

MILKY WAY MASS AND POTENTIAL RECOVERY USING TIDAL STREAMS IN A REALISTIC HALO

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ABSTRACT

We present a new method for determining the Galactic gravitational potential based on forward modeling of tidal stellar streams. We use this method to test the performance of smooth and static analytic potentials in representing realistic dark matter halos, which have substructure and are continually evolving by accretion. Our FAST-FORWARD method uses a Markov Chain Monte Carlo algorithm to compare, in six-dimensional phase space, an “observed” stream to models created in trial analytic potentials. We analyze a large sample of streams that evolved in the Via Lactea II (VL2) simulation, which represents a realistic Galactic halo potential. The recovered potential parameters are in agreement with the best fit to the global, present-day VL2 potential. However, merely assuming an analytic potential limits the dark matter halo mass measurement to an accuracy of 5%–20%, depending on the choice of analytic parameterization. Collectively, the mass estimates using streams from our sample reach this fundamental limit, but individually they can be highly biased. Individual streams can both under- and overestimate the mass, and the bias is progressively worse for those with smaller perigalacticons, motivating the search for tidal streams at galactocentric distances larger than 70 kpc. We estimate that the assumption of a static and smooth dark matter potential in modeling of the GD-1- and Pal5-like streams introduces an error of up to 50% in the Milky Way mass estimates.

Key words: dark matter – Galaxy: halo – Galaxy: structure

Online-only material: color figures

1. INTRODUCTION

The mass density profile, shape, and extent of the dark matter halos that surround galaxies provide important clues to how galaxies form, and test the properties of the dark sector and the fundamental cosmological model. Given the vast stellar data sets and unique perspective on our own Galaxy, we might expect the mass distribution for the Milky Way to be well understood. However, estimates for the overall mass of the Galaxy are currently uncertain by a factor of two (see Figure 7 in Barber et al. 2014 and references therein).

A major hurdle in measuring the Milky Way mass has been the scarcity of tracers in the outer halo. So far, the best measurements have been obtained from satellite galaxy (e.g., Watkins et al. 2010) and halo star kinematics (e.g., Deason et al. 2012). These methods usually implicitly assume that the stellar halo is a relaxed structure with objects on randomized orbits.

However, extended substructures discovered in the Galactic halo reveal that the Milky Way halo is neither dynamically old nor relaxed (e.g., Majewski et al. 2003; Belokurov et al. 2006; Jurić et al. 2008; Bonaca et al. 2012). The presence of tidal debris invalidates gravitational potential recovery methods based on the assumption of random orbits, as neither orbits nor orbital phases are equally represented in an unrelaxed halo. This has been shown through simulations of Milky Way-like halos, which suggest that even large samples of hundreds of outer halo stars may actually be on only a handful of independent orbits (Bullock & Johnston 2005). Assuming that orbits in a truly unrelaxed halo are randomized introduces a bias of ~20% in mass estimates (Yencho et al. 2006).

Like satellite galaxies and halo stars, tidal streams can be used as tracers of the gravitational potential. In particular, their extent over tens of degrees on the sky offers a unique glimpse into the Galactic halo shape. The largest identified tidal debris in the Galactic halo is the currently disrupting Sagittarius dwarf spheroidal (Sgr; Ibata et al. 2001a). The Sgr stream is an example of a dynamically hot stream, due to its large width in both position ($\gtrsim 20^\circ$; Belokurov et al. 2006) and velocity space ($\gtrsim 20 \text{ km s}^{-1}$; Koposov et al. 2013). It is also very long; its tails have been traced to wrap more than 360 degrees using M giants, blue horizontal branch, and turn-off stars (Majewski et al. 2003; Belokurov et al. 2014; Pila-Díez et al. 2014). Despite this wealth of observational data, there is little agreement on the halo shape from Sgr models (Helmi 2004; Johnston et al. 2005; Law & Majewski 2010; Deg & Widrow 2013; Vera-Ciro & Helmi 2013; Ibata et al. 2013; Gibbons et al. 2014). Most significantly, uncertainties in the amount of progenitor rotation (Peñarrubia et al. 2010), the presence of bifurcations in both leading and trailing arms (Koposov et al. 2012), and orbit evolution due to dynamical friction (Ibata et al. 2001b) affect the recovery of the Galactic potential using Sgr.

