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Determination of correlation coefficient of the statistical and formative time delay in nitrogen

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Online at stacks.iop.org/JPhysD/42/212001**Abstract**

The determination of the correlation coefficient between the statistical and formative time delay of dc electrical breakdown in nitrogen is presented in this paper. Starting from a bivariate normal (Gaussian) distribution of two random variables, the analytical distribution of the electrical breakdown time delay is theoretically founded on correlation of the dependent statistical and formative time delay, contrary to convolution of independent variables. Within the limits, a Gaussian density distribution of the electrical breakdown time delay goes to a Gaussian of the formative time or to a Gaussian of the statistical time delay depending on electron yields (preionization level) in the interelectrode space, while the correlation coefficient is determined in the transition region from t_f to t_s dominated statistics.

The time that elapses from the application of voltage greater than the static breakdown voltage U_s to breakdown is the electrical breakdown time delay t_d . It comprises the statistical t_s and the formative time delay t_f [1]. The statistical time delay is the time which elapses from the application of sufficient voltage and appearance of a free electron initiating breakdown. The time interval from this moment to the collapse of applied voltage and occurrence of self-sustained current is the formative time delay [1]. The stochastic nature of breakdown time delay was experimentally proven by Zuber [2] and the exponential distribution was theoretically derived by von Laue [3]. The two new distributions of the statistical time delay in nitrogen were presented in [4]. Depending on the level of preionization, the Gauss-exponential and Gaussian distribution of the statistical time delay were obtained and theoretically based on a binomial distribution for the occurrence of initiating electrons. The dependence of the formative time on working voltages and relaxation time in nitrogen is presented in [5]. The formative time was determined from measured time delay data in the manner

described in [6] and fitted by the model based on exponential growth of the electron current density from a cathode [5].

The complex structure of the breakdown time delay distributions was observed in the transient hollow cathode discharges [7], since the formative time delay has small fluctuations and a large mean value, while the statistical time delay has a small mean value and large fluctuations. Also, the minimization and control of the breakdown time delay are relevant to some other applications, such as spark counters [8], ECR ion sources [9] or micro-discharges [10]. An attempt to separate the formative and statistical time delay in neon and krypton was made in [11–15], on the basis of the time delay measurements, numerical simulation and by irradiation. It was assumed [12–15] that the statistical and formative time delays are independent variables and the breakdown time delay is obtained as their convolution. However, the statistical and formative time delay are not independent variables, but the statistical time delay dispersion in those papers is erroneously ascribed to the formative time, as thoroughly discussed in [4]. Taking into account that the statistical time delay is the waiting time for the occurrence of successful electron(s) that produces breakdown, the formative and statistical time delay

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are dependent variables if $t_s < t_f$ [16]. The degree of their dependence is determined by the preionization level or effective electron yield $Y_{\text{eff}} = Y P$, where Y is the electron yield or the rate of electron production and P is the breakdown probability of one electron to cause the breakdown. At high effective electron yields the correlation coefficient tends to unity [16].

In a mathematical sense, the dependence of two continuous random variables X and Y implies that one variable, say Y , either increases or decreases as X changes. Two measures of dependence are the covariance and the correlation coefficient $\rho(X, Y)$ [17]. If X and Y are independent continuous variables, then their covariance and correlation coefficient are equal to zero, but the opposite statement is true only in the special case of a bivariate normal distribution. It can be shown that the correlation coefficient satisfies the inequality $-1 \leq \rho \leq 1$. Thus, a positive correlation coefficient indicates that Y increases as X increases, a negative ρ implies that Y decreases as X increases and $\rho = 0$ implies zero covariance and no correlation [17]. The characteristic case of the joint density function of two continuous random variables is the earlier mentioned bivariate normal distribution, sometimes also called a bivariate Gaussian distribution. If we regard the statistical and formative time delay as continuous random variables t_s and t_f , then their bivariate Gaussian density distribution $f(t_s, t_f)$ is given by

