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A mathematical model for the melting rate in welding with a multiple-wire electrode

Janez Tušek

Faculty of Mechanical Engineering, University of Ljubljana, Aškerčeva 6, 1000 Ljubljana
Institut za varilstvo (Welding Institute), Ptujska 19, 1000 Ljubljana, Slovenia

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Abstract. The paper deals with the development of a mathematical model for the calculation of melting rates obtained in gas-shielded arc welding with a multiple-wire electrode or in submerged arc welding with a multiple-wire electrode. The first part provides a very general and short description of welding with a multiple-wire electrode, the main advantages and characteristics, as well as variants of welding with a multiple-wire electrode applicable in practice to various cases. The major part of the paper treats the development of the mathematical model for the calculation of the melting rate on the basis of the physical principles of the welding arc and of the wire extension heating due to current conduction and mutual influence of the welding arcs. Finally, a comparison is made between the melting rate results obtained by practical measurements and those obtained theoretically by the mathematical model. The mathematical model for the calculation of melting rates in welding with a multiple-wire electrode is accurate enough to be used in practice and for further studies.

1. Introduction

In general, several methods of the modelling of technological, chemical, physical and other processes are well known. In consumable-electrode welding, two different principles have asserted themselves for calculation of the melting rate, i.e. of the weight of filler material melted in a unit of time.

The first applies a statistical method. This model is based on the large amount of data that has been obtained by experimental means. In this case, we are not interested in the process itself, but in the input and output data which form the basis for statistical processing and the calculation of statistical models. In the experimental work, care should be taken to ensure that the experiments concerned are real and that the results obtained represent an actual probability.

The second method is based on the mathematical and physical principles of the process. In this case, one should know in detail the process itself, as well as the physical and chemical principles applicable.

In our case, i.e. fusion arc welding, the second method will be applied. On the basis of physical laws we will try to mathematically describe the process and on the basis of input data, i.e. the welding parameters and physical properties of the materials in our case, to predict the quantity of filler material that is melted in welding with a multiple-wire electrode.

For single-wire electrode welding there are various mathematical models to be found in literature for the calculation of the melting rate [1–8]. However, a model for welding with a twin-wire electrode and a multiple-wire electrode has yet to be found.

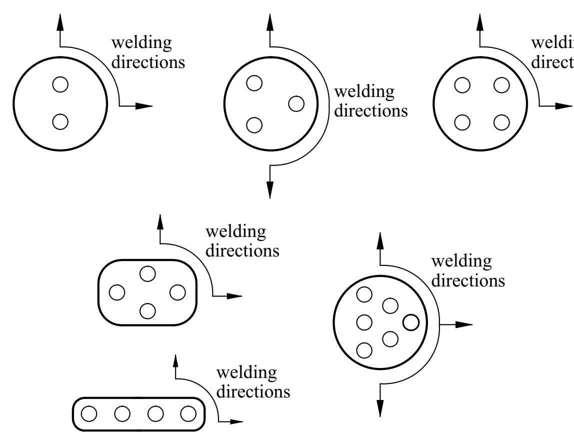

Figure 1. The contact nozzle in which the wires are arranged optionally.

2. A short description of welding with a multiple-wire electrode

We have a case of welding with a multiple-wire electrode when two or more wires are travelling simultaneously through the contact nozzle. All the wires concerned are supplied from the same power source, have the same wire feed speed and the same regulation (figure 2). Wires in the contact nozzle can be arranged in a different manner, which is most often dependent on the purpose and on the shape of the welded joint. Some cases of application are shown in figure 1.

The device for multiple-wire welding is mostly applied in submerged arc welding or electro-slag welding, and