Cold streams might be expected to provide more stringent constraints on the potential as compared to the Sgr stream (Lux et al. 2013). These streams originate from smaller stellar systems such as lower-mass dwarf galaxies and globular clusters (e.g., Odenkirchen et al. 2003; Grillmair & Dionatos 2006; Belokurov et al. 2006; Grillmair 2009; Bonaca et al. 2012; Koposov et al. 2014; Martin et al. 2014; Bernard et al. 2014). Both classes of stellar systems are dispersion dominated, so their disruption mechanism is better understood and the potential recovery is less affected by the assumptions on the progenitor’s properties. The warmer Orphan stream yielded estimates of the total mass

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within 60 kpc (Newberg et al. 2010; Sesar et al. 2013), while Koposov et al. (2010) used the six-dimensional (6D) phase space information from the colder GD-1 stream to constrain both the halo mass and shape inside the Solar circle.

Irrespective of their dynamical temperature, tidal streams have so far been modeled exclusively in parametric potential forms which are smooth and static. However, N -body simulations of structure formation show that dark matter halos are clumpy and evolving. For example, Milky Way-mass halos have accreted on the order of $\sim 10\%$ of their present mass in the last 6 Gyr (Diemand et al. 2007). Furthermore, $\sim 10\%$ of their total halo mass is in subhalos (e.g., Diemand et al. 2008), which can influence the progenitor orbit (e.g., Vera-Ciro & Helmi 2013), heat up streams upon close encounters (e.g., Yoon et al. 2011), or create density inhomogeneities (e.g., Siegal-Gaskins & Valluri 2008).

In this paper, we test biases introduced into recovering the gravitational potential when potential complexity is not taken into account. To do this, we compare two sets of “test” streams: the first is formed by inserting cold stellar streams into a realistic, high-resolution dark matter-only simulation, and the second is formed through evolution in a simplified, smooth, and static potential. We recover the gravitational potential using these sets of streams.

The structure of the paper is as follows. We start by describing the Via Lactea II (VL2) simulation that we use as a realistic, Milky Way-like dark matter halo in Section 2. We test how well commonly used profiles represent the VL2 potential, and provide triaxial potential fits (Section 2.2). We also explore how the recovered best-fit parameters depend on the radial extent of the data, and how they change over time as the potential evolves (Section 2.3). Next, we describe a large sample of stellar streams created in the VL2 (Section 3.1) and an analogous sample of streams created in analytic potentials (Section 3.2). Analytic potential parameters were obtained by fitting the VL2 potential field. We present a novel method for forward modeling of stellar streams, the FAST-FORWARD method, in Section 4. The results of potential recovery on both stream samples are presented in Section 5, while the differences between the samples are discussed in Section 6. We also put our results in the context of the known Milky Way streams in Section 6.2, and conclude in Section 7.

2. TESTBED FOR THE GALACTIC POTENTIAL

High-resolution simulations are essential for interpreting Galactic studies in a cosmological context. We use the Via Lactea II to represent the Galactic potential in which tidal streams are formed. Here, we provide a brief overview of the simulation in Section 2.1 and analytic fits to its potential in Section 2.2 and Section 2.3. Stream formation is described in the next Section 3.

2.1. Via Lactea II Simulation

Via Lactea II (VL2) is a high-resolution dark matter simulation of a Milky Way-like halo (Diemand et al. 2008). The simulation contains 1.1 billion particles, each with a mass of $4.1 \times 10^3 M_\odot$, evolved using the PKDGRAV2 tree-code (Stadel 2001) and a force resolution of 40 pc. The simulation was run from redshift $z \simeq 100$ to the present with adaptive time steps set to 1/16 of the local dynamical time. The main halo experienced no recent major merging events, but grew through minor mergers for the past 6 Gyr. The final mass within the radius en-

closing 200 times the mean matter density, $r_{200} = 402$ kpc, is $M_{200} = 1.9 \times 10^{12} M_\odot$. This falls in the range of most Milky Way mass determinations (Barber et al. 2014). At a distance of ~ 800 kpc from the main VL2 halo, there is another massive halo ($6 \times 10^{11} M_\odot$; Teyssier et al. 2012); thus, VL2 is similar to the Milky Way not only in terms of its mass and accretion history, but also as a halo in a Local Group-like environment.