$$f(t_s, t_f) = \frac{1}{2\pi\sigma_{t_s}\sigma_{t_f}\sqrt{1-\rho^2}} \exp\left[-\frac{1}{2(1-\rho^2)} \times \left(\frac{(t_s - \bar{t}_s)^2}{\sigma_{t_s}^2} - 2\rho\frac{(t_s - \bar{t}_s)(t_f - \bar{t}_f)}{\sigma_{t_s}\sigma_{t_f}} + \frac{(t_f - \bar{t}_f)^2}{\sigma_{t_f}^2}\right)\right], \quad (1)$$

with the marginal Gaussian density functions $f_{t_f}(t_f)$ and $f_{t_s}(t_s)$ which can be obtained by integration of equation (1) for all t_s and t_f , respectively. \bar{t}_s and \bar{t}_f denote the mean values of the marginal distribution of t_s and t_f , σ_{t_s} and σ_{t_f} are their standard deviations and ρ is the correlation coefficient of t_s and t_f . Taking into account the above-mentioned dependence between t_s and t_f , the density function of the breakdown time delay $t_d = t_s + t_f$ can be calculated by integration [18]:

$$f(t_d) = \int_0^\infty f(t_s, t_d - t_s) dt_s, \quad (2)$$

which in the case of the bivariate Gaussian distribution (equation (1)) gives [18]

$$f(t_d) = \frac{1}{\sqrt{2\pi(\sigma_{t_s}^2 + 2\rho\sigma_{t_s}\sigma_{t_f} + \sigma_{t_f}^2)}} \exp\left[-\frac{1}{2} \frac{[t_d - (\bar{t}_s + \bar{t}_f)]^2}{\sigma_{t_s}^2 + 2\rho\sigma_{t_s}\sigma_{t_f} + \sigma_{t_f}^2}\right]. \quad (3)$$

Equation (3) implies that the breakdown time delay t_d has a Gaussian density distribution with the mean value $\bar{t}_d = \bar{t}_s + \bar{t}_f$ and the standard deviation $\sigma_{t_d} = \sqrt{\sigma_{t_s}^2 + 2\rho\sigma_{t_s}\sigma_{t_f} + \sigma_{t_f}^2}$, from which the correlation coefficient ρ can be determined.

In order to check the earlier hypothesis of the dependence of the statistical and formative time delay and to demonstrate

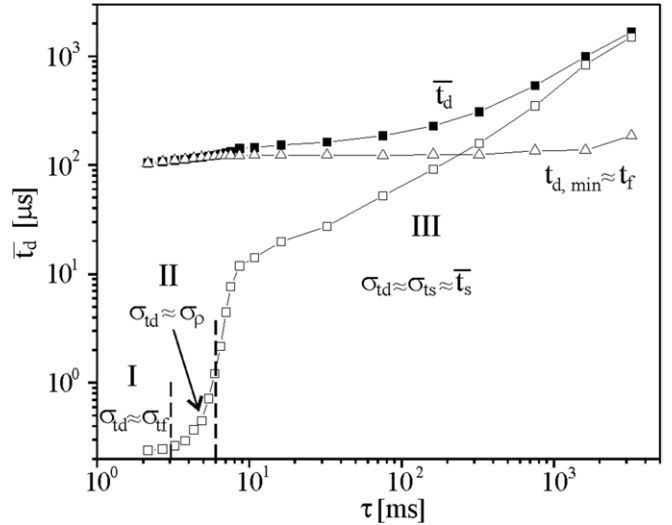


Figure 1. The memory curve, the formative and statistical time delay and their standard deviations as a function of relaxation times in nitrogen.

