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Correction of astigmatism in a Czerny-Turner spectrograph using a plane grating in divergent illumination

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Abstract A method for eliminating the astigmatism of a Czerny-Turner spectrograph is described in which the grating is used in divergent illumination. Expressions are given for the distances of the tangential and sagittal astigmatic images from the camera mirror in terms of the separation between the entrance slit and the collimating mirror and the condition for the elimination of astigmatism is derived. These results are compared with ray tracing data for a practical instrument. Experimental tests of spatial and spectral resolution for an f/15 spectrograph of 0·6 m focal length are given at wavelengths 5461 and 2800 Å. Over a spectral range of 100 Å centred on 2800 Å a spatial resolution along the slit in excess of 50 cycles mm^{-1} has been achieved for a flat inclined image plane. For the same spectrograph with the grating in collimated light the spatial resolution is only 3 cycles mm^{-1}.

1 Introduction

A proposed balloon-borne spectroheliograph to operate at 2800 Å, the wavelength of the Mg II resonance lines, requires an instrument of 0·2 Å spectral resolution capable of resolving 20 cycles mm^{-1} along the entrance slit. The instrument is to be used at f/15 with a 0·9 m focal length telescope, and the spatial resolution requirement corresponds to the accuracy of the stabilization of the solar image on the entrance slit, which is expected to be approximately ± 5 seconds of arc r.m.s. Payload size restrictions impose a limit of 0·6 m on the focal length of the spectrograph mirrors.

A disadvantage of the Czerny-Turner spectrograph arises from the fact that it is not completely stigmatic and as a consequence it is not useful for those studies which demand a good spatial resolution. Whilst the astigmatism is small for an instrument of small aperture, for an f/15 spectrograph the spatial resolution along the image of the slit is only of the order of a few cycles per mm. Several methods have been advanced for the reduction or elimination of this defect which include the use of: convex compensating mirrors (Rosendahl 1962); toroidal collimating and camera mirrors (Shafer 1967); distortion of the plane grating (Dalton 1966) and the use of correcting lenses (Foreman 1968).

When illuminated in collimated light there is no astigmatism introduced by the grating and the contributions from the two mirrors are additive. For a plane grating in noncollimated light however, there is a large astigmatism term in the aberration expansion which vanishes when the angles of incidence and diffraction are equal and large coma terms which do not vanish except at the zero order position (Welford 1965). Theories of the plane grating in convergent or in divergent light have been given by several authors and discussed by Murty (1962) who suggested possible methods of eliminating the coma at one wavelength.

Instruments employing plane gratings in noncollimated light using a single focusing element have been described by Monk (1928), Gillieson (1949), Richards, Thomas and Weinstein (1957), Dalton (1962) and Schroeder (1966). Such a spectrograph is stigmatic but suffers from considerable coma and if the tolerance condition for coma is not to be exceeded then the speed of the instrument is necessarily extremely small. This disadvantage is discussed in detail by Richards *et al.* (1957) and by Murty (1962) who shows that the limitation on ruled grating width W is approximately $W/2\,(\text{f number})^2 \leqslant \sigma$, where σ is the grating spacing. Seya, Namioka and Sai (1967) have treated the Monk–Gillieson mounting by taking into consideration the effect of imperfectly convergent light containing the aberrations due to a spherical mirror. Analytical expressions for the mirror–grating system give a remedy for eliminating the coma at a chosen wavelength.

Because of the necessity for a high spatial resolution we have examined the performance of a Czerny-Turner spectrograph in which the grating is used in noncollimated illumination. It is shown that at a given wavelength the astigmatism can be eliminated if the separation between entrance slit and collimating mirror is correctly chosen. Although the grating is in noncollimated illumination and is no longer a coma-free element the divergence angle of the light falling on to the grating is small. Because of this small angle a larger grating width and thus a greater spectrograph speed can be achieved than that possible with a single focusing element and plane grating.