Dark matter halos have a universal spherically averaged density profile, regardless of their mass or cosmological parameters (NFW; Navarro et al. 1997). The density profile of VL2 is also well described by a cusped, Navarro–Frenk–White (NFW) like profile (Diemand et al. 2008). Lifting the requirement of spherical symmetry when modeling dark matter halos, Jing & Suto (2002) showed that they are, in general, triaxial. In VL2, Zemp et al. (2009) detected triaxiality in halo shape from deviations of local density from the spherically averaged values. For ease of calculation, most potential recovery methods introduce triaxiality in the potential, instead of density. In the remainder of this section, we provide fits of the triaxial NFW and logarithmic potentials to the Via Lactea II simulation (Section 2.2). Furthermore, we show how they have evolved during the last 6 Gyr (Section 2.3).

2.2. Parametric Fits to the VL2 Potential

The true form of dark matter halo profiles is unknown. Historically, NFW and logarithmic profiles have been the analytic potential forms most commonly used to represent dark matter halos. Logarithmic halos are motivated by the observed flat-rotation curves of galaxies (Rubin et al. 1980), while the NFW profile provides a good fit to the spherically averaged profiles of simulated dark matter halos (Navarro et al. 1997). In this section, we assess the performance of both of these potential forms on the Via Lactea II potential.

We studied the VL2 potential on a random sample of 100,000 simulation particles extracted at redshift $z = 0$. The exact gravitational potential at each particle position was calculated by summing up the contributions from all the other particles, treating each as a Plummer sphere with a characteristic scale of 50 pc. Next, the VL2 potential was modeled with the following commonly used parameterizations:

1. a logarithmic potential of the form

$$\Phi_{\log} = V_c^2 \ln(r^2 + R_c^2), \quad (1)$$

where V_c is the circular velocity and R_c is the core radius; and

2. an NFW potential of form

$$\Phi_{\text{NFW}} = -V_h^2(R_h/r) \ln(1 + r/R_h), \quad (2)$$

where R_h is the scale radius, and V_h is the circular velocity at R_h .

In both potential forms, the radial distance r takes into account halo triaxiality:

$$r^2 = C_1 x^2 + C_2 y^2 + C_3 xy + (z/q_z)^2. \quad (3)$$

We follow Law & Majewski (2010) in defining the constants C_i :

$$\begin{aligned} C_1 &= \cos^2 \phi / q_1^2 + \sin^2 \phi, \\ C_2 &= \cos^2 \phi + \sin^2 \phi / q_1^2, \\ C_3 &= 2 \sin \phi \cos \phi (1/q_1^2 - 1), \end{aligned}$$