the determination of the correlation coefficient, we performed new experimental measurements of the breakdown time delay. The dependence of the mean values of the breakdown time delay $\bar{t}_d(\tau)$ on the relaxation time τ (the well-known ‘memory curve’ [19–21]) was measured on a gas tube made of borosilicate glass (8245, Schott technical glass) equipped with a planar copper electrode. The diameter of the electrode was $D = 2.2$ cm and the interelectrode gap $d = 0.8$ cm. The tube was filled with research grade nitrogen at a pressure of 2.66 mbar (Matheson Co. with oxygen impurities of below 1 ppm.) Before the measurements, the cathode surface was prepared by running a glow discharge for about 30 min and several thousand breakdowns. The static breakdown voltage was $U_s \approx 330$ V. The time delay measurements were carried out by applying rectangular (step) pulses to the discharge tube, at the glow current $I_g = 0.1$ mA, glow time $t_g = 1$ s, working voltage $U_w = 500$ V and at different relaxation times τ . The mean values of the breakdown time delay times were established from a series of 200 measurements. The measurements were performed by using an electronic system [4, 16] which reached the voltage rise time and resolution limit below $0.2 \mu\text{s}$. The measured memory curve is presented in figure 1. It is divided into three characteristic regions. In the first region (I in figure 1), below $\tau = 3$ ms, the t_d standard deviation is nearly equal to the t_f one ($\sigma_{t_d} \approx \sigma_{t_f}$), since σ_{t_s} can be neglected. In the second region (II in figure 1) the contributions of σ_{t_f} and σ_{t_s} to σ_{t_d} are of comparable value, and the correlation coefficient between t_s and t_f will be determined. In region III of the memory curve, σ_{t_s} dominates ($\sigma_{t_d} \approx \sigma_{t_s}$) and the characteristic distributions of the statistical time delay appear (Gaussian, Gauss-exponential and exponential) [4]. In region III of the memory curve the good estimation for the formative time delay is $t_{d,\text{min}}$, which represents the minimal value of the time delay obtained in the series of 200 measurements.

The $\sigma_{t_s}(\tau) = 1/Y(\tau)$ dependence (with $P \approx 1$ at a given working voltage) to about 10 ms of region III of the memory

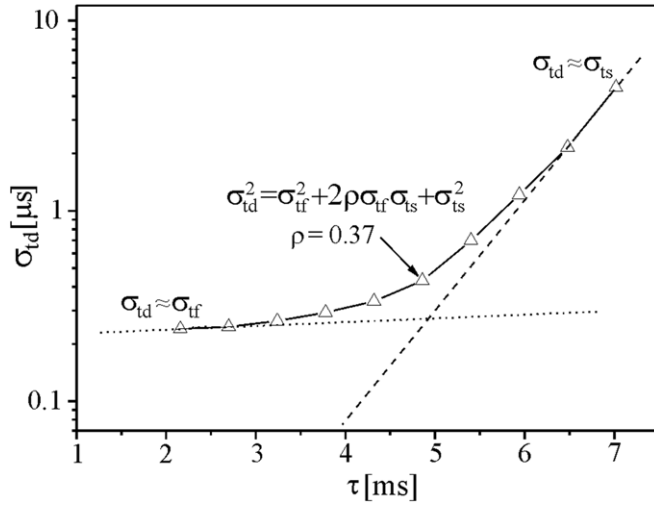


Figure 2. The $\sigma_{id}(\tau)$ dependence in region II of the memory curve (figure 1): symbols—experimental data, dotted line—linear fit of $\sigma_{tf}(\tau)$, dashed line—linear fit of $\sigma_{ts}(\tau)$.

curve (figure 1) is approximately linear in semilog scale confirming that a secondary electron yield in the interelectrode space is determined by the nitrogen ions decaying by diffusion [5, 6]. After that, in region III, the dominant emission mechanism of electrons initiating breakdown is the surface recombination of nitrogen atoms remaining from the preceding glow [5, 21].

