measurement equation (4), mass will be a quantity derived from time. Consequently, any change in the second definition or in its realization will affect, conceptually, the kilogram realization as well.

The kilogram is basically a unit for mass measurements in a classical framework and at a macroscopic scale; its link with the fundamental constants is given by N_{A}. Though m_{u} (1/12 of the ^{12}C mass) is the reference for the atomic and molar mass scales, it may be a point of discussion as to whether the mass of the electron is a better basis for the kilogram definition [33, 40]. Both are sufficiently universal and invariable for the intended use, and the extent to which of these masses is more fundamental is a less important question. In principle, once the kilogram is defined on an atomic mass basis, the Planck constant and equation (4) will be the base for a new definition of the second. A fixed N_{A} value is advantageous for the definition of the mole, because the number of entities in 1 mol will be fixed as well.

4. New definitions of measurement units by fixing the numerical values of fundamental constants

If the definition of a unit fixes the numerical value of a fundamental constant of nature, or of a combination of several constants, this fixed value is exact until a significant breakthrough imposes a substantial revision of the way nature is conceived. Because physical laws establish relations between the fundamental constants, the way the measurement units are defined will also affect the uncertainties of constants that are not specifically referred to in the unit definitions; moreover, uncertainties may become smaller. However, before new definitions are enforced, careful verifications are necessary to check whether the system of units is consistent with the laws of physics. In the following, some options for new definitions of the kilogram will be discussed, see also [17, 20, 32–34, 38–51]. The appendix gives an overview of definitions together with the relative uncertainties of some fundamental constants which might ensue if enforced.

4.1. The kilogram

The measurement of the Avogadro constant, N_{A}, is the objective of an international project which relies on the use of a pure near-perfect single crystal of ^{28}Si [3]. The relation between N_{A} and the unit kilogram, as well as possible redefinitions and realizations of the kilogram, are presented in the following.

According to the definition of the mole, the molar mass of ^{12}C is $M(^{12}\mathrm{C}) = 12\,\mathrm{g\,mol^{-1}}$, exactly. For any substance S, the molar mass is

$$M(\mathrm{S}) = N_{\mathrm{A}}m(\mathrm{S}), \tag{6}$$

where

$$m(\mathrm{S}) = A_{\mathrm{r}}(\mathrm{S})m_{\mathrm{u}} \tag{7}$$

is the mass of the S atom in kg, $A_{\mathrm{r}}(\mathrm{S})$ is the relative atomic mass and $m_{\mathrm{u}} = m(^{12}\mathrm{C})/12$—the atomic mass constant—is the mass of a ^{12}C atom divided by 12. From (6) and (7) we may write

$$m_{\mathrm{u}}N_{\mathrm{A}} = 10^{-3}\,\mathrm{kg\,mol^{-1}} \tag{8}$$

or

$$1\,\mathrm{kg} = 10^{3}\{N_{\mathrm{A}}\}m_{\mathrm{u}}. \tag{9}$$

The Si crystal experiment aims at setting up directly the ratio between a macroscopic and an atomic volume [31,52] or, equivalently, between a macroscopic and an atomic mass. The relevant measurement equation is

$$N_{\mathrm{A}} = \frac{V(M_{\mathrm{Si}}/m)}{v_0/n_{\mathrm{Si}}} = \frac{M_{\mathrm{Si}}}{m_{\mathrm{Si}}}, \tag{10}$$

where M_{Si} is the mean molar mass of the three Si isotopes in the Si crystal, m_{Si} the mean mass of the three Si isotopes, v_0 the volume of the unit cell in the Si crystal, n_{Si} the number of Si atoms in this cell, m and V the mass and the volume of the crystal, yielding its density $\rho = m/V$ [21]. A correction for the binding energy of the Si atoms in the crystal (4.6 eV) or its equivalent mass is neglected in (10) because the relative change in N_{A} (1.8×10^{-10}) is smaller by about three orders of magnitude than its present relative uncertainty. M_{Si} is evaluated from the mean relative atomic mass of Si; this requires the measurement of three isotope abundances, i.e. three amount-of-isotope fractions. Because the sum of the abundances equals 1 exactly, the three abundances are obtained from the measurement of two ratios of amount-of-substance fractions [21]. The calibration of these ratio measurements requires ‘synthesizing’ a known amount ratio prepared via mass ratios [53].

The Si crystal experiment would allow a redefinition of the kilogram on the basis of a fixed value of N_{A}, whereby the atomic mass constant would be fixed as well. Accordingly, the kilogram would then be related to the mass of the ^{12}C atom as

$$1\,\mathrm{kg} = \frac{\{N_\mathrm{A}\}}{0.012} m(^{12}\mathrm{C}). \tag{11}$$

Once the kilogram is related to a fixed value of N_A and m_u, any experiment that derives a weighable mass from an atomic mass may realize the unit kilogram, for example, the ion beam experiment [4]. The relative atomic mass, A_r, which must be known for such experiments, can be measured today by using Penning traps with relative uncertainties down to 10^{-11} [28, 54].

A kilogram definition which bypasses the problem of the direct application of (4) to a macroscopic body is

$$1\,\mathrm{kg} = \frac{\{N_\mathrm{A}\}}{0.012} \nu_\mathrm{X} \frac{h}{c_0^2}. \tag{12}$$

Accordingly, 1 kg is the mass of $\{N_\mathrm{A}/0.012\}$ particles whose creation energy equals the energy of a photon having the frequency ν_X. With some abuse of terminology, this frequency is the Compton frequency of the quoted particles; strictly speaking, the Compton frequency concept applies only to field quanta, i.e. point-like and structureless particles. In (12), the particle number $\{N_\mathrm{A}/0.012\}$ is set in such a way that the mass of the particle, $m(\mathrm{X}) = h\nu_\mathrm{X}/c^2$, is, to within the present combined uncertainty, that of the ^{12}C atom. Instead of the ^{12}C atom, in (12) one could address the electron. In such a case,

$$1\,\mathrm{kg} = \frac{\{N_\mathrm{A}\}}{10^{-3} A_\mathrm{r}(\mathrm{e})} \nu_\mathrm{X} \frac{h}{c_0^2}. \tag{13}$$

It is worth noting that, while avoiding associating a Compton frequency directly with a 1 kg body as proposed in [17, 20], definitions (12) and (13) sum up the mass of particles instead of (somewhat problematically) adding photon frequencies. They link the kilogram to the mass of a particle (though unphysical) and it is this atom-scale mass that is given in terms of Compton frequency. Finally, such definitions seem more easily deliverable to a general audience than that proposed in [17, 20]. In (12) and (13) the particle-number choice is suggested by the need to identify the quoted $h\nu_\mathrm{X}/c^2$ mass, for practical purposes, with the mass of a real particle.