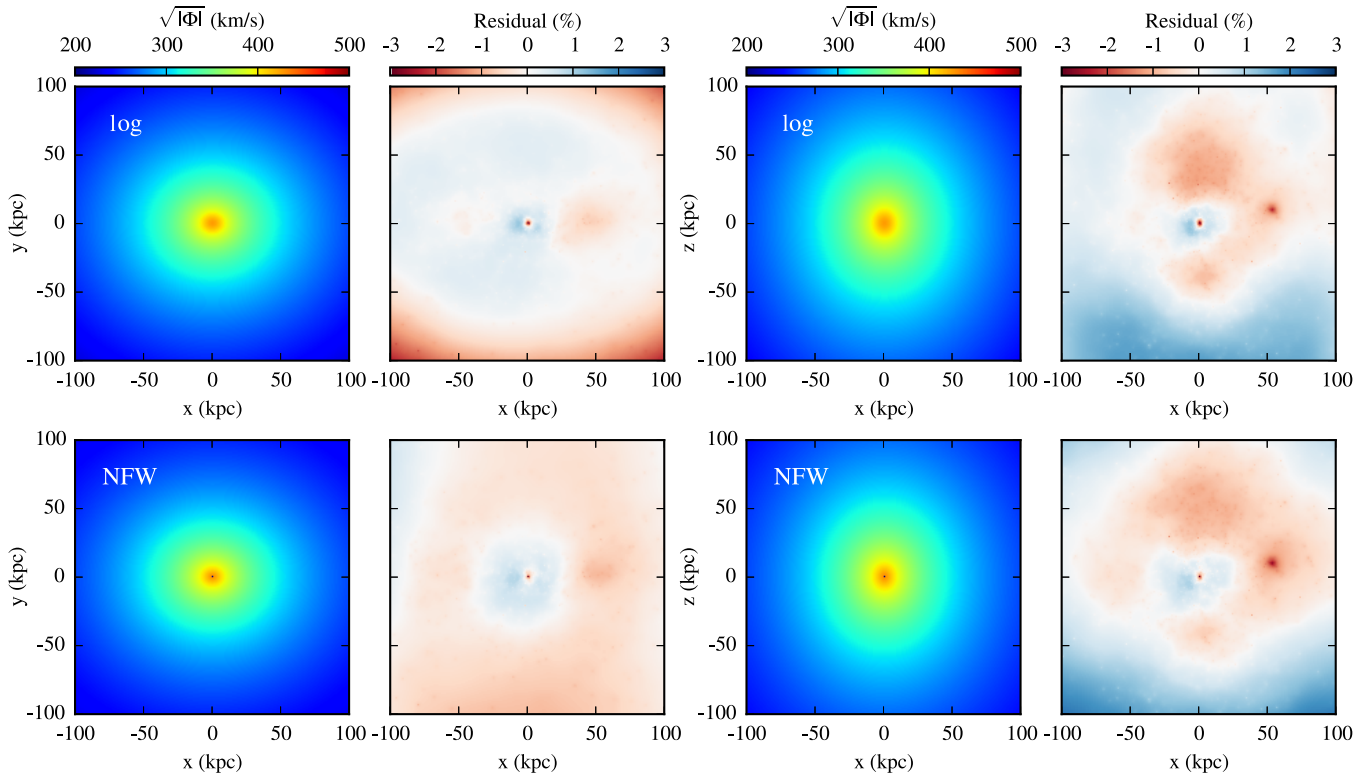


Figure 1. Triaxial parametric fits to the present-day Via Lactea II potential, with logarithmic in the top row and NFW on the bottom. The best fit to the $z = 0$ plane is shown in the first column, color-coded by $\sqrt{|\Phi|}$ (km s^{-1}), with the residuals normalized to the exact VL2 potential in the second column. Similarly, the third column shows the best fit to the $y = 0$ plane and the fourth column has the corresponding residuals. Although there are inhomogeneities in the residuals, they are on the order of $\lesssim 3\%$.

(A color version of this figure is available in the online journal.)

where q_1 is the ratio between the x and y axes, q_z is the ratio between the z and y axes, and ϕ is the rotation angle around the z axis. The chosen coordinate system is aligned with one of the dark matter halo symmetry planes. A general orbit in these triaxial potentials does not necessarily have conserved integrals of motion (e.g., actions Binney & Tremaine 2008). In that sense, they are not integrable, and the action-angle-based potential recovery methods are not guaranteed to always work with these potential forms.

Both potential forms have five free parameters: two defining the typical circular velocity and scale radius of the halo and three for its shape and orientation. The best-fit parameters were found by a least-squares minimization of the difference between the analytical model and the exact potential at the positions of the simulation particles. The minimization was performed 500 times on a bootstrapped sample of simulation particles, thus providing both the best-fit parameters and their uncertainties. The best-fit analytic models to the inner 100 kpc are shown in Figure 1 with the logarithmic potential on the top and the NFW profile on the bottom. The first column shows the potential in the x – y plane, color-coded by a square root of the gravitational potential, $\sqrt{|\Phi|}$. The second column has normalized, $(\text{model} - \text{VL2})/\text{VL2}$, potential residuals in the same plane. Similarly, the potential in the x – z plane is shown in the third column with the corresponding residuals in the fourth. Both the logarithmic and NFW forms are very good approximations to the exact VL2 potential on the global scale; the deviation of the logarithmic best-fit model from the exact potential is on the order of 3%, while the NFW fit is even better with residuals smaller than 1%.