2 Theory

The design of the optical system under consideration (figure 1) has been greatly influenced by the requirement for an instrument of compact size suitable for assembly on to a balloon-borne, biaxial pointing control. For compactness the angle of incidence θ of the central ray on the collimating mirror M_1 and the camera mirror M_2 are made equal and as small as possible (approximately 4°). The radius of curvature of M_1 ($R_1 = 1199$ mm) and M_2 ($R_2 = 1118$ mm) have also been chosen from practical considerations. The instrument uses a 58 mm square Bausch and Lomb replica grating with 2160 grooves per mm blazed at 3000 Å. In order to produce an approximately flat image plane the grating is placed at the noncritical distance of $0{\cdot}84\,f$ from the mirrors, where f is the mean focal length (Mielenz 1964).

The positions of the tangential S_t and sagittal S_s astigmatic

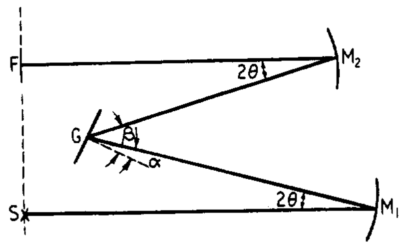


Figure 1 Schematic plan of the optical system under consideration. M_1 and M_2, the collimating mirror and camera mirror respectively; G, the grating; S, the entrance slit; F the film plane

images referred to the different optical components are obtained using the light path expansion given by Beutler (1945). Expressions are then derived which give the distances of the final astigmatic images from the camera mirror in terms of the separation S between the entrance slit and the collimating mirror. Equation (3) gives the required value of S for the elimination of astigmatism at a chosen wavelength. In these formulae the distances between the grating and the two mirrors have been neglected for simplicity. This omission is discussed below.

2.1 *Astigmatism*

The distances of the sagittal S_s and tangential S_t images from the camera mirror are given by

$$S_s = \frac{R_1 R_2 S}{2S(R_1+R_2)\cos\theta - R_1 R_2} \quad (1)$$

which is independent of wavelength and

$$S_t = R_1 R_2 S \bigg/ \left\{ 2S \sec\theta \left(R_1 + R_2 \frac{\cos^2\alpha}{\cos^2\beta} \right) - R_1 R_2 \frac{\cos^2\alpha}{\cos^2\beta} \right\} \quad (2)$$

where α and β are the angles of incidence and diffraction. Astigmatism vanishes when $S_s = S_t$ giving for any chosen wavelength

$$2S = \frac{R_1 R_2 (\cos^2\alpha/\cos^2\beta - 1)}{\sec\theta\,(R_1 + R_2\cos^2\alpha/\cos^2\beta) - (R_1+R_2)\cos\theta}. \quad (3)$$

Substitution for S in equations (1) or (2) then gives the distance of the stigmatic images from the camera mirror.

Figure 2 shows the calculated relationship between S_s, S_t and S for two wavelengths (5461 and 2800 Å) using the component parameters given above. Values of α and β are as follows: 5461 Å: $\alpha = 28°\ 34'$, $\beta = 44°\ 34'$; 2800 Å: $\alpha = 9°\ 48'$, $\beta = 25°\ 48'$.

For the grating in collimated light ($S \simeq R_1/2$) the separation of the tangential and sagittal images is approximately 5·6 mm. This value is in agreement with that calculated using an expression given by Rosendahl (1962) and illustrates the limitation on spatial resolution unless an instrument of small aperture is used. Figure 2 shows that a reduction in the value of S and an increase in the separation between camera mirror and image plane can produce a stigmatic image. In the present optical configuration the necessary conditions for elimination of astigmatism are

	5461 Å	2800 Å
S (calculated)	586·8 mm	569·4 mm
S (ray tracing)	586·4	568·0
$S_s = S_t$ (calculated)	573·2	590·5
$S_s = S_t$ (ray tracing)	572·9	589·2

It is seen that the calculated values and those derived from ray tracing (discussed in §3) are in fair agreement at 5461 Å but the agreement is poorer at 2800 Å. This is because the above formulae do not include intercomponent separations and in addition higher order terms in the light path expansion have been omitted. When the component separation is taken into account the calculated and ray tracing results agree within 100 μm. At shorter wavelengths the value of S decreases and the image of the slit formed by the collimating mirror lies closer to the grating. Thus with decreasing wavelength the distance between the collimating mirror and the grating becomes more important. Whilst the above formulae are only approximate they do provide a useful preliminary guide for designing a stigmatic instrument.