With reference to (12) and (13), if the second were to be defined by the frequency of a photon whose energy is equal to the m(X) creation energy, we could set $m(^{12}\mathrm{C}) = m(\mathrm{X})$ or $m_\mathrm{e} = m(\mathrm{X})$ exactly. However, since the second is not defined in such a way, m(X) can be slightly different from $m(^{12}\mathrm{C})$, the ^{12}C (or the electron) mass must be measured and an uncertainty must be associated with it.

If $m(^{12}\mathrm{C})$ is used in the realization of the kilogram, for example, in the silicon crystal or the ion accumulation work, an uncertainty must be attributed to this mass.

The definition (11) fixes the mass of the ^{12}C atom and, consequently, N_A—the number of atoms in 12 g of ^{12}C, but it does not require any correction to (6). In contrast, if in addition to (12) we also fix N_A, for instance in a new definition of the mole, the 'molar mass factor'

$$1 + \kappa = \frac{\nu_{^{12}\mathrm{C}}}{\nu_\mathrm{X}}, \tag{14}$$

where $\nu_{^{12}\mathrm{C}} = m(^{12}\mathrm{C})c^2/h$, must be introduced. It will appear in

$$M(\mathrm{S}) = (10^{-3}\,\mathrm{kg\,mol^{-1}})(1 + \kappa)A_\mathrm{r}(\mathrm{S}) \tag{15}$$

and in

$$(10^{-3}\,\mathrm{kg\,mol^{-1}})(1 + \kappa)\frac{1}{N_\mathrm{A}}. \tag{16}$$

At the time of the redefinition, the molar mass factor will be set exactly to one—but its uncertainty, being that of $\nu_{^{12}\mathrm{C}}$, will not be zero. Since

$$\nu_{^{12}\mathrm{C}} = \nu_\mathrm{Y} \frac{m(^{12}\mathrm{C})}{m(\mathrm{Y})} = \nu_\mathrm{Y} \frac{A_\mathrm{r}(^{12}\mathrm{C})}{A_\mathrm{r}(\mathrm{Y})}, \tag{17}$$

where Y is any particle or atom, and since no smaller uncertainty than that of

$$\nu_\mathrm{e} = \frac{2cR_\infty}{\alpha^2} \tag{18}$$

is available, the particle Y can be the electron. The relative uncertainty, 1.5×10^{-9}, of ν_e and $1 + \kappa$ is dominated by that of the square of the fine structure constant α^2 [27, 55].

The kilogram realization requires a link with the macroscopic scale; one is an object whose number of atoms is counted with the necessary accuracy, another, since (13) implies a fixed h value, is the watt balance experiment. As regards the first approach, the Avogadro project aims to count the atoms in a Si crystal-sphere. Deriving from (13), (14) and (16) that

$$m_\mathrm{u} = \frac{h\nu_\mathrm{e}}{c_0^2 A_\mathrm{r}(\mathrm{e})} \tag{19}$$

the relevant measurement equation is

$$m = \frac{n_\mathrm{Si} V}{v_0} \frac{A_\mathrm{r}(\mathrm{Si})}{A_\mathrm{r}(\mathrm{e})} \frac{h\nu_\mathrm{e}}{c_0^2}. \tag{20}$$

The uncertainty of this kilogram realization will combine both the counting and ν_e uncertainties. A realization via ion accumulation is similarly based on the mass of an atom and its uncertainty will similarly depend on that of ν_e.

Instead of relying on (18), the Compton frequency of the atomic mass constant, ν_u, can be directly measured by nuclear spectroscopy [56]. The measurement is based on the fact that, in a neutron capture reaction,

$$\mathrm{Y} + \mathrm{n} \to \mathrm{Z}^* \to \mathrm{Z} + \sum \gamma, \tag{21}$$

the daughter isotope Z is slightly lighter than the ensemble formed by the mother isotope and the neutron n, the capture-state Z*. This mass difference is equal to the binding energy, which can be measured by determining the frequencies of the γ-ray cascade in the decay scheme of the capture-state. Though the decay frequency $\nu_{\mathrm{Z}^*\to\mathrm{Z}}$ is different from ν_u, this last frequency can be obtained, similarly to (17), by scaling the former according to the difference between the relative atomic masses of the capture and ground states. Without pretension of completeness and rigour, the relevant measurement equation can be written as

$$\nu_\mathrm{u} = \frac{m_\mathrm{u} c^2}{h} = \frac{\nu_{\mathrm{Z}^*\to\mathrm{Z}}}{A_\mathrm{r}(\mathrm{Y}) + A_\mathrm{r}(\mathrm{n}) - A_\mathrm{r}(\mathrm{Z})}. \tag{22}$$

Equivalent, low energy, ν_u measurements are those of the m_u/h ratio, such as the measurements of recoil frequency [57] or of the de Broglie wavelength of a thermal neutron [49].

4.2. *Ampere or volt*

The elementary charge e and the Planck constant h are essential in the volt and ohm realizations via the Josephson voltage and the quantum Hall resistance. Since 1990, conventionally fixed values

$$K_{\mathrm{J-90}} \approx \frac{2e}{h} \tag{23}$$

and

$$R_{\mathrm{K-90}} \approx \frac{h}{e^2} \tag{24}$$

have been given to the Josephson and the von Klitzing constants [37]. The conventional values were chosen to be the closest possible approximations to the SI values of e/h and h/e^2, based on the experimental evidence then available. The experimental representations of these references have much smaller uncertainties than the realization of the ampere, but they are not expressed in SI units. If both h and e were fixed in new definitions, the Josephson voltage and the quantum Hall resistance should be exact realizations of the relevant SI units, but μ_0 will become a quantity to be measured. Because the electrical units are not independent of each other, only one of them can become a base unit. This might be the ampere realized, for example, by electron counting.

If the definition of the kilogram does not fix h, an alternative could be to substitute the volt definition (based on a fixed value of e/h and thus K_J) for the ampere definition. In this case, μ_0 becomes a quantity to be measured and R_K has a small uncertainty. This option (see also appendix A) would satisfy the needs of the majority of electrical measurements.

4.3. *The mole*

It is useful to reconsider the concept ’amount of substance’, as well as the definition of the unit mole. From a didactical point of view, students have great problems in understanding the real purpose of the mole [58]. In addition, it is often hard for scientists to understand the role of the unit mole among the other units of measurement. During the discussions in the Comité consultatif des unités (CCU) and the CIPM in 1969 and 1970, i.e. before the formal definition of the mole in 1971, several arguments against its definition as a base unit were put forward [59–61]. The main reasons for a definition were requests from the International Organization for Standardization/Technical Committee 12 (ISO/TC12) and other international organizations, but also a longstanding tradition in chemistry, as well as a lack of clarity in relating chemical measurement results to SI units. Remaining problems with the mole have been discussed recently, for example, in the Comité consultatif pour la quantité de matière: métrologie en chimie (CCQM) [62, 63].