In order to determine the correlation coefficient between t_s and t_f , the $\sigma_{id}(\tau)$ dependence in the first 7 ms is presented in figure 2. The experimental points below 3 ms (region I in figure 1) refer to the formative time delay and their linear extrapolation to longer τ gives the estimation of σ_{tf} in regions II and III. Similarly, the extrapolation of the last two experimental points in figure 2 gives the estimation of σ_{ts} in regions I and II. The experimental point at $\tau = 4.9$ ms with $\sigma_{id} \approx 0.43 \mu\text{s}$ was chosen for the determination of ρ (marked with arrow in figure 2). The extrapolated values of t_s and t_f dispersions at this τ are $\sigma_{ts} = 0.25 \mu\text{s}$ and $\sigma_{tf} = 0.27 \mu\text{s}$, respectively. From the dispersion of the density distribution given by equation (3), ρ can be expressed in the form

$$\rho = \frac{\sigma_{id}^2 - (\sigma_{tf}^2 + \sigma_{ts}^2)}{2\sigma_{tf}\sigma_{ts}}, \quad (4)$$

which gives the value of the correlation coefficient $\rho = 0.37$, which means that the statistical and formative time delay are medium correlated. The correlation coefficient, determined on the finite sample size ($n = 200$), represents the estimation of the population correlation coefficient ρ between t_s and t_f . In order to determine the confidence level of ρ we have applied the standard statistical test for the correlation coefficient on a large sample [17], which results in a probability greater than 99% that the population correlation coefficient is $\rho > 0$. Thus, we can reject the hypothesis that t_s and t_f are uncorrelated, independent variables.

It is clear from the above analysis that neither the statistical and formative time delay are independent variables nor the density distribution of t_d can be calculated like their

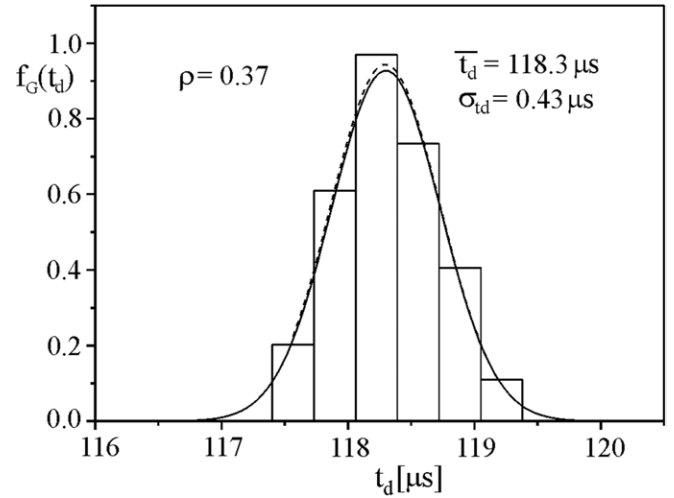


Figure 3. The Gaussian density distribution of the breakdown time delay at $\tau = 4, 9$ ms: solid line—fit based on the correlation of t_s and t_f (equation (3)), dashed line—the analytical expression (equation (5)).

convolution [12–15]. This statement will be tested through analysis of the experimentally obtained Gaussian density distribution $f_G(t_d)$ presented in figure 3. The fit of $f_G(t_d)$ by theoretical density (3) with the parameters $\bar{t}_d = 118.3 \mu\text{s}$, $\rho = 0.37$, $\sigma_{ts} = 0.25 \mu\text{s}$, $\sigma_{tf} = 0.27 \mu\text{s}$ is presented by the solid line in figure 3. The agreement between experimental and theoretical distributions is obvious and confirms our earlier statements. Also, the experimental density $f_G(t_d)$ is fitted by an analytical Gaussian distribution of the form

$$f_G(t_d) = \frac{1}{\sqrt{2\pi}\sigma_{id}} \exp\left[-\frac{(t_d - \bar{t}_d)^2}{2\sigma_{id}^2}\right], \quad (5)$$