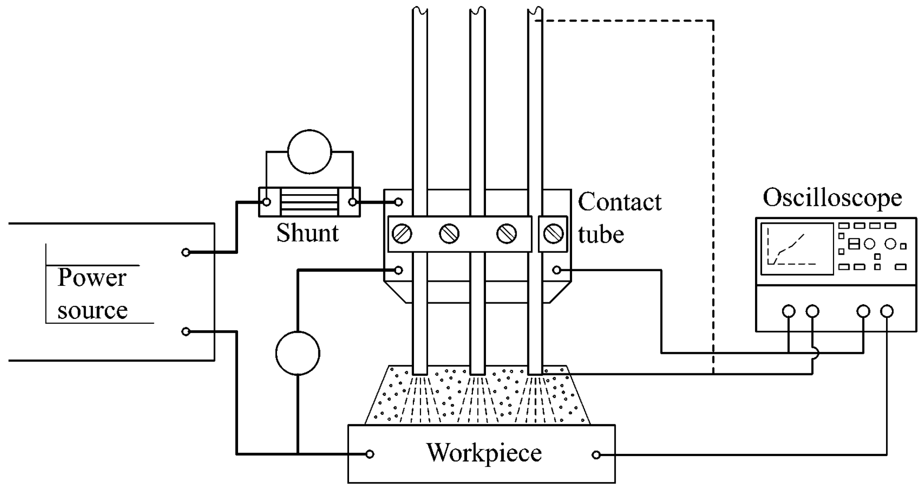

Figure 2. A schematic representation of an apparatus for voltage drop measurement in the wire extension in welding with a multiple-wire electrode.

possibly also in gas-shielded arc welding. In the latter case, the welding parameters need to be selected very carefully, the wire feed should be constant, without oscillation, and the wires should always be fed to the 'same' welding spot. This means that very stable conditions need to be maintained. In opposite conditions, strong spatter, irregular motion of the molten pool as well as an irregular final layer and undercuts may occur [9–13].

3. Description of the research work performed

It was stated in the introduction that the mathematical model for the calculation of the melting rate would be based on the physical processes occurring during welding with a multiple-wire electrode. Since the melting rate is expressed by the quantity of filler material that is melted per unit of time, it is necessary to define the energies consumed in welding-wire fusion while taking efficiencies into consideration. The thermal energy consumed in arc welding fusion is generated in two different ways. An electric current flows through the wire extension and heats it in accordance with Ohm's law. The quantity of heat generated is a function of welding current intensity, wire diameter, wire extension length and the kind of material. The second part of the energy consumed in filler material fusion is generated in the welding arc. The welding arc is also an electric conductor with high electrical resistance and high temperature. Investigations and theoretical calculations show that, in order for fusion to occur, the heat generated in the electrode region of the welding arc needs to be consumed [13–15]. Having detached from the wire, a droplet is already in a liquid state. Its traverse through the welding arc only overheats it and reduces the efficiency of the welding process.

In general, the energy equation for the melting speed could be written in a thermodynamic form

$$(Q_0 + Q_w)\eta = Mc_p\,\Delta T + H \tag{1}$$

Equation (1) tells us that the heat generated in the arc Q_0 (W) and in the wire extension Q_w (W) melts the filler material M (kg h^{-1}) with specific heat c_p (J g^{-1} K^{-1}). The heat losses into the environment, the powder or the shielding gas and other losses are contained in the efficiency (η), while material losses in heating due to various transformations are contained in the latent heat.

If equation (1) is to be solved, one should know the heat energy generated in the wire extension, the heat energy in the electrode region of the arc, the thermal efficiency of these two forms of energy and—the most problematic—the functional relations of c_p (T), H (T) and ρ (T) for the filler material used.

For the calculation of the heat energy in the wire extension, the most important heat is that which is generated due to ohmic resistance. In general, this heat can be defined by

$$Q_j = \frac{I^2 L}{s}\rho_r(T) \tag{2}$$

where I (A) is the current intensity, L (mm) is the wire extension length, s (mm^2) is the wire cross section and ρ_r (Ω) is the specific resistance.

With the specific resistance being dependent on temperature and the temperature being dependent on the wire extension length, the voltage drop in the wire extension, i.e. between the contact nozzle and the arc, was measured experimentally. In the literature available, no such measurements could be found so a contact nozzle was adapted to welding with a triple-wire electrode, and the system for voltage drop measurement shown in figure 2 was established [12].