A case occurring in chemical practice is that of weighing a specified substance and dividing its mass (expressed in gram) by its molar mass to obtain its amount-of-substance (expressed in mol). There is no mystery or conceptual difficulty with such measurements, performed in a measurement method called gravimetry. Only mass values in terms of the gram and molar mass in terms of gram per mole are necessary. Molar mass values for all elements have carefully evaluated uncertainties and provided a link between mass and amount-of-substance [64]. But, in general, chemists only need to know the number ratio of a number of molecules of a substance A and a number of molecules of a substance B. The ratio between two amounts-of-substance, and ultimately the amount-of-substance, can be measured by means of various measurement methods such as gravimetry, mass spectrometry, electrolytic measurements, spectrometric methods [21] or methods involving pressure and temperature measurements [65].

With $\{N_\mathrm{A}\}$ being a fixed number, chemists would know that a mole of a substance is a defined number $\{N_\mathrm{A}\}$ of specified entities. Note that N_A is put in braces, because, in the present mole definition, it is a quantity with dimension mol^{-1}. There have always been good reasons to use it as a number and it has been known for a long time as the ‘Avogadro number’. With the mole being the unit of the quantity amount-of-substance, chemists could simply keep the name mole for this number (just a defined number of entities such as atoms or ions or molecules). If the kilogram is defined as in section 4.1, one mol of $^{12}\mathrm{C}$ has a mass of 0.012 kg or of $(1+\kappa)\times 0.012$ kg. The amount-of-substance, n, is the number ratio between the number of entities in a sample and the number of entities in one mole $\{N_\mathrm{A}\}$. According to the terminology of [66], amount-of-substance is regarded as a dimensionless quantity, or a quantity of dimension one, with the unit one, 1. Even though [66] explains a dimensionless quantity within the frame of derived SI units, the amount-of-substance is in fact a real base quantity, as is used in chemistry, and the mole is a SI base unit.

Using N_A henceforth as the ‘Avogadro number’ (with the unit one), its relation to the atomic mass constant, m_u, would be written in the form

$$1\,\mathrm{kg} = 10^3 N_\mathrm{A} m_\mathrm{u}. \tag{25}$$

In addition to other advantages of the proposed new concept of the mole, the above equation makes the relation between the kilogram, the Avogadro number and the atomic mass constant clearer and simpler.

The molar mass M of a substance, usually given in terms of g/mol, is a quantity used for measuring the amount-of-substance by gravimetry only. But in mass spectrometry, electrolytic measurements, spectrometric measurement or other measurement methods, molar mass is not a reference quantity (see also [65]). In the case of the substance $^{12}\mathrm{C}$, if N_A has the unit one, we may write

$$m(1\,\mathrm{mol}\ ^{12}\mathrm{C}) = M(^{12}\mathrm{C}) = A_\mathrm{r}(^{12}\mathrm{C}) N_\mathrm{A} m_\mathrm{u} = 12 N_\mathrm{A} m_\mathrm{u}. \tag{26}$$

N_A can still be written as

$$N_\mathrm{A} = \frac{M(\mathrm{S})}{m(\mathrm{S})}, \tag{27}$$

where N_A is the scaling factor linking the mass of one mol of a substance S on the macroscopic level to the mass of its constituent atomic particle with mass $m(\mathrm{S})$, e.g. one molecule of substance S.

The Si Avogadro experiment, indeed, consists of measurements of number ratios between quantities [21]:

$$N_\mathrm{A} = \frac{M_\mathrm{Si}}{m}\,\frac{V}{(v_0/n_\mathrm{Si})} \tag{28}$$

with

$$M_{\mathrm{Si}} = \sum_{28}^{30} \frac{M({}^i\mathrm{Si})R_{i/28}}{\sum_{28}^{30} R_{i/28}} \tag{29}$$

and the measured ratios $R_{i/28} = f_i/f_{28}$, where f_i is the abundance of the ^{i}Si isotope (there are three: ^{28}Si, ^{29}Si and ^{30}Si) with $\sum f_i = 1$.

We would still continue to say: 'The amount (of substance) of ^{12}C is *n* mol', because one 'mol' is a defined number of specified entities and '*n*' is a multiple of this number.

The metrological traceability of amount-of-substance measurement results to the SI is a more philosophical question. If the result of such a measurement is the ratio of two quantities, these must have the same unit, whatever it may be. For example, if the measured mass of a substance is converted to obtain its amount-of-substance by means of its molar mass, traceability means that the measured mass value and the mass value of one mole must each be traceable to the same unit of mass, the kilogram. On the other hand, the ratio of counted ions in a mass spectrometer, originating from two substances, gives the number ratio between their amounts-of-substance whereby proof of traceability to any SI unit is unnecessary.

The new definition of the mole would be far clearer and more transparent in chemistry. It would also bring greater clarity to the whole system of SI units.

5. Proposals for the formulations of new definitions

As already mentioned, it is foreseen that the kilogram, the ampere, the kelvin and possibly the mole will be redefined at the same time. Electrical measurements are made by the use of the Josephson effect and the quantum Hall effect, which are based on conventionally fixed values of $2e/h$ and h/e^2, respectively. Therefore, fixing of both the Planck constant and electron's charge values in the kilogram and ampere definitions, as proposed in [20], has the advantage of fixing as well the Josephson voltage and Hall resistance values. In the following subsections, alternative redefinitions of the kilogram are discussed together with their implications for electrical metrology.

5.1. Kilogram and volt

(kg-N_A) 'The kilogram is $(6.022\,141\,5 \times 10^{23}/0.012)$ times the rest mass of the ^{12}C atom in the ground state.'

(kg-N_e) 'The kilogram is $1.097\,769\,24 \times 10^{30}$ times the rest mass of the electron.'

(V-K_J) 'The volt is equal to the difference between two electrical potentials within which the energy of a pair of electrons equals that of a photon whose frequency is $4.835\,978\,79 \times 10^{14}$ Hz.'

Definition (kg-N_A), see also [17], fixes the values of N_A and m_u and enables realizations, for instance, via the Avogadro and the ion accumulation experiments. A watt balance realization is also possible, but it requires the value of *h* as input data. This value can be calculated, for instance, from the molar Planck constant $N_A h$, which is known to within 1.5×10^{-9} relative uncertainty [27, 55]. An important feature of (kg-N_A) is that it refers to a single unbound atom. As a consequence of binding energy, which is about 7.4 eV, a 1 kg ^{12}C crystal with the same number of atoms as specified in (kg-N_A) is about 7×10^{-10} kg lighter than 1 kg.

Since

$$m_e = A_r(e)m_u = (10^{-3}\,\mathrm{kg\,mol^{-1}})A_r(e)\frac{1}{N_A}, \tag{30}$$

the definition (kg-N_e) fixes the $A_r(e)/N_A$ value; N_A and m_u will then have the same 4.4×10^{-10} relative uncertainty as $A_r(e)$ [27, 67]. Therefore, a realization based on the Si crystal experiment will comprise this negligibly small additional uncertainty.