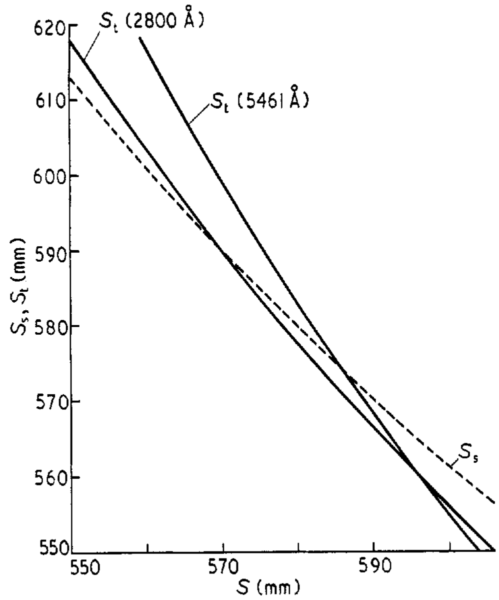


Figure 2 Calculated values of tangential S_t and sagittal S_s image distances from the camera mirror as a function of the separation S between the entrance slit and the collimating mirror. Results are given for λ5461 and 2800 Å

2.2 *Coma*

It has been shown that astigmatism in a Czerny-Turner spectrograph can be eliminated at a chosen wavelength if the grating is employed in divergent illumination. Since (as shown by Richards *et al.* 1947) this method of illumination will introduce coma then a restriction will be placed on the maximum width of grating W. An approximate value for W may be obtained from the light path expansion given by Murty (1962). Using the Rayleigh limit, then in our notation the tolerance for coma requires a restriction on the grating width given by

$$W^3 \leqslant 4\lambda r^2 \bigg/ \cos^2\alpha \left\{ 2\sin\alpha + \sin\beta \left(\frac{\cos^2\alpha}{\cos^2\beta} \right) + \sin\alpha \left(\frac{\cos^4\alpha}{\cos^4\beta} \right) \right\} \quad (4)$$

where r is the distance between the grating and the image of the entrance slit formed by the collimating mirror. Substituting the parameters for the present system in equation (4) gives $W \leqslant 89$ mm at 5461 Å and $W \leqslant 48$ mm at 2800 Å.

These values for W are probably pessimistic since a quarter wave tolerance has been used following Murty and Beutler. This tolerance limit may be relaxed and the Strehl intensity tolerance employed as discussed by Welford (1963, 1965).

For their calculation of grating width Richards *et al.* (1947) used a tolerance for coma of 0·64 λ. Using this same limit the above values for W would be approximately 122 mm and 65 mm at 5461 Å and 2800 Å respectively.

Richards *et al.* and Murty have shown that in convergent illumination the tolerance conditions for coma can only be met if the grating width is extremely small or the f/number extremely large. In the Czerny-Turner arrangement these restrictions are greatly relaxed since the divergence angle of the wavefront falling on to the grating is small.

For the present system the collimating mirror forms an image of the slit located at a distance r of approximately 30 m from the grating at 5461 Å and 12 m at 2800 Å. With decreasing wavelength the separation between the entrance slit and the collimating mirror must be reduced in order to eliminate astigmatism. Thus the divergence angle of the wavefront falling on to the grating will be increased at shorter wavelengths and this will lead to a reduction in permissible grating width and spectrograph speed if the coma tolerance is not to be exceeded.

3 Ray tracing results

In order to provide a comparison with the above analysis and to examine the optical system in greater detail, ray tracing techniques have been used to determine best geometrical focus conditions. The program which has been developed and used extensively by one of us (A C N) is based on a system described by Spencer and Murty (1962). Thirty rays are traced from a given object point along the entrance slit and the number density of these rays striking the first surface is chosen to give a representation of a uniform intensity distribution. Several image planes are sampled for each chosen object distance in order to determine the best focus plane.