Figure 2 shows a scheme of the contact nozzle through which three wires travel simultaneously at the same speed. The three wires are supplied from one power source. In order to be able to measure the voltage drop in the wire extension, the contact nozzle was cut as shown in figure 2 so that a tungsten wire, which was fixed to the wire before welding at the spot also shown in figure 2, could travel through the gap. During welding the tungsten wire travelled together with the welding wire to the contact nozzle and then to the arc. When the tungsten wire reached the contact nozzle, the voltage drop measurement was started and it continued until the tungsten

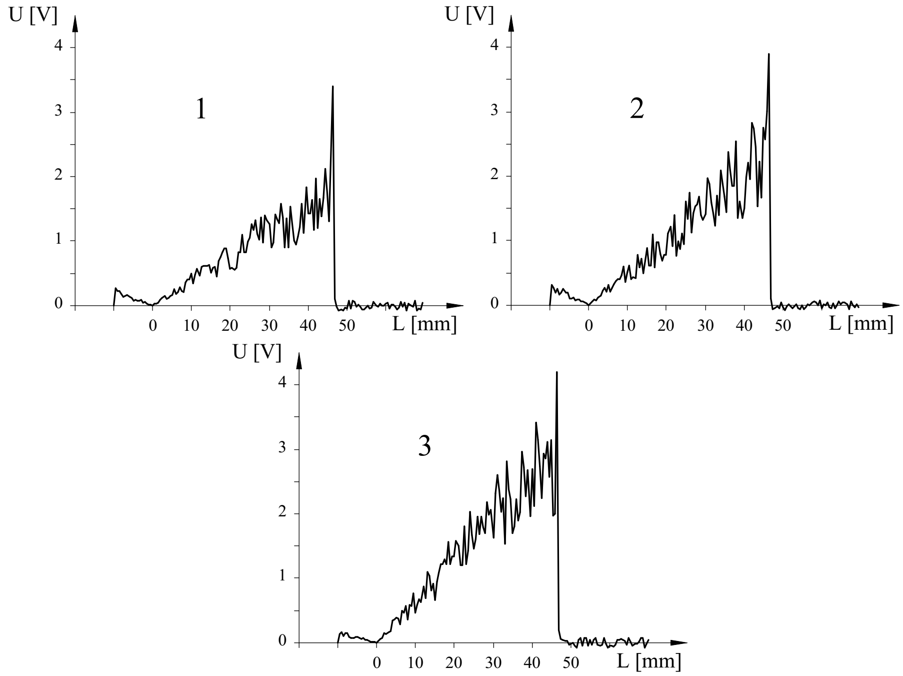

Figure 3. Curves of voltage drops in the wire extension in welding with a single-wire electrode (1), double-wire electrode (2) and triple-wire electrode (3): $U = 30$ V, $L = 50$ mm, $b = 9$ mm, $I = 400$ A/wire, $d = 3$ mm with positive electrode.

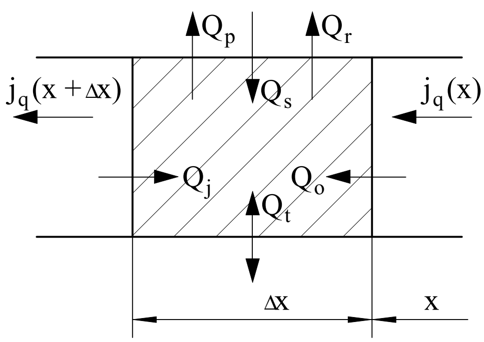

Figure 4. A schematic representation of the wire extension with heat inventory.

wire reached the arc. In the arc, the tungsten wire did not melt but fell off; the measurement was thus ended. Curves of voltage drops in the wire extension, i.e. between the contact nozzle and the arc, in welding with single-wire, double-wire and triple-wire electrodes are shown in figure 3.

The curves of the voltage drops show what the voltage is as a function of the contact nozzle and also indicate the voltage oscillation frequency which is changing and dependent on the number of droplets detached. A difference in the voltage drops can be observed in the case of the single-wire, the double-wire and the triple-wire electrodes. The greater the number of wires, the greater is the voltage drop. The voltage oscillation frequency is also higher in the case of a triple-wire electrode than in the case of a single-wire or double-wire electrode.

Consequently, it can be concluded that the resistance in the wires increases with the increase in the number of wires when other welding parameters per wire are maintained as constant. There are two reasons for the increase in resistance. The first is a thermal influence. The mutual influence of the welding arcs additionally heats the wires, which results in a temperature increase, and consequently in an increase of electric resistance. The second is a mutual electromagnetic influence, which also increases the electric resistance in the wire extension.

During welding, all the welding parameters, in addition to the voltage drops, i.e. the wire feed speed, the surfacing mass, the quantity of powder consumed in submerged arc welding and the gas flow in gas-shielded arc welding, were measured, while on the basis of curves of the welding current and welding voltage, an approximate evaluation of the number of droplets detached from each wire per unit of time could be made.