Definition (V-K_J), a modified version of that proposed in [48], could replace the present ampere definition, giving a fixed value to

$$K_J = \frac{2e}{h}. \tag{31}$$

Consequently, the μ_0 value is no longer fixed, but must be measured. Definition (V-K_J) allows a volt realization based on the Josephson effect whereas the ohm realization based on the quantum Hall effect will have a relative uncertainty of 1.5×10^{-9} [27, 55]. It is independent of the N_A or *h* choice as the basis for the kilogram definition and it also improves the conversion of γ- and x-ray energies from eV units into Hz units.

5.2. Kilogram and ampere

(kg-N_C,h) 'The kilogram is $(6.022\,141\,5 \times 10^{23}/0.012)$ times the rest mass of a particle whose creation energy equals that of a photon whose frequency is $[(0.012/6.022\,141\,5 \times 10^{23}) \times 299\,792\,458^2/(6.626\,0693 \times 10^{-34})]$ Hz.'

(kg-N_e,h) 'The kilogram is $[6.022\,141\,5 \times 10^{23}/(5.485\,799\,0945 \times 10^{-7})]$ times the rest mass of a particle whose creation energy is that of a photon whose frequency is $[(5.485\,799\,0945 \times 10^{-7}/6.022\,141\,5 \times 10^{23}) \times 299\,792\,458^2/(6.626\,0693 \times 10^{-34})]$ Hz.'

(A-e) 'The ampere is the electric current in the direction of the flow of $6.241\,509\,48 \times 10^{18}$ elementary charges per second.'

Definitions (kg-N_C,h) and (kg-N_e,h) consist of two statements; their combination defines the kilogram. Basically, they fix the Planck constant value; their rationale, as already explained in section 4.1, is to make reference to a material (though unphysical) particle instead of to a photon collection. A new kilogram definition cannot fix *h* and the number of ^{12}C atoms in 1 kg, because the second would then be redefined. The present mole definition fixes the molar mass of the ^{12}C atom to exactly 12 g and N_A must be measured using the kilogram. If a new mole definition (see below) leaves the relation to the kilogram and fixes the numerical value of N_A, the same as in (kg-N_C,h) and (kg-N_e,h), then the ^{12}C molar mass is no longer exactly 12 g; however, it is sufficiently close to this value for all practical purposes. Both definitions then imply the introduction of a molar mass factor, whose present relative uncertainty is 1.5×10^{-9} [27, 55]. For all practical purposes, in (kg-N_C,h) the quoted particle is the ^{12}C atom, whereas in (kg-N_e,h) it is the electron. A similar definition is possible, which addresses a particle whose mass equals the atomic mass constant.

Definition (A-e), see also [20], is very clear and it fixes the elementary charge e by the inverse of the given number. Because the kilogram fixes the value of h the values of K_J and R_K have fixed values as well, and volt and ohm will become SI units based on fixed numerical values of $2e/h$ and h/e^2, respectively.

5.3. The mole

(mol-$\mathrm{N_A}$) 'The mole is the unit of amount-of-substance. It is equal to $6.022\,141\,5 \times 10^{23}$ specified identical entities. The entities may be atoms, ions, molecules or other particles.'

Compared with the present definition of the mole, (mol-$\mathrm{N_A}$) makes clear that the unit mol is another name for an Avogadro number of entities. Hence, the Avogadro number is re-symbolized as N_A instead of $\{N_\mathrm{A}\}$. (mol-$\mathrm{N_A}$) leaves off any relation to mass and the kilogram [21, 31]. As already mentioned, the chemist does not count numbers such as N_A but is interested in the ratios of numbers of different entities. This can be done by amount-of-substance spectrometry or by electrolytic measurements without requiring a value for the mass of one mole or for the exact value of N_A. In gravimetry, the relative atomic mass or the molar mass of the substance must be known. Continuity with the present mole definition requires that the kilogram is the mass of $N_\mathrm{A}/0.012$ ^{12}C atoms. Therefore, if the kilogram is redefined as the mass of $N_\mathrm{A}/0.012$ ^{12}C atoms, the number given in definition (mol-$\mathrm{N_A}$) must be exactly 0.012 times the ^{12}C number in the definition (kg-$\mathrm{N_A}$). Also, the definition (kg-$\mathrm{N_C}$,h) displays explicitly this continuity, provided the quoted frequency is as close as possible to the Compton frequency of ^{12}C and the number given in (mol-$\mathrm{N_A}$) is exactly 0.012 times the scale factor given in (kg-$\mathrm{N_A}$,h). Therefore, it is clear that the molar mass of ^{12}C is sufficiently close to 0.012 kg for all practical applications. Such information is not explicitly displayed in the Mills *et al* definitions [20].

6. *Mise en pratique* (realization of a unit)

The first *mise en pratique* was established for the realization of the metre, redefined in 1983. The particular formulation of the metre definition was chosen, because at a first glance it is readily understandable. The experimental realization with an uncertainty required in metrology could only be achieved by measuring the time light takes to travel over astronomical distances but not if measured under laboratory conditions. In practice, the relation $c = \lambda\nu$, where λ is the wavelength and ν the frequency of light, was used for measuring the wavelength of a laser beam of visible light by using its corresponding frequency up-scaled from the frequency of an atomic clock that realizes the second. This procedure is a possible realization of the metre definition. Instead of measuring the time light takes to travel one metre, the time it takes to travel one wavelength is measured, which is $1/\nu = \lambda/c$. By the use of interferometers the laser wavelength is up-scaled to end gauges and line gauges. The use of several laser wavelengths are the procedures described in the *mise en pratique* for the metre, for example, using the measurement of the wavelength of a specified radiation of the krypton lamp. The wavelength of the krypton lamp radiation is a realization of the metre provided its combined uncertainty contains the measurement uncertainty of a measurement according to the definition, for example a measurement of a laser wavelength as described above.

Mises en pratique are also foreseen for new definitions of the kilogram, the ampere, the kelvin and possibly the mole [18]. This means, whatever the formulation of the definitions, all relevant experimental techniques for realizing the unit will have to be described. A *mise en pratique* of any new definition of the kilogram is expected to take into account not only the watt balance experiment and the Avogadro experiment, but also the international prototype of the kilogram, all with quoted measurement uncertainties at the time of publication. As measurement uncertainties in the experiments improve, the *mise en pratique* will be revised as was the case with that of the metre in 1992, 1997 and 2001. The definition itself will remain unchanged.

As with the metre definition, at least one of the cited realization procedures must realize the kilogram according to the definition being based on accepted laws of physics and achieving an uncertainty that meets the requirements for practical measurement results. Other realizations must be linked to such a realization. If, for example, the kilogram is defined by a fixed value of N_A or h, the Avogadro experiment and the watt balance experiment are equivalent realizations of the definition of the kilogram, because the value for $N_\mathrm{A}h$ as derived from other fundamental constants is used for converting N_A into h or vice versa with a corresponding negligible additional uncertainty contribution. The international prototype of the kilogram is not a realization according to such a definition of the kilogram. It has therefore to be linked to one of the above-described realizations comprising their uncertainties.