Although the residuals are small, they show a radial structure indicating that some of the potential parameters have radial dependence. In Figure 2, we investigate this dependence by probing progressively larger VL2 radii ($20 \text{ kpc} < r_{\text{max}} < 150 \text{ kpc}$) when fitting model potentials. This encompasses the radial range probed by our simulated streams (for more details, see Section 3). As before, we start by calculating the potential at each particle due to all other particles in the simulation. We then select particles within a maximum radius r_{max} and fit the potential form to just those particles. The radial dependence of the best-fit parameters, including their 1σ uncertainties, is shown in Figure 2 (black points) for the logarithmic on the left and the NFW potential on the right. The values of all potential parameters change with radius on the order of $\lesssim 10\%$. The only exception is the scale radius for a logarithmic fit, which varies 40%. The rate of change is similar for both potential forms used. Furthermore, the shape parameters have very similar values in the logarithmic and NFW best-fit models. The halo profile is mildly triaxial, with the x axis being the shortest and z the longest. The axis ratios q_i have their extremes at 40–60 kpc, beyond which q_1 increases and q_z decreases with radius, making the halo more spherical on average. The rotation of the x – y plane around the z axis is $\phi \simeq 90^\circ$, i.e., the coordinate system is roughly aligned with the halo axes. The variations in the recovered potential parameters presented in Figure 2 indicate that even if an exact measurement of the potential is available, fitting a parametric potential form will yield different results depending on the radial extent of the available data.

The first goal of gravitational potential recovery is to determine a halo’s mass and shape. Once the potential form is known,

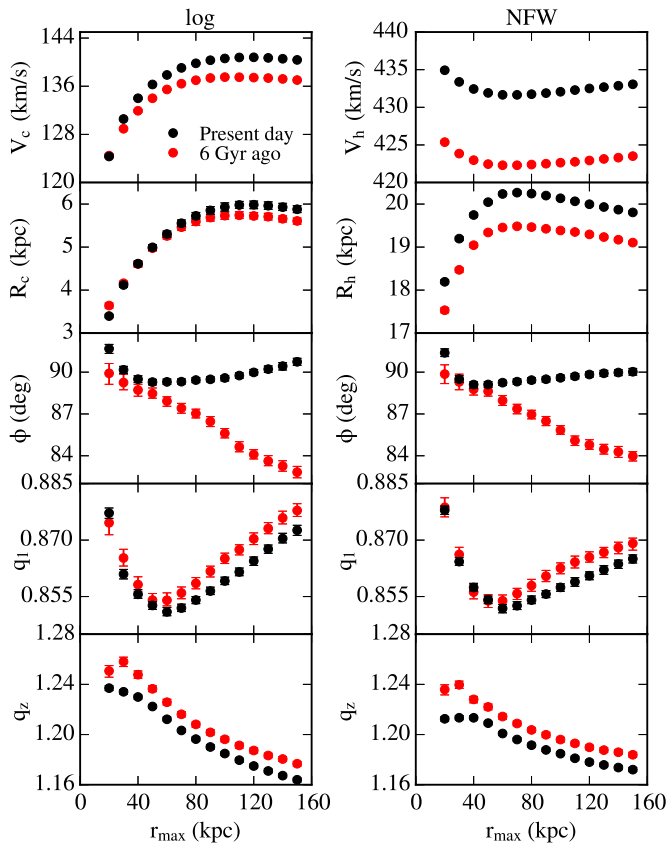


Figure 2. VL2 potential is parameterized with circular velocity (V_c , V_h), characteristic radius (R_c , R_h), rotation angle (ϕ), and axis ratios (q_1 and q_z), as defined in Section 2.3. We show the best-fit parameters for the triaxial logarithmic (left) and NFW (right) potential to the Via Lactea II simulation, with associated 1σ uncertainties, measured within a radius r_{\max} . Even when a complex potential is known exactly, the best-fit parametric form changes with radial extent of the available data. The most significant change in potential parameters from 6 Gyr ago (red) to present day (black) is observed in the normalization of the velocity and radius parameters with minimal changes in the shape parameters.