4. Heat energy input into wire extension in welding with a multiple-wire electrode

As mentioned in the introduction, a sufficient quantity of heat should be introduced into the filler material so that the latter may heat up to the melting point. The total heat inventory in the wire portion in welding with a triple-wire electrode can be illustrated by figure 4 and the following equation

$$Q_j + Q_s + Q_0 - Q_r - Q_p \pm Q_t = \rho S \Delta x \int_T^{T+\Delta T} c_p(T)\,\mathrm{d}t \quad (3)$$

where ρ (g cm^{-3}) is the welding wire density, S (mm^2) is the wire cross section, Δx (mm) is the wire portion and c (J g^{-1} K^{-1}) is the specific heat. Q_r is the heat loss due to radiation, Q_p is the heat loss due to heat transfer from the wire into the shielding medium (powder or gas), Q_t is the heat generated in the wire extension due to the Thompson effect, Q_j is the heat energy generated in the wire extension due to ohmic resistance and

$$Q_s = \frac{Ld}{6}\sigma(T_0^4 - T_w^4)\frac{1}{[(1/\varepsilon_1) + ((1/\varepsilon_2) - 1)]} \times \left[\sqrt{1 + \left(\frac{b}{d}\right)^2} - \frac{b}{d}\right] \quad (4)$$

is the heat energy introduced into the wire due to radiation of the neighbouring arcs in joules. Here L (mm) is the wire extension, d (mm) is the wire diameter, b (mm) is the distance between the wires, $\sigma = 5.67 \times 10^{-8}$ W m^{-2} k^{-4} is Stefan's constant, T_0 (K) is the arc temperature, T_w (K) is the wire temperature, ε_1 is the arc emissivity and ε_2 is the absorption factor of the wire.

Q_0 is the heat energy transferred from the arc to the wire extension due to conduction. Owing to the constant wire feed speed and constant melting rate the process as such can be considered stationary, not dependent on time and can be defined by

$$Q_0 = -\lambda(T)\frac{\partial T}{\partial x} \quad (5)$$

where λ (J g^{-1} K^{-1}) is the thermal conductivity of the welding wire.

It is also presumed that heat conduction in the directions Z and Y is negligible and that the temperature in the wire cross section is constant.

Q_r is the heat loss in the wire due to radiation. This heat can also be defined by a generalized Stefan's law

$$Q_r = \varepsilon\sigma T^4 \, \Delta x \, \pi d \quad (6)$$

where ε is the wire emissivity, T (K) is the average wire temperature, L, d are as in equation (4) and Δx (mm) is the wire portion.

Q_p is the heat loss due to heat transfer from the wire into the shielding medium which is defined by

$$Q_p = \alpha_T \, \Delta x \, 1.5d(T_w - T_p) \quad (7)$$

where α_T (J m^{-2} K^{-1}) is the heat transfer coefficient, T_p (K) is the temperature of the shielding medium and Δx, T_w are as in equations (4) and (6).

By means of experimental measurements it was established that the polarity affects the wire extension heating. This statement, which is not described in the welding literature available, can be explained by the Thomson effect and is mathematically defined by [7]

$$Q_t = \mu(T) I \, \Delta x \frac{\partial T}{\partial L} \quad (8)$$

where μ (V K^{-1}) is the Thomson coefficient and I (A) is the electric current intensity. The Thomson coefficient μ can be positive or negative. If the electric current is flowing in the direction of the temperature gradient, the coefficient is positive, while in the opposite case it is negative.

In addition to the heat energy sources mentioned, which heat the wire extension, and heat losses in the wire, there are some more sources and losses which, however, do not particularly affect the melting rate of the filler material.

On the basis of experimental results, the physical and chemical properties of materials and the literature data concerning welding with a single-wire electrode in various shielding media, equation (3) can be somewhat simplified.

According to reference data [16] on metal inert gas (MIG) welding, heat losses due to radiation amount to 4% and those due to gas flow to 5%. It is known that current densities in wires in submerged arc welding are lower than in MIG welding, therefore the above-mentioned losses are even smaller. The Thomson effect is dependent on polarity and reaches important values with higher arc voltages, higher current densities and increased wire extensions. Consequently, in submerged arc welding, the Thompson effect may be neglected [16–20].