When the kilogram has been redefined, the current work in NMIs on the experimental realization of the kilogram has to be continued [18] and possibly the *mise en pratique* to be revised when measurement results have improved. For practical applications and long-term maintenance, the present experimental designs and procedures should be simplified, especially in the case of the Si Avogadro work.

Considering that the *mise en pratique* will comprise all the relevant realizations, a particular definition of a SI unit may be chosen for other reasons than the smallest measurement uncertainty in the results of any particular experiment discussed previously, as long as adequate fitness for the intended use of the unit is achieved. Good reasons may be its clarity and the transparency of its practical realization [51]. Another reason could be the degree of invariance of the reference to which the unit is related.

7. Discussion

7.1. Scientific and metrological implications

Nature as viewed by modern science is characterized by

(1) statistical thermodynamics, which implies the equality of energy and temperature as expressed by the Maxwell–Boltzmann equation $E = k_\mathrm{B}T$,

(2) relativistic invariance, which implies the equality of energy and mass as summarized by the Einstein equation $E = mc_0^2$,
(3) quantum mechanics, which implies the equality of energy and frequency as summarized by the Planck equation $E = h\nu$.

Interactions are described by coupling constants which can be used to allow an independent unit to be set for the interaction charge. In the case of the electromagnetic one, this freedom, together with the charge quantization, can be exemplified by the equation $E = e\Phi$, which gives the energy of an electron in an electrical potential Φ.

Since energy is conserved—it is converted from one form to another, but the energy of a closed system does not change—in order to get a sufficient knowledge of the world, it is sufficient to perform energy measurements. In practice, however, we have no simple and universal instrument for measuring the energy. However, it appears in different forms, as described by the previous equations, related to different measurable quantities: frequency, mass, temperature, voltage and so on; fundamental constants such as c_0, h, k_B and e convert the values of these measurable quantities into energy units.

In the case of mass, weighing by comparing an unknown mass with a reference one is still the most accurate measurement method. However, the international prototype of the kilogram is no longer considered the best reference, nor is it a suitable one. Comparing mass values in Penning traps is the most accurate measurement method for atoms. Although a better choice might be the mass of an elementary particle such as the electron, the mass of the ^{12}C atom is sufficiently invariant for use as a metrological reference. Therefore, the problem of mass measurements is to find an invariant common reference for the mass measurements at both the atomic and macroscopic scales.

Time and frequency measurements are based on the energy difference between two hyperfine levels of the ^{133}Cs ground state; this reference is applicable to both the atomic and macroscopic measurements and can be reproduced with a very small uncertainty. Temperature measurements are based on the triple point of water T_{TPW}, which is not as fundamental and invariant as the Boltzmann constant. Electrical measurements use the Josephson voltage and the quantum Hall resistance as conventional references.

Going back to the question of what is the best reference to serve in the definition of a unit, we should consider that each quantity requires a measurement principle ensuring that it is compared with a reference of its same kind: mass measurements require a mass reference, time measurements a time reference, current (charge over time) measurements a charge reference, temperature measurements a temperature reference, and so on. For the SI units envisaged for redefinition, straightforward references are an atomic mass for the kilogram and the electron charge for the coulomb. In contrast to mass, time or charge, which are 'extensive' quantities, temperature is an 'intensive' quantity that cannot be evaluated by additive procedures from a reference. Temperature references for the kelvin are nevertheless required; these could be the dielectric constant gas thermometer (DCGT) or acoustic gas thermometer (AGT). Consequently, the constants fixed in their definitions should be the mass of an atom or of the electron for the kilogram, the elementary charge for the coulomb and k_B for the kelvin. The atomic mass unit would replace the prototype of the kilogram as a direct reference and experiments such as the Avogadro one would establish the ratio between mass values on the atomic and the macroscopic levels. For the ampere, the flow of a given number of elementary charges per second would replace its present definition, where a combination of Josephson and quantum Hall effects is a possible realization. Energy equations would be the basis for the kelvin [68]:

$$k_B T = p\frac{\alpha_0}{\varepsilon - \varepsilon_0}, \qquad (32)$$

where k is the Boltzmann constant, p the gas pressure, α_0 the static electric dipole polarizability and ε the dielectric constant of the ideal gas, in the case of the DCGT, or

$$k_B T = \frac{M u_0^2}{N_A \gamma_0}, \qquad (33)$$

where M is the molar mass of the gas, u_0 the zero frequency, zero pressure limit of the speed of sound and $\gamma_0 = c_p/c_V$ the zero pressure limit of the ratio between heat capacities at constant pressure, c_p, and at constant volume, c_V, in the case of the AGT.

If the kilogram is defined by fixing the Planck constant and by addressing a photon collection, we would not have any evident mass reference; speaking of mass in terms of frequency would be difficult to understand. Furthermore, even though the watt balance is a possible realization of this definition, it does not realize a direct link between macroscopic and microscopic masses. In fact, the measurement equation (5) relies on many quantities, none of them being a mass. The formulation of a new definition would be much more clear and understandable if it relates the kilogram to a reference mass [51] or, at least, if it recalls a well-known relation between quantities measurable in practice. A definition relating the kilogram to an atomic particle and also fixing h could be a compromise, although, in order to avoid fixing the second as well, that particle cannot be a real one. Practical considerations suggest a wording of this definition such that, for all practical purposes, the quoted particle is the ^{12}C atom or the electron. This definition can be realized by experiments that perform a measurement of h (such as the watt balance) or that count the number of atoms in a macroscopic object (such as the Avogadro and ion accumulation experiments). In these second realizations, the mass ratio between the counted atom and the particle quoted in the kilogram definition must also be measured.

7.2. *Implications for society*

Measurement units have been established for practical use in technology, trade and science. In all these applications measurement standards of the same kind are required with which the unknown quantities are compared. These standards are by themselves metrologically traceable to a realization of the definition of the unit. A careful user of a measurement standard or of a measuring instrument would want to know the traceability chain of his results, the laboratory that realizes the unit involved and, ultimately, the definition of that unit.

Moreover, knowledge of the definitions of those units which are important in everyday life, such as the kilogram, metre and second, is part of the general education of students and they are expected to be known by everyone to a considerable extent [63]. The definition of the metre is a good example of a formulation which is both correct and simple, even though the understanding of its practical realization requires a much higher education than that supplied by primary and secondary schools. Nobody has any problem in the understanding of the present definition of the kilogram. A future definition will require some additional knowledge of modern science. The kilogram as the mass of a number of atomic or elementary particles will certainly be understandable, but not as the mass that is given just by the numerical value of the Planck constant. A definition based on an atomic particle whose mass is, in turn, fixed in terms of its Compton frequency could be understood, but it requires some basic knowledge of modern physics, namely that elementary particles can be created provided conservation laws (energy, momentum, charge, etc) are satisfied and a sufficient amount of energy is available, for instance when a photon, with an energy of at least $2 \times mc_0^2$, interacts with an atomic nucleus, allowing the production of two particles of mass m, e.g. an electron and a positron.