(A color version of this figure is available in the online journal.)

the density is easily obtained using Poisson’s equation. Integrating the density field out to a spherical radius, we can obtain an estimate of the enclosed mass. This estimate might be different than that obtained when fitting the density field directly. However, the transition from potential to density is often made in Galactic potential studies, and here we assess its accuracy. The VL2 halo mass inside 150 kpc estimated using the best-fit logarithmic potential is $M_{\text{tot,log}} = 1.37 \times 10^{12} M_{\odot}$, and for the NFW potential equals $M_{\text{tot,NFW}} = 1.09 \times 10^{12} M_{\odot}$. A direct measure of the mass in the inner 150 kpc of the VL2 yields $M_{\text{tot,VL2}} = 1.16 \times 10^{12} M_{\odot}$. Logarithmic and NFW estimates are thus 20% higher and 5% lower, respectively, than the true mass. These set the upper limit on the accuracy of measuring the total mass of the VL2 halo when assuming these parametric potential forms.

2.3. Potential Evolution in the Last 6 Gyr

Dark matter halos experience rapid growth at early times, which slows down after redshift $z \simeq 1$. We create streams in Via Lactea II during the slow growth phase, starting from a snapshot 6 Gyr ago (described in the next section). At a final snapshot, streams from this sample are distributed in the inner 150 kpc. During the 6 Gyr time span, the VL2 mass has increased $\sim 5\%$

within 150 kpc, which is typical for halos in this mass range (Zemp 2014; M. Zemp 2014, private communication). In this section, we show how the gravitational potential was affected.

We analyze a sample of 100,000 randomly selected simulation particles at the $z = 0.69$ snapshot, corresponding to a ~ 6 Gyr lookback time. Analogously to the results of the previous section, we obtain the best-fit parameters for the triaxial logarithmic and NFW potentials as a function of radius, and compare them to the present day values in Figure 2 (red). The most prominent change is observed in the growth of the halo circular velocity and the scale radius. On the other hand, surprisingly little evolution is seen in the shape parameters during this time.

3. SIMULATED STREAMS

In this work, we focus on cold streams originating from globular clusters. The physics of globular cluster disruption has been extensively studied in direct N -body simulations (Heggie & Hut 2003; Baumgardt & Makino 2003; Küpper et al. 2008b, 2010; Just et al. 2009), so the formation of their tidal tails is well understood and easier to model than the disruption of dwarf galaxies. These simulations show that clusters on a wide range of orbits in a galactic tidal field experience mass loss through evaporation driven by two-body relaxation. Evaporated stars occupy a range of orbits similar to, though not quite the same as, the progenitor’s orbit, thus forming a thin and cold stellar stream (Eyre & Binney 2011).

The disruption of globular clusters has been studied in N -body simulations that directly solve forces between the star particles, but assume a smooth, time-independent galactic potential. This is at odds with the predictions from the cosmological simulations of structure formation—galactic potentials are clumpy and constantly evolving (e.g., Springel et al. 2005). So far, it has been unfeasible to couple simulations of globular cluster disruption and galaxy formation. The former require high temporal and spatial resolution (\sim year, \sim parsec) to correctly capture the details of evaporation, while the latter follow phenomena on much longer (~ 10 Gyr) and larger (~ 1 Mpc) scales. A recent step toward bridging this gap follows the dynamical evolution of globular clusters using a direct N -body simulation, with the tidal forces along their orbits taken from a larger simulation of galaxy formation (Renaud & Gieles 2013; Rieder et al. 2013). While this approach accurately represents internal globular cluster evolution, the galactic potential and its influence on the resulting stream is represented only coarsely by interpolating between the simulation outputs.