with the fitting parameters $\bar{t}_d = 118.295 \mu\text{s}$ and $\sigma_{id} = 0.431 \mu\text{s}$ obtained by nonlinear least square fitter with minimized χ^2 value and $R^2 \approx 0.99$ [17]. Going to shorter relaxation times τ , the statistical time delay and its dispersion become negligible compared with the formative time dispersion due to the high level of residual ionization, i.e. $\bar{t}_s \ll \bar{t}_f$ and $\sigma_{ts} \ll \sigma_{tf}$ (figure 1 and figure 2), and the experimental Gaussian density distribution function of the formative time at $\tau = 2.7$ ms is obtained, $f_G(t_f)$ (figure 4). This is the first experimentally obtained t_f Gaussian distribution in nitrogen. It is fitted by an analytical Gaussian distribution of the form

$$f_G(t_f) = \frac{1}{\sqrt{2\pi}\sigma_{tf}} \exp\left[-\frac{(t_f - \bar{t}_f)^2}{2\sigma_{tf}^2}\right], \quad (6)$$

with the fitting parameters $\bar{t}_f = 110.5 \mu\text{s}$ and $\sigma_{tf} = 0.30 \mu\text{s}$ obtained by nonlinear least square fitter with minimized χ^2 value and $R^2 \approx 0.96$. The ratio $\sigma_{tf}/\bar{t}_f = 2.7 \times 10^{-3}$ is in good agreement with values obtained for the formative time delay distributions in neon [16, 22]. The $f_G(t_f)$ density in figure 4 can be fitted by the theoretical distribution (3) neglecting terms with σ_{ts} and assumptions $\bar{t}_s \ll \bar{t}_f$ and $t_d \approx t_f$. Moreover, the t_f Gaussian distribution (figure 4) is obtained

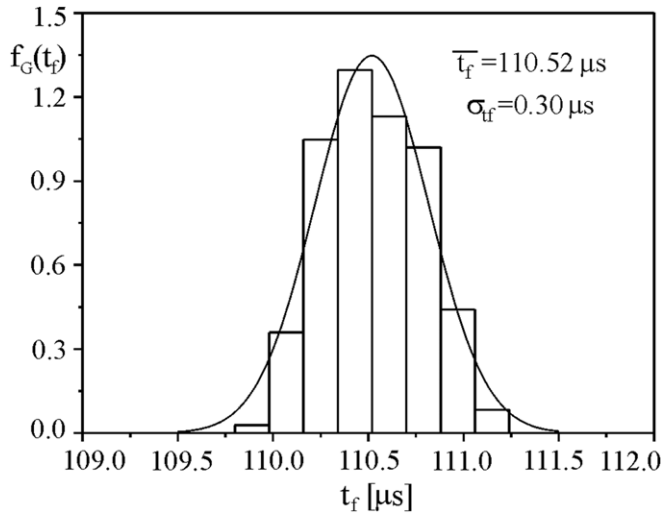


Figure 4. The Gaussian density distribution of the formative time delay at $\tau = 2, 7$ ms; solid line—the analytical expression (equation (6)).

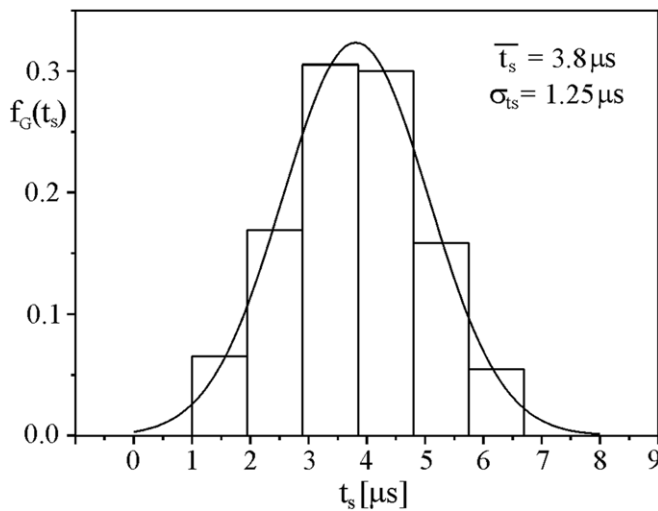


Figure 5. The Gaussian density distribution of the statistical time delay at $\tau = 6$ ms; solid line—the analytical expression (equation (7)).