7.3. The best time for new definitions

It is probable that the international prototype of the kilogram is unstable in time. But so far no evidence for such a change or for the value of a change has been noticed in practical measurements [69], e.g. through complaints about offsets in recalibrations of standards or through the observation of systematic drifts in replicate measurements of fundamental constants. From this point of view, the redefinition of the kilogram is not really urgent. Measurements of electrical quantities are not subject to drifts because they are linked to ratios of fundamental constants, such as the conventional values of $K_{\mathrm{J-90}}$ and $R_{\mathrm{K-90}}$. Also, the desire to redefine the ampere or other electrical units to integrate the Josephson volt or the quantum Hall resistance into the SI is justified but not urgent. The deviations of the conventional values of these constants from their SI values since 1990 are almost certainly based on measurement errors and are probably not subject to systematic drifts in time.

The time for redefining the SI units will come when the experimental results of the realizations related to redefinitions are 'indeed acceptable' [18], in the case of the kilogram, when the Avogadro and watt balance projects deliver compatible results with relative measurement uncertainties of 2×10^{-8} or less.

8. Conclusion

In any future redefinition of the SI unit kilogram, there are good reasons for fixing a number as the ratio between the kilogram and the mass of an atomic particle, such as the mass of a carbon-12 atom or of the electron. Compared with other proposals such as fixing a numerical value of the Planck constant or of the frequency being equivalent to a kilogram, a link to a mass on the atomic level is more obvious and intrinsic to the unit of mass. Moreover, such an atomic definition would establish a fixed link between atomic masses and SI units, which would enable one to introduce a fixed value for the Avogadro constant in a new mole definition and would be understandable, at least in principle, by the general public. Other options for the definition of the kilogram, such as fixing the Planck constant, trace it back to an atomic mass. These can preserve the concept of mass ratios and have clear advantages for electrical units, but involve a higher level of understanding of basic physics to appreciate the nature of the elementary mass particle. It is proposed that the Avogadro constant be converted to a number, the 'Avogadro number', and that the mole be linked to this number. The unit of the amount-of-substance would be this particular number of specified, identical entities. This would not only bring greater clarity and simplicity to the SI, but would also lead to a better understanding of the mole by the physics and chemistry communities, as well as by the general public.

Acknowledgments

The authors wish to express their appreciation to E O Goebel for his kind support and helpful discussions, to R S Davis, W Woeger, B Siebert and H Bettin for useful suggestions for improving the manuscript and to M Tanaka for encouraging them to publish their ideas for redefinitions.

Appendix 1. Options for new definitions and their impact on fundamental constants

Table A1 shows the relative uncertainties of some fundamental constants as a result of values fixed in new definitions, based on the CODATA 2002 values [27] except the value of the fine structure constant α, which is taken from [55]. The sixth column shows the data according to the proposal of [20] together with the new proposal of this paper. Table A2 shows the underlying formulae for the calculation of these uncertainties.

Table A1. Relative uncertainties given in 10^{-8} of some constants for different options of fixed values. The corresponding CODATA 2002 [27] uncertainties are also given for comparison. N_{A} is the Avogadro constant, h the Planck constant, e the elementary charge, m_{u} the atomic mass constant ($=m(^{12}\mathrm{C})/12$), m_{e} the electron mass, m_{p} the proton mass, F the Faraday constant, R_{K} the von Klitzing constant, K_{J} the Josephson constant, μ_{B} the Bohr magneton, γ_{p} the proton gyromagnetic ratio, $\{m_{\mathrm{u}}c_0^2/e\}$ the conversion factor between particle mass in unit of u and energy in unit of eV, $1+\kappa$ the molar mass factor (see [20] and if $\nu_{\mathrm{Y}} = \nu_{\mathrm{e}}$, see section 4.1) and μ_0 the magnetic constant (permeability of vacuum).

Constant	CODATA 2002	N_{A} and e fixed	N_{A} and h/e fixed	h and e fixed	h, N_{A} and e fixed
N_{A}	17	0	0	0.15	0
h/e	8.5	0.15	0	0	0
h	17	0.15	0.15	0	0
$N_{\mathrm{A}}h$	0.67	0.15	0.15	0.15	0
e	8.5	0	0.15	0	0
m_{u}	17	0	0	0.15	0.15
m_{e}	18	0.044	0.044	0.15	0.14
m_{p}	17	0.013	0.013	0.15	0.15
F	8.6	0	0.15	0.15	0
R_{K}	0.33	0.15	0.15	0	0
K_{J}	8.5	0.15	0	0	0
μ_{B}	8.6	0.15	0.21	0.14	0.14
γ_{p}	8.6	1.0	1.0	1.0	1.0
$\{m_{\mathrm{u}}c_0^2/e\}$	8.6	0	0.15	0.15	0.15
$1+\kappa$	—	—	—	—	0.15
μ_0	0	0.16	0.08	0.07	0.07
Defining		kg, ampere, mole	kg, volt, mole	kg, ampere	kg, ampere, mole

Table A2. Formulae on which the uncertainties of table A1 are based, data in brackets from CODATA 2002 [27] and [55] and the others calculated from data of the same column. $M_u = 10^{-3}\,\mathrm{kg\,mol^{-1}}$.

Constant	CODATA 2002	N_A and e fixed	N_A and h/e fixed	h and e fixed	h, N_A and e fixed
N_A		N_A	N_A	$\frac{c[A_r(e)]M_u[\alpha]^2}{2[R_\infty]h}$	N_A
h/e		$\frac{2}{K_J}$	$\frac{h}{e}$	$\frac{h}{e}$	$\frac{2}{K_J}$
h		$\frac{c[A_r(e)]M_u[\alpha]^2}{2[R_\infty]N_A}$	$\frac{c[A_r(e)]M_u[\alpha]^2}{2[R_\infty]N_A}$	h	h
N_Ah	$\frac{cA_r(e)M_u\alpha^2}{2R_\infty}$	N_Ah	N_Ah	N_Ah	N_Ah
e		e	$\frac{h}{(h/e)}$	e	e
m_u		$\frac{M_u}{N_A}$	$\frac{M_u}{N_A}$	$\frac{M_u}{N_A}$	$\frac{(1+\kappa)M_u}{N_A}$
m_e		$[A_r(e)]m_u$	$[A_r(e)]m_u$	$[A_r(e)]m_u$	$\frac{2h[R_\infty]}{c[\alpha]^2}$
m_p		$[A_r(p)]m_u$	$[A_r(p)]m_u$	$[A_r(p)]m_u$	$\left[\frac{m_p}{m_e}\right]m_e$
F		eN_A	eN_A	eN_A	eN_A
R_K		$\frac{h}{e^2}$	$\frac{(h/e)^2}{h}$	$\frac{h}{e^2}$	$\frac{h}{e^2}$
K_J		$\frac{2e}{h}$	$\frac{2}{(h/e)}$	$\frac{2e}{h}$	$\frac{2e}{h}$
μ_B		$\frac{eh}{4\pi m_e}$	$\frac{eh}{4\pi m_e}$	$\frac{e[\alpha]^2[c]}{8\pi[R_\infty]}$	$\frac{eh}{4\pi m_e}$
γ_p		$\frac{e}{m_e}\left[\frac{\mu_p}{\mu_B}\right]$	$\frac{e}{m_e}\left[\frac{\mu_p}{\mu_B}\right]$	$\frac{e}{m_e}\left[\frac{\mu_p}{\mu_B}\right]$	$\frac{e}{m_e}\left[\frac{\mu_p}{\mu_B}\right]$
$m_uc_0^2/e$	$\left(\frac{m_e}{e}\right)\frac{c^2}{A_r(e)}$	$\frac{m_u[c]^2}{e}$	$\frac{m_u[c]^2}{e}$	$\frac{m_u[c]^2}{e}$	$\frac{m_u[c]^2}{e}$
$1+\kappa$		—	—	—	$\frac{2[R_\infty]N_Ah}{[c][\alpha]^2[A_r(e)]M_u}$
μ_0	$4\pi 10^{-7}$	$\frac{2h[\alpha]}{[c]e^2}$	$\frac{16N_A[R_\infty]}{K_J^2}[A_r(e)][c]^2M_u[\alpha]$	$\frac{2h[\alpha]}{[c]e^2}$	$\frac{2h[\alpha]}{[c]e^2}$
Defining		kg, ampere, mole	kg, volt, mole	kg, ampere	kg, ampere, mole