By neglecting the above-mentioned sources and sinks and by considering equations (2), (4) and (5), equation (3) takes the following form:

$$\lambda\frac{\partial^2 T}{\partial x^2} + \frac{\partial T}{\partial x}c_p\frac{M}{S} + \frac{I^2\rho_r(T)}{S^2} + Q_s(T) = 0 \quad (9)$$

where M (g s^{-1}) is the melting rate, λ is as in equation (5) and S, c_p are as in equation (3).

The specific electric resistance is dependent upon temperature. As regards the filler materials used, this dependence was established experimentally and can be, in a somewhat simplified form, defined by

$$\rho_r = K_1 T_z. \quad (10)$$

The heat influence of the neighbouring arc can be, somewhat simplified, defined as a linear function by

$$Q_s = K_2 T_z. \quad (11)$$

By rearranging equation (9) and by considering equations (10) and (11), one can obtain

$$\frac{\partial^2 T}{\partial x} + C_1\frac{\partial T}{\partial x} + C_2 T = 0. \quad (12)$$

Constants C_1 and C_2 are expressed by

$$C_1 = \frac{c_p M}{S\lambda} \quad (13)$$

$$C_2 = \frac{I^2 K_1}{S^2\lambda} + \frac{K_2}{\lambda}. \quad (14)$$

Equation (12) is a homogeneous, partial, differential quadratic equation which is easy to solve if the limiting conditions are known.

By solving this equation, one can establish the temperature distribution in the wire extension during welding with a multiple-wire electrode.

5. Total heat energy input into the filler material and elaboration of a mathematical model for calculation of the melting rate

For fusion of the filler material, the heat energy generated in the wire extension and a portion of the arc energy are consumed.

Table 1. Experimentally obtained values of the factors α, β Q and U_E for some of the filler materials used in submerged arc welding with a single-wire electrode.

Wire	Wire diameter, ∅ (mm)	α (Ω m)		β (J g^{-1})		U_E (V)		Q_k (J g^{-1})	
		(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)
A	3	1.19	1.08	505	505	4.96	8.2	1705	1870
B	2	1.22	1.22	586	586	4.92	8.0	1705	1870
C	1.6	1.26	1.26	610	610	4.8	7.8	1705	1870
D	1.2	1.26	1.26	615	615	4.75	7.7	1705	1870
E	1.2	1.22	1.22	625	625	4.75	7.7	1705	1870
F_1	3.2	1.38	1.30	1430	1430	5.6	9.4	1705	1870
F_2	3.2	1.47	1.47	1080	1080	—	—	—	—

Table 2. Experimentally obtained values of the factors α, β Q and U_E in submerged arc welding with double-wire and triple-wire electrodes.

Wire	Wire diameter, ∅ (mm)	α (Ω m)		β (J g^{-1})		Q_k (J g^{-1})		U_E (V)	
		(+)	(−)	(+)	(−)	(+)	(−)	(+)	(−)
A	3 double	1.34	1.19	505	505	1560	1795	5.5	8.7
A	3 triple	1.56	1.35	505	505	1490	1705	6.1	9.6

If in equation (9) the term which describes heat transfer from the arc to the wire extension (the quantity of heat being very low due to constant droplet melting) is deleted, we obtain

$$\frac{c_p}{\rho}\int_0^L \mathrm{d}T = \frac{I^2 L}{MS} = f(Q_w). \qquad (15)$$

Equation (15) includes the physical properties of the welding wires, the welding parameters and the melting rate. The actual heat input Q_w into the welding wires applied as a function of the parameters I (A), L (mm), S (mm^2), ρ (g cm^{-3}) and v_w (mm s^{-1}) was investigated experimentally.

The heat energy in the wire extension, which is defined by equation (9), but in which the term describing heat transfer from the arc to the wire extension can be deleted, can also be defined by

$$\mathrm{d}Q_w = \frac{RI^2}{SL}\,\mathrm{d}t. \qquad (16)$$

By integration and rearrangement, we obtain

$$Q_w = \frac{j}{L\rho}\int_0^t U_L(t)\,\mathrm{d}t \qquad (17)$$

where j (A mm^{-2}) is the current density.

