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2006 Metrologia 43 L21

(http://iopscience.iop.org/0026-1394/43/5/L01)

INSTITUTE OF PHYSICS PUBLISHING METROLOGIA

Metrologia **43** (2006) L21–L22 doi:10.1088/0026-1394/43/5/L01

LETTER TO THE EDITOR

Exact calculation of the coverage interval for the convolution of two Student's *t* distributions

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Received 5 July 2006
Published 6 September 2006
Online at stacks.iop.org/Met/43/L21

Abstract
Fotowicz (2006 *Metrologia* **43** 42–5) presented an approximation for computing the coverage interval of the convolution of two Student's t distributions. This approximation appeared to perform well for a limited range of values of the degrees of freedom. In this letter, we provide an eight-line program for *exact* computation of the coverage interval with no limitations. Several advantages of our approach are noted.

1. Introduction

Suppose Y is a symmetric random variable specified by the probability density function $g(\cdot)$ and the cumulative distribution function $G(\cdot)$. Its coverage interval corresponding to coverage probability $p = 1 - \alpha$ is defined by $[y_{\mathrm{low}}, y_{\mathrm{high}}]$, where $y_{\mathrm{low}} = G^{-1}(\alpha/2)$ and $y_{\mathrm{high}} = G^{-1}(1-\alpha/2)$. The recent paper by Fotowicz (2006) presented novel approximations for computing the coverage interval of 'the convolution of standard distributions attributed to input quantity values, such as Student's t, normal, rectangular, ...' These approximations appeared to perform well. However, when the convolution of two Student's t random variables was considered the proposed approximations appeared to perform poorly for small degrees of freedom (see figure 3 in Fotowicz (2006)). In fact, figure 3 in Fotowicz (2006) only considered cases with degrees of freedom greater than or equal to 3.

In this letter, we provide a simple computer program for calculating the coverage interval for the convolution of *any* two Student's t random variables. The program is written in R (Ihaka and Gentleman 1996) because, unlike other software, it is easy to implement and freely downloadable from the Internet (http://www.r-project.org).

2. Program

Suppose X_1 and X_2 are independent Student's t random variables specified by the probability density functions:

$$g_1(\xi)\frac{1}{\sqrt{\nu_1}B(\nu_1/2, 1/2)}\left(1+\frac{\xi^2}{\nu_1}\right)^{-(1+\nu_1)/2}$$

and

$$g_2(\xi)\frac{1}{\sqrt{\nu_2}B(\nu_2/2, 1/2)}\left(1+\frac{\xi^2}{\nu_2}\right)^{-(1+\nu_2)/2},$$

respectively, for $-\infty < \xi < \infty$. Let Y denote the convolution $Y = X_1 + X_2$.

The following R program computes the *exact* coverage interval, $[y_{\mathrm{low}}, y_{\mathrm{high}}]$, for given input values of α, ν_1 and ν_2.

```
f<-function (y)
{ff<-function (x) {pt(y-x,df=nu1)*dt(x,df=nu2)}
return(integrate(ff,lower=-Inf,upper=Inf)
    $value-alpha/2)}
ylow<-uniroot(f,lower=-100,upper=100)$root
f<-function (y)
{ff<-function (x) {pt(y-x,df=nu1)*dt(x,df=nu2)}
return(integrate(ff,lower=-Inf,upper=Inf)
    $value-(1-alpha)/2)}
yhigh<-uniroot(f,lower=-100,upper=100)$root.
```

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Note that there are no restrictions on the input values except that $\nu_1 > 0$, $\nu_2 > 0$ and $0 < \alpha < 1$.

3. Conclusions

We have provided a simple program in R (freely available statistical software) for computing the coverage interval of the convolution of *any* two Student's t distributions. This program has several advantages over that suggested by Fotowicz (2006) because: (1) no restrictions are imposed on the input parameters ν_1, ν_2 and α; (2) the program computes the *exact* coverage interval and avoids the need for any approximation; (3) the program is simple; (4) the program is easy to implement and (5) the software is freely available on any platform. The electronic version of the computer program can be obtained by contacting the author.

References

Fotowicz P 2006 An analytical method for calculating a coverage interval *Metrologia* **43** 42–5

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