References

[1] BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML and ILAC 1993 *The International Vocabulary for Basic and General Terms in Metrology* (Geneva: ISO) (revised 2006)

[2] Eichenberger A, Jeckelmann B and Richard P 2003 Tracing Planck's constant to the kilogram by electromechanical methods *Metrologia* **40** 356–65

[3] Becker P 2003 Tracing the definition of the kilogram to the Avogadro constant using a silicon single crystal *Metrologia* **40** 366–75

[4] Glaeser M 2003 Tracing the atomic mass unit to the kilogram by ion accumulation *Metrologia* **40** 376–86

[5] Girard G 1994 The third periodic verification of national prototypes of the kilogram (1988–1992) *Metrologia* **31** 317–36

[6] Davis R 2003 The SI unit of mass *Metrologia* **40** 299–305

[7] Davis R S 1989 The stability of the SI unit of mass as determined from electrical measurements *Metrologia* **26** 75–6

[8] Steiner R L, Williams E R and Newell D B 2005 Towards an electronic kilogram: an improved measurement of the Planck constant and electron mass *Metrologia* **42** 431–41

[9] Fujii K, Waseda A, Kuramoto N, Mizushima S, Tanaka M, Valkiers S, Taylor P, Kessel R and de Bièvre P 2003 Evaluation of the molar volume of silicon crystals for a determination of the Avogadro constant *IEEE Trans. Instrum. Meas.* **52** 646–51

[10] Becker P, Bettin H, Danzebrink H-U, Gläser M, Kuetgens U, Nicolaus A, Schiel D, de Bièvre P, Valkiers S and Taylor P 2003 Determination of the Avogadro constant via the silicon route *Metrologia* **40** 271–87

[11] Fujii K *et al* 2005 Present state of the Avogadro constant determination from silicon crystals with natural isotopic compositions *IEEE Trans. Instrum. Meas.* **54** 854–9

[12] Kibble B P, Robinson I A and Belliss J H 1990 A realization of the SI Watt by the NPL moving-coil balance *Metrologia* **27** 173–92

[13] Clothier W K, Sloggett G J, Bairnsfather H, Currey M F and Benjamin D J 1989 A determination of the volt *Metrologia* **26** 9–46

[14] Funck T and Sienknecht V 1991 Determination of the volt with the improved PTB voltage balance *IEEE Trans. Instrum. Meas.* **40** 158–61

[15] Shiota F, Miki Y, Fujii Y, Morokuma T and Nezu Y 2000 Evaluation of equilibrium trajectory of superconducting magnetic levitation system for the future kg unit of mass *IEEE Trans. Instrum. Meas.* **49** 1117–21

[16] Riski K, Heikkinen P, Kajastie H, Manninen J, Rossi H, Nummila K, Frantsuz E and Khavinson V 2001 Design of a superconducting magnetic levitation system *Proc. 17th Int. Conf. IMEKO TC 3, Istanbul, 17–21 September 2001* pp 239–46

[17] Mills I M, Mohr P J, Quinn T J, Taylor B N and Williams E R 2005 Redefinition of the kilogram: a decision whose time has come *Metrologia* **42** 71–80

[18] CIPM 2005 *94th meeting (2005)* Recommendation 1 (CI-2005) (Sèvres: BIPM) http://www.bipm.org/en/committees/cipm/publications_cc.html

[19] CCM 2005 *Rapport de la* 7^{e} *session* (Sèvres: BIPM) http://www.bipm.org/en/committees/cc/ccm/publications_cc.html

[20] Mills I M, Mohr P J, Quinn T J, Taylor B N and Williams E R 2006 Redefinition of the kilogram, ampere, kelvin and mole: proposed approach to implementing CIPM recommendation 1 (CI-2005) *Metrologia* **43** 227–46

[21] De Bièvre P 2002 The Avogadro constant: the unit and the scaling factor *Atti del convegno celebrativo, 'Amedeo Avogadro di Quaregna' (Oropa (Italia) 09–11 June 2002)* Università degli Studi del Piemonte Orientale 'Amedeo Avogadro', and Istituto Nazionale di Ricerca Metrologica, Torino

[22] De Bièvre P, Valkiers S and Taylor P D P 1998 The importance of the Avogadro constant for amount-of-substance measurements *Fresenius J. Anal. Chem.* **361** 227–34

[23] Tazzari S and Ferrario M 2003 Trends in high energy particle accelerators *Rep. Prog. Phys.* **66** 1045–94

[24] Okun L 1989 The concept of mass *Phys. Today* **42** 31–6

[25] Wilczek F 2006 The origin of mass *Mod. Phys. Lett.* A **21** 701–12
Wilczek F 2004 Whence the force of $F = ma$? I: culture shock *Phys. Today* (October) 10
Wilczek F 2004 Whence the force of $F = ma$? II: Rationalizations *Phys. Today* (December) 10
Wilczek F 2005 Whence the force of $F = ma$? III: Cultural diversity *Phys. Today* (July) 11
Wilczek F 2005 Nobel lecture: asymptotic freedom: from paradox to paradigm *Rev. Mod. Phys.* **77** 857–70

[26] Bernardeau F 2003 Dark matter, dark energy, gravitational lensing and the formation of structure in the universe *Rep. Prog. Phys.* **66** 691–736

[27] Mohr P J and Taylor B N 2005 CODATA recommended values of the fundamental physical constants: 2002 *Rev. Mod. Phys.* **77** 1–107