The functional relation between the heat introduced into the wire Q_w (J g^{-1}) and f (Q_w) (J g^{-1} Ω^{-1} mm^{-1}) was investigated by means of numerous experiments performed on eight different types of welding wires. The diagram in figure 5 shows the results obtained with two 3 mm wires used in submerged arc welding and with a 3.2 mm cored wire also used in submerged arc welding.

The functional relation between Q_w and $f(Q_w)$ is linear up to a certain value which is still applicable in practice and is important. It can be defined by

$$Q_w = \alpha f(Q_w) - \beta. \qquad (18)$$

The values of the constants α (Ω mm) and β (J g^{-1}) for the materials applied can be obtained for two welding wires from the diagrams in figure 5 [1] and for some other welding wires from table 1.

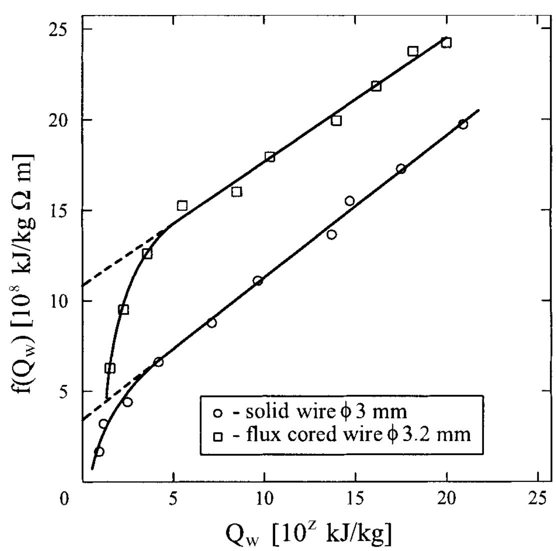

Figure 5. The relationship between heat input into the welding wire and the parameters which generate this heat for a 3.2 mm cored wire and a 3 mm solid wire in submerged arc welding.

The constant α is mainly dependent on the kind of material and represents the specific electric resistance of the filler material at higher temperatures, i.e. at the wire extension tip. The constant α is also a function of polarity and of the wire diameter. It can be further established that the value of α is much higher with cored wires than with solid wires. Since a cored wire has low specific heat, the covering of the cored wire is consequently heated to a higher temperature and has a higher specific resistance.

The second constant in equation (18) is β (J g^{-1}), which is the heat energy in the wire at room temperature. Following theoretical considerations, β should be dependent only on the kind of material used. Experiments, however, have shown that this constant is also dependent on the wire diameter and welding wire type.

The second portion of heat by which the filler material is melted is obtained from the welding arc. The quantitative value of arc energy can also be determined experimentally.

In cases where only arc energy is consumed for melting of the filler material, i.e. welding is carried out without the wire extension, the energy input into a unit of the filler material can be determined by

$$Q_A = \frac{U_E I}{M} \tag{19}$$

when

$$f(Q_w) = \frac{Lj^2 S}{M} \tag{20}$$

is entered into equation (18), we obtain

$$Q_w = \alpha \frac{Lj^2 S}{M} - \beta. \tag{21}$$

Then we sum up equations (19) and (21), consider the number of wires in the contact tube, rearrange the terms of the equation obtained, and thus obtain a mathematical model for calculation of the melting rate in welding with a multiple-wire electrode

$$\boldsymbol{M} = \frac{\boldsymbol{I} \cdot \boldsymbol{n} \cdot (\boldsymbol{U_E} + \alpha \cdot \boldsymbol{L} \cdot \boldsymbol{j} \cdot \boldsymbol{n}^{-0.2})}{\boldsymbol{Q_k} + \beta}. \tag{22}$$

In equation (22), 'n' is the number of wires, Q_k (J g^{-1}) is the energy per unit of mass of the filler material which is contained in the droplet right after its detachment from the wire and U_E (V) is the voltage drop in the cathode and anode regions of the welding arc.

Similarly to constants α in β, the values Q_k and U_E are also obtained experimentally. Table 1 shows these values for some of the filler materials used in welding with a single-wire electrode, while table 2 shows them for a 3 mm wire used in welding with double-wire and triple-wire electrodes.

6. Comparison of experimentally measured and theoretically calculated values of melting rate

On the basis of numerous experiments and on the known physical properties of the filler materials, the mathematical model for calculation and prediction of the melting rate in welding with a multiple-wire electrode, as defined by equation (22), was elaborated. A larger number of repeated experiments in welding under various welding conditions were carried out. The results obtained were compared to the calculated values. The comparison is shown in figure 6.