The alternative approach, tailored for studying cold tidal streams in a realistic galaxy potential, is to couple an approximate treatment of the stream formation to a high-resolution simulation of galaxy formation. We follow Küpper et al. (2012) and create stream models using the *streakline* method. Since clusters experience constant mass loss due to two-body interactions and decreasing unbinding energy, we can assume that stars escape the cluster at every simulation time step and trace their paths in the galactic tidal field. The stars escape the globular cluster from a tidal radius, r_t , which can be calculated analytically (King 1962) for a wide range of galactic potentials and cluster orbits using

$$r_t = \left(\frac{GM}{\Omega^2 - \partial^2 \Phi / \partial R^2} \right)^{1/3}, \quad (4)$$

where M is the cluster’s mass, Φ is the galactic potential, R is the cluster’s galactocentric distance, and Ω is its instantaneous angular velocity. Escaping stars are given velocities to match the cluster’s angular velocity, as this choice was shown to reproduce stream morphologies from direct N -body simulations (Küpper et al. 2012).

To assess the role of different aspects of the galactic potential in tidal stream evolution, we use a large set of streams whose progenitors are on a variety of orbits. In the rest of this section, we describe the two sets of streams: *VL2 streams*, evolved in a live dark matter potential (Section 3.1); and *analog streams*, a control set evolved in a parametric potential (Section 3.2).

3.1. Via Lactea Streams

A sample of 12,800 streams was created in a resimulation of the Via Lactea II using the streakline method, starting from particle positions 6 Gyr ago (A. H. W. Küpper et al., in preparation). All streams have progenitors with the same Palomar 5-like properties; a mass of $20,000 M_\odot$ and an effective radius of 20 pc. Palomar 5 was chosen as an exemplary Milky Way globular cluster that features long tidal tails (Odenkirchen et al. 2003). Initially, the clusters were uniformly placed at their orbital apocenters in the 15–150 kpc range. Their initial velocities were also uniformly distributed in the $0.25\text{--}1 V_c$ range, where V_c is the local circular velocity. The angular position and the direction of the velocity vector were randomly chosen on a sphere. The clusters’ orbits were integrated together with the dark matter particles for 6 Gyr with time steps fixed to $dt = 1$ Myr. At each time step, two star particles were released from the cluster’s Lagrange points, with the distance from the cluster given by Equation (4) and the gravitational potential evaluated numerically. To create more realistic streams, the streakline method described above was modified to reflect the spread in velocities of stars escaping the cluster (Lane et al. 2012). An initial radial velocity offset is randomly drawn from a Maxwell–Boltzmann distribution with dispersion equal to the velocity dispersion in the progenitor (Küpper et al. 2008a), which is equal to $\sigma \simeq 2 \text{ km s}^{-1}$ for our choice of clusters. Note that as the stream gets older, its velocity dispersion naturally decreases due to phase-mixing (Helmi & White 1999). The stream stars were treated as massless particles which evolved in a potential created by dark matter particles and globular clusters but which affected neither. The cluster masses remain constant throughout the simulation.

In this study, we analyze a sub-sample of the 256 streams described above, selected to cover the original sample’s range in apocentric distance and orbital eccentricity. Two streams from this sample are projected in an x – z plane in the first column of Figure 3. These streams are representative of the sample, chosen to illustrate how streams differ depending on their orbital phases (near pericenter on top, near apocenter on bottom). The stars forming the leading tail are shown in warm colors, and those in the trailing tail are drawn in cold. In both schemes, the darker colors represent older parts of the stream. The progenitor is marked with a white circle. The top stream is near its pericenter, and thus is very extended. The bottom stream, close to its apocenter, spans a much smaller distance range. The density along the stream is non-uniform in both cases, which is due to the combined effects of the epicyclic motion of stream stars with respect to the progenitor’s orbit (e.g., Küpper et al. 2008b) and the encounters of stream stars with the dark matter subhalos (e.g., Carlberg 2013). The former effect is a consequence of the