[28] Audi G, Wapstra A H and Thibault C: 2003 The AME2003 atomic mass evaluation *Nucl. Phys.* A **729** 337–676 and references therein (^{16}O: van Dyke 2001Va33, 2003Va.A)

[29] Coplen T B 1999 Atomic weights of the elements *J. Phys. Chem. Ref. Data* **30** 701–12

[30] Lévy-Leblond J M 1979 The importance of being (a) constant *Proc. Int. School of Physics E. Fermi: Problems in the Foundations of Physics, vol 19, Course LXXII (Varenna, Italy 25 July–6 August 1977)* ed G Toraldo di Francia (Amsterdam: North-Holland)

[31] De Bièvre P and Peiser S 1994 Calling for research of measurement, a process exemplified by redeterminations of the Avogadro constant *Proc. 1994 NCSL Workshop & Symp.* (Chicago, USA) pp 565–78 (Session 7A)

[32] Bordé C J 2005 Métrologie fondamentale: unitées de base et constantes fondamentales *C.R. Physique* **5** 813–20

[33] Bordé C 2005 Base units of the SI, fundamental constants and modern quantum physics *Phil. Trans. R. Soc.* A **363** 2177–201

[34] Wignall J W G 2005 An absolute replacement for the standard kilogram *Meas. Sci. Technol.* **16** 682–4

[35] Rainville S *et al* 2005 A direct test of $E = mc^2$, *Nature* **438** 1096–97

[36] Millikan R A 1916 A direct photoelectric determination of Planck's h *Phys. Rev.* **7** 355–88

[37] CIPM 1988 Recommendation 1 and Recommendation 2 *Metrologia* **26** 69–70

[38] Jeckelmann B and Jeanneret B 2001 The quantum Hall effect as an electrical resistance standard *Rep. Prog. Phys.* **64** 1603–55

[39] Piquemal F and Genevès G 2000 Argument for a direct realization of the quantum metrological triangle *Metrologia* **37** 207–11

[40] Schwitz W, Jeckelmann B and Richard P 2004 Towards a new kilogram definition based on a fundamental constant *C. R. Physique* **5** 881–92

[41] Petley B W 1996 The mole and the unified atomic mass unit *Metrologia* **33** 261–4

[42] Taylor B N and Mohr P J 1999 On the redefinition of the kilogram *Metrologia* **36** 63–4

[43] Tuminsky V S 1999 Unit systems based on fundamental constants *Metrologia* **36** 9–14

[44] Kose V, Siebert B R L and Woeger W 2003 General principles for the definition of the base units in the SI *Metrologia* **40** 146–53

[45] Moldover M R and Ripple D C 2003 Comment on 'General principles for the definition of the base units in the SI' *Metrologia* **40** L9–10

[46] Flowers J L and Petley B W 2005 The kilogram redefinition—an interim solution *Metrologia* **42** L31–4

[47] Leonard B P 2006 Note on invariant redefinitions of SI base units for both mass and amount of substance *Metrologia* **43** L3–5

[48] Taylor B N 1991 The possible role of the fundamental constants in replacing the kilogram *IEEE Trans. Instrum. Meas.* **40** 86–91

[49] Krüger E, Nistler W and Weirauch W 1999 Re-evaluation of a precise measurement of h/m_{n} *Metrologia* **36** 147–8

[50] Cabiati F, Mana G and Peuto A 1993 Le costanti fondamentali e la Metrologia della massa *Nuovo Saggiatore* **9** 51–61

[51] Valdès J 2005 Features and future of the International System of Units *Advances in Imaging and Electron Physics* vol 138, ed P Hawkes (New York: Academic) pp 251–320

[52] De Bièvre P *et al* 1997 The molar volume of silicon *IEEE Trans. Instrum. Meas.* **46** 592–5

[53] De Bièvre P, Lenaers G, Murphy T J, Peiser H S and Valkiers S 1995 The chemical preparation and characterization of specimens for 'absolute' measurements of the molar mass of an element, exemplified by silicon, for redeterminations of the Avogadro constant *Metrologia* **32** 103–10

[54] Rainville S, Thompson J K and Pritchard D E 2004 An ion balance for ultra-high-precision atomic mass measurements *Science* **303** 334–8

[55] Gabrielse G, Hanneke D, Kinoshita T, Nio M and Odom B 2006 New determination of the fine structure constant from the electron g-value and QED *Phys. Rev. Lett.* **97** 030802 1–4

[56] Becker P, Jentschel M, Mana G and Zosi G 2006 Towards an atomic realization of the kilogram: the measurement of N_A and $N_A h$ *Proc. Int. School of Physics E Fermi: Metrology and Fundamental Constants, Course CLXVI (Varenna, Italy 18–28 July 2006)* ed S Leschiutta, T Hänsch and A Wallard (Amsterdam: North-Holland) submitted

[57] Wicht A, Hensley J M, Sarajlic E and Chu S 2002 A preliminary measurement of the fine structure constant, based on atom interferometry *Phys. Scr.* T **102** 82–8

[58] Lybeck L, Marton F, Strömdahl H and Tullberg A 2003 Phänomenologische Untersuchungen zum ‘Molbegriff’ *Chim. Didactica* **29** 61–96

[59] Comité Consultatif des unités 1970 *2^e session 1969* (Sèvres: BIPM)

[60] Comité International des Poids et Mesures 1971 *Procès-Verbaux des séances, 2^e série—tome 37, 58^e session—1969* (Sèvres: BIPM)

[61] Comité International des Poids et Mesures 1971 *Procès-Verbaux des séances, 2^e série—tome 38, 59^e session—1970* (Sèvres: BIPM)

[62] Milton M J T and Quinn T J 2005 Primary methods for the measurement of amount of substance *Metrologia* **38** 289–96

[63] CCQM 2005 *Report on the 11th meeting (2005)* Recommendation Q1 2005 On the possible redefinition of the kilogram (Sèvres: BIPM) http://www.bipm.org/en/committees/cc/ccqm/publications_cc.html

[64] De Bièvre P and Peiser H S 1997 The reliability of values of molar mass, the factor that relates measurements expressed in two base units (mass and amount-of-substance) *Metrologia* **34** 49–59

[65] McGlashan M L 1994/1995 Amount of substance and the mole *Metrologia* **31** 447–55

[66] BIPM 2006 *The International System of Units* 8th edn (Sèvres: BIPM) http://www.bipm.org/en/si/si_brochure/general.html

[67] Farnham D L, van Dyke R S Jr and Schwinberg P B 1995 Determination of the electron’s atomic mass and the proton/electron mass ratio via Penning trap mass spectroscopy *Phys. Rev. Lett.* **75** 3598–601

[68] Fischer J and Fellmuth B 2005 Temperature metrology *Rep. Prog. Phys.* **68** 1043–94

[69] Davis R S 2005 Possible new definitions of the kilogram *Phil. Trans. R. Soc.* A **363** 2249–64