The diagram in figure 6 shows the functional relations between the measured melting rates in submerged arc welding with double-wire and triple-wire electrodes using different polarities and the calculated melting rates. The diagram shows a very high degree of similarity between the measured and the theoretically calculated values of the melting rate. A particularly high degree of likeness can be observed in welding with a positive double-wire electrode and also in welding with a triple-wire electrode. Somewhat less favourable results, i.e. a somewhat stronger discrepancy in results, is observed in welding with a negative triple-wire electrode and with higher melting rates.

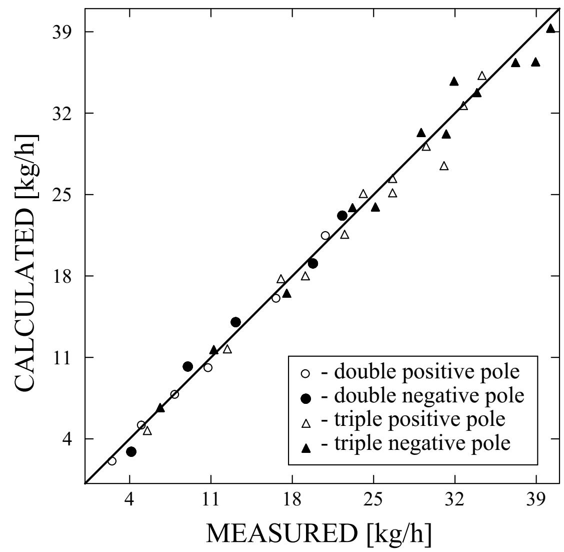

Figure 6. A comparative diagram of the theoretically calculated melting rates in submerged arc welding with double-wire and triple-wire electrodes (diameter of 3 mm, with positive electrode and negative electrode) and the calculated values.

7. Conclusions

On the basis of the results obtained, it can be established that the mathematical model for calculation of the melting rate in welding with a multiple-wire electrode is accurate enough for practical application. The only weakness of the model is the fact that four different coefficients should be known for each filler material used. For filler materials these coefficients should be determined experimentally.

References

[1] Wilson J L, Claussen, G E and Jackson C E 1956 *Welding J.* **35** 1S
[2] Lesnewich A 1958 *Welding J.* **37** 343S
[3] Jackson C E 1960 *Welding J.* **39** 177S
[4] Robinson M H 1961 *Welding J.* **40** 503S
[5] Demjancevic V P 1974 *Avtomatičeskaja Svarka* **27** 50
[6] Holmøj E 1979 *Proc. Int. Conf. on Arc Physics and Weld Pool Behaviour* (London: The Welding Institute)
[7] Waszink J H and Van den Heuvel G J P M 1982 *Welding J.* **61** 269S
[8] Chandel R S 1987 *Welding J.* **66** 135S
[9] Tušek J 1990 *Document 212-772-90* (Montreal: International Institute of Welding)
[10] Tušek J 1993 *Proc. Int. Conf. on Joining of Metals JOM-6* (Helsingør: European Institute for the Joining of Materials)
[11] Kralj V and Tušek J 1993 *Document 212-843-93* (Glasgow: International Institute of Welding)
[12] Tušek J 1991 *Document 212-791-91* (Den Haag: International Institute of Welding)
[13] Tušek J 1994 *Int. J. Joining Mater.* **6** 105
[14] Van Adrichem T J 1966 *Document 212-178-66* (Delft: International Institute of Welding)
[15] Eichhorn F, Felleisen F R and Huber W 1976 *Vorträge des Hochschulskolloquium 'Physik des Schweißlichtbogens'* (Düsseldorf: DVS-Verlag)
[16] Waszink J H and Van den Heuvel G J P M 1979 *Proc. Int. Conf. on Arc Physics and Weld Pool Behaviour* (London: The Welding Institute)
[17] Robinson M H 1961 *Welding J.* **40** 503
[18] Belschuk G A 1977 *Schweißtechnik, Berlin* **27** 452
[19] Willgoss R A 1984 *Welding Metal Fabrication* **53** 340
[20] Stenbacka N and Persson K A 1989 *Welding J.* **68** 41