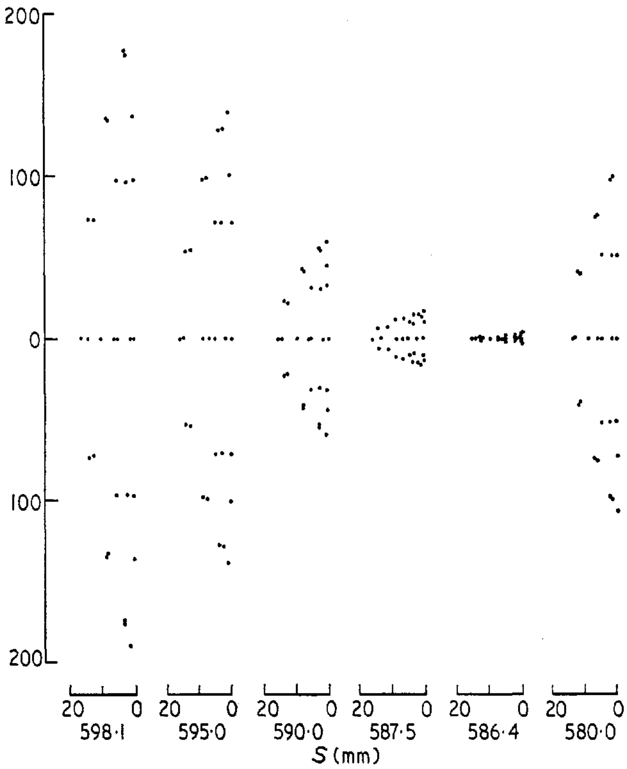
In figure 3(a) spot diagrams are given for an f/15 spectrograph described in §2. The diagrams are for an object point at the slit centre and the wavelength is 5461 Å. It is seen how a reduction in the separation S between the entrance slit and the collimating mirror leads to a reduction in astigmatism. Each spot diagram corresponds to the best geometrical focus for each value of S. Spot diagrams for an object point 4 mm above the slit centre are shown in figure 3(b). This particular case was examined since for the spectroheliograph project this would correspond to the radius of the solar disk imaged on the entrance slit.

Figures 4(a) and (b) show spot diagrams for the same f-number instrument at 2800 Å. These diagrams show very clearly the improvement in spatial resolution to be expected in the Czerny-Turner spectrograph by departing from the normal collimated light configuration. For best focus conditions all the rays lie within a circle of 15 μm diameter. The half-width of the diffraction pattern of an ideal system would be approximately 4 μm. For an object point at the slit centre the intersection of all the rays with the image plane lies within a circle of diameter less than the width of the diffraction pattern.