for $Y_{\text{eff}} \gtrsim 10^8 \text{ s}^{-1}$, similarly to the t_f Gaussian distributions in neon [16, 22] obtained for $Y_{\text{eff}} \gtrsim 10^9 \text{ s}^{-1}$ (or the statistical time delay dispersion $\sigma_{t_s} \approx 1/Y_{\text{eff}} \sim \text{ns}$).

On the other hand, for $\tau \gtrsim 6$ ms, the formative time standard deviation can be neglected, i.e. $\sigma_{t_d} \approx \sigma_{t_s}$ and the experimental distribution functions describe the statistical time delay [4]. The Gaussian density function $f_G(t_s)$ obtained at $\tau = 6$ ms from the measured time delay data by subtracting $t_f \approx t_{d,\text{min}} = 120 \mu\text{s}$ is presented in figure 5. Also, it is fitted by an analytical Gaussian distribution of the form

$$f_G(t_s) = \frac{1}{\sqrt{2\pi}\sigma_{t_s}} \exp\left[-\frac{(t_s - \bar{t}_s)^2}{2\sigma_{t_s}^2}\right], \quad (7)$$

with the fitting parameters $\bar{t}_s = 3.8 \mu\text{s}$ and $\sigma_{t_s} = 1.25 \mu\text{s}$ obtained by nonlinear least square fitter with minimized χ^2 value and $R^2 \approx 0.997$. From relation $\bar{t}_s = \kappa \cdot \sigma_{t_s}$ [16], $\kappa \approx 3$

is obtained, which confirms that the Gaussian distribution in nitrogen (figure 5) refers to the statistical time delay. Also, earlier in our paper [4] it was shown that the statistical time delay in nitrogen has a Gaussian density distribution for $Y_{\text{eff}} \gtrsim 10^6 \text{ s}^{-1}$, which is now confirmed in the case of the t_s distribution presented in figure 5. As in the case of the t_f distribution, the $f_G(t_s)$ density in figure 5 can be fitted by the theoretical distribution (3) neglecting terms with σ_{t_f} and the assumption $t_f \approx \bar{t}_f \approx t_{d,\text{min}}$.

The presented theory and determination of the correlation coefficient of t_s and t_f in nitrogen can generally be applied to single species or a mechanism dominating breakdown. In the case of the air breakdown, as well as of gas mixtures, the different species or mechanisms are involved in the breakdown initiation. Therefore, our initial distributions have to be generalized in order to describe the correlation of double-Gaussian distributions of the breakdown time delay, such as those obtained in [23] for an air and nitrogen/oxygen mixture or in [16] for a neon/nitrogen mixture.

In conclusion we may add that the hypothesis of the independence between the statistical and formative time delay of electrical breakdown [12–15] completely failed. The experimental density distribution function of breakdown time delay t_d was modelled by the theoretical distribution of the sum of two random variables, based on the bivariate Gaussian density distribution of the dependent statistical and formative time delay. The value $\rho = 0.37$ of the correlation coefficient between t_s and t_f in nitrogen was determined from the measured values of σ_{t_d} , σ_{t_s} and σ_{t_f} . In the limiting cases, the Gaussian density distribution functions of the formative and statistical time delay were experimentally obtained and successfully modelled. What the convolution hypothesis additionally disqualifies are the flat $t_f(\tau)$ dependences [12, 15] (which should increase due to the charged particle decay in afterglow), as well as the flat $t_f(U)$ dependences [11, 15] whose decreasing behaviour is well established when the voltage increases [22].

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