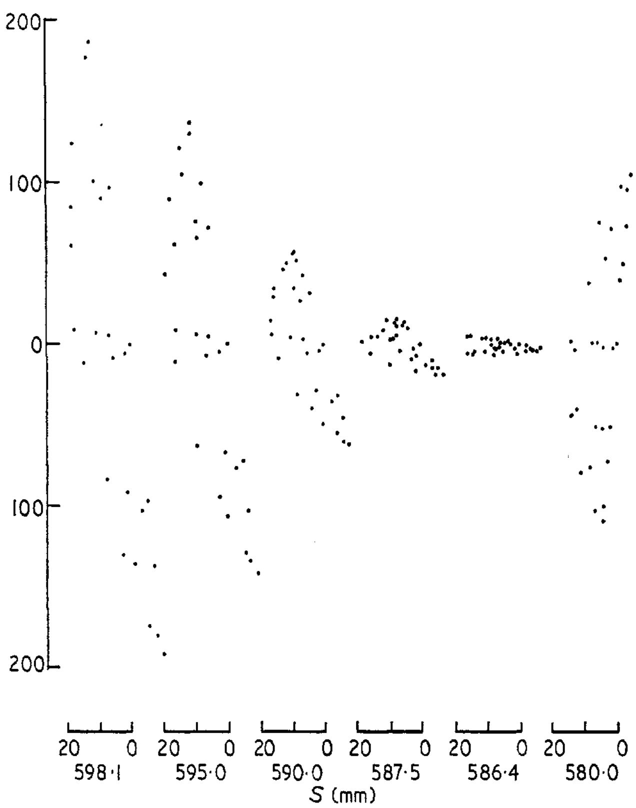
4 Experimental tests

Spatial resolution parallel to the slit has been measured in the visible and ultraviolet using a USAF 1951 resolving power target on a quartz substrate (W and L E Gurley, New York) placed in contact with a 1 mm wide entrance slit. Images of the entrance slit and test target were recorded on Kodak spectrum analysis No. 1 and Ilford fine grain safety positive at 2800 Å and on Ilford Micro-neg Pan and Adox KB 14 at 5461 Å. For optimum focusing conditions the following results for overall resolution (cycles per mm) were obtained for an f/15 spectrograph.

The values given are for an intensity modulation in the recorded image ($I_{max}-I_{min}/I_{max}+I_{min}$) of 0·11 corresponding



(a)



(b)

Figure 3 Computed spot diagrams for several separations S between the entrance slit and the collimating mirror; (a) corresponds to an object point at the centre of the entrance slit, (b) to an object point 4 mm above the slit centre. The wavelength is 5461 Å and the numerical scales are in micrometres

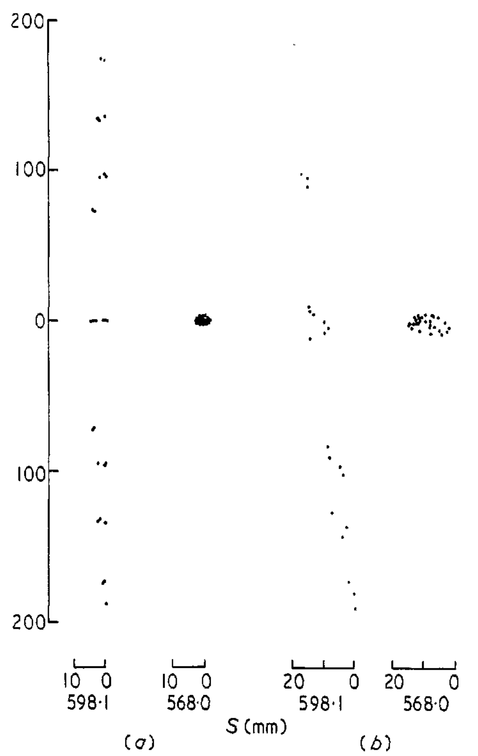


Figure 4 Computed spot diagrams at 2800 Å for the two values of S corresponding to the grating in collimated light and the grating in divergent light optimized for the elimination of astigmatism; (*a*) object point at the centre of the entrance slit; (*b*) object point 4 mm above the slit centre. The numerical scales are in micrometres

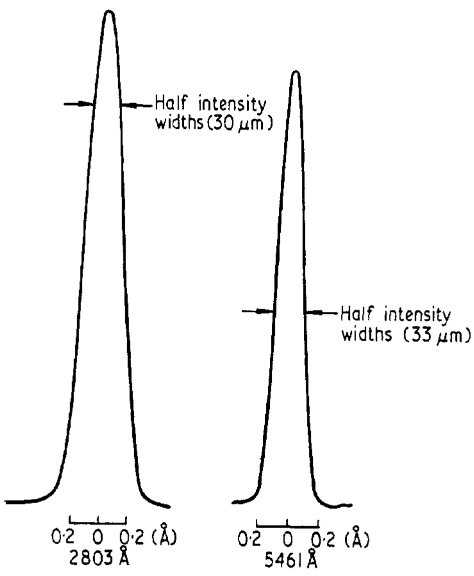


Figure 5 Densitometer traces on a linear density scale of recorded spectral profiles at 5461 and 2800 Å. The half intensity widths of these profiles are indicated

to the Rayleigh limit. These were obtained from densitometer traces of the recorded image and do not include any allowance for emulsion resolution.

	2800 Å	5461 Å
Slit centre	114	81
4 mm above slit centre	81	81

For the normal Czerny-Turner configuration with the grating in collimated light the recorded resolution was only 3 cycles mm^{-1}.

Densitometer traces on a linear density scale of recorded spectral profiles obtained using a ^{198}Hg source are shown in figure 5. These profiles are for a 26 μm entrance slit corresponding to a spectral width of approximately 0·2 Å. It is seen from these traces that although the grating is in non-collimated light the profiles show no evidence of asymmetric broadening. These records were made on Kodak spectrum analysis No. 1 emulsion at 2803 Å and Ilford Micro-neg Pan at 5461 Å.

Whilst it is possible to eliminate astigmatism at only one wavelength the ray tracing results indicated that an acceptable overall resolution would be obtained over a wavelength range of some 100 Å by tilting the image plane. This has been examined experimentally at wavelengths 2852, 2802 and 2753 Å. The spectrograph was adjusted for optimum resolution at 2802 Å and the spatial resolution was determined at the two other wavelengths by moving the image plane towards and away from the camera mirror. The required tilt of the image plane for best overall focus was found to be 4°, which is in close agreement with ray tracing data. Values of spatial resolution (cycles per mm) corresponding to this image plane for an intensity modulation of 0·11 in the recorded image are as follows.

	2753 Å	2802 Å	2852 Å
Slit centre	81	114	81
4 mm above entrance slit	64	81	51

5 Conclusions

The spatial resolution of a Czerny-Turner spectrograph with the grating in collimated and in divergent light has been measured and results are comparable with those predicted by theory and ray tracing data. The astigmatism can be eliminated at a chosen wavelength by using the grating in divergent light without imposing severe restrictions on the grating width and spectrograph speed. Over a spectral range of 100 Å centred on this wavelength an acceptable spatial resolution in excess of 50 cycles mm^{-1} can be achieved on a flat, inclined image plane using an f/15 system.

The spectral and spatial resolutions needed for the spectroheliograph have been easily realized using a design configuration and component parameters which will meet the restrictions imposed by the payload size. No attempt has been made to determine an optimum design. Improvements in the performance of the spectrograph could possibly be obtained by altering the radii of curvature of the two mirrors as discussed by Shafer, Megill and Droppleman (1964). A lateral displacement of the grating perpendicular to the rulings may also produce some amelioration though the actual magnitude of such a movement is in some dispute (Chandler 1968, Rouse *et al.* 1969).

Apart from the planned spectroheliograph which instigated this investigation a compact spectrograph of this type will be of value whenever information on the spatial variation of the source is required over a limited wavelength range. An example of such an application is the use of the spectrograph in combination with a Fabry-Perot etalon.

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References

Beutler H G 1945 *J. Opt. Soc. Am.* **35** 311–50

Chandler G G 1968 *J. Opt. Soc. Am.* **58** 895–99

Dalton M L 1962 *Rev. Sci. Instrum.* **33** 928–9

Dalton M L 1966 *Appl. Opt.* **5** 1121–3

Foreman W T 1968 *Appl. Opt.* **7** 1053–9

Gillieson A H C P 1949 *J. Sci. Instrum.* **26** 335–9

Mielenz K D 1964 *J. Res. Nat. Bur. Stand.* **68C** 205–13

Monk G S 1928 *J. Opt. Soc. Am.* **17** 358–64

Murty M V R K 1962 *J. Opt. Soc. Am.* **52** 768–73

Richards E W T Thomas A R and Weinstein W 1947 *AERE Report* No. C/R 2152

Rosendahl G R 1962 *J. Opt. Soc. Am.* **52** 412–5

Rouse P E Brixner B and Kline J V 1969 *J. Opt. Soc. Am.* **59** 955–60

Seya M Namioka T and Sai T 1967 *Sci. Light* **16** 138–57

Schroeder D J 1966 *Appl. Opt.* **5** 545–8

Schafer A B Megill L R and Droppleman L 1964 *J. Opt. Soc. Am.* **54** 879–87

Shafer A B 1967 *Appl. Opt.* **6** 159

Spencer G H and Murty M V R K 1962 *J. Opt. Soc. Am.* **52** 672–8

Welford W T 1963 *Optica Acta* **10** 121–7

Welford W T 1965 *Progress in Optics* **4** 243–80