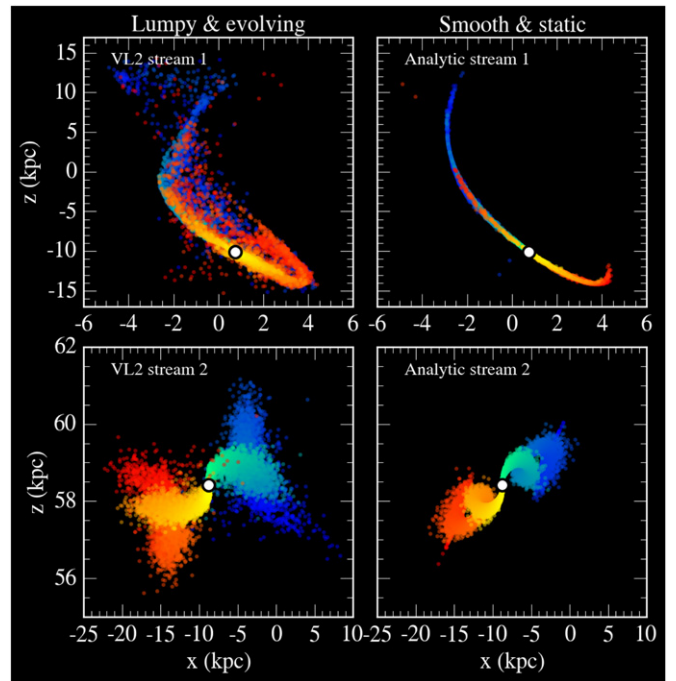


Figure 3. Top: example of a stream close to the pericenter formed in a live dark matter simulation (left) and in an analytic potential (static and smooth, right). The leading tail is shown in warm colors, the trailing tail in cold colors, and the progenitor in white. Darker colors (blue and red) correspond to older parts of the stream, while lighter colors (green and yellow) indicate the most recently stripped stars. Bottom: same as above, but for a different stream near its apocenter. Analytic streams have a general shape similar to the VL2 streams, but are generally narrower.

(A color version of this figure is available in the online journal.)

cluster disruption mechanism, and so it is also present in smooth potentials. The latter effect is inherent to clumpy potentials.

3.2. Analytic Stream Sample

To illustrate the impact of a live dark matter potential on the stream morphology, we contrast our sample with one evolved in a static and smooth potential. This control sample consists of 256 streams with progenitors currently at the same position and orbital phase as in the original sample, but evolved in an analytic potential. For the analytic potential, we choose the best-fit logarithmic and NFW form to the global, present day VL2 potential (black points at 150 kpc in Figure 2). The progenitors’ orbits were traced back for 6 Gyr in the chosen potential. Once the initial position and velocity of the progenitor had been determined, the streakline method was used to create streams analogous to the VL2 sample, with the same progenitor properties as described above.

In the right column of Figure 3, we show smooth streams analogous to the realistically evolved ones in the left column. The range spanned by the streams and their general shape are remarkably similar between the samples. The major difference is the streams’ widths, with streams from the VL2 simulation being wider due to the combined effects of potential evolution and heating by the dark matter subhalos.

4. STREAM MODELING

Our goal is to use streams to infer the galactic potential. In this section, we present a method for measuring the potential using the “forward modeling” of tidal debris from globular

clusters. Our FAST-FORWARD method makes models of a stellar stream in a test potential and compares the resulting 6D phase-space distributions to the observed stream. The probability of an observational data set being generated from these distributions is used as the likelihood in a Markov Chain Monte Carlo (MCMC) algorithm to efficiently search for the underlying probability distributions of the potential parameters. We are primarily interested in the accuracy of the realistic potential recovery when assuming a simplified form. Therefore, we apply the FAST-FORWARD method to our stream samples assuming both positional and velocity information is present, and in the absence of observational errors. However, the method can be naturally extended to include those errors and deal with missing phase-space dimensions. A similar method was used by Varghese et al. (2011) and Gibbons et al. (2014) to model debris originating from more massive progenitors and within the action-angle framework by Bovy (2014) and Sanders (2014). Below, we describe the method in more detail (Section 4.1) and test it on an N -body simulation of a Palomar 5-like stream (Section 4.2).

4.1. The FAST-FORWARD Method

The first step in the FAST-FORWARD method for constraining the potential is to create a model of the stream under consideration. Assuming the current position and velocity of the progenitor are known accurately, we first track the progenitor back along its orbit in a test potential for 6 Gyr, the length of time the streams have been forming in our sample. Then, the streakline method is used to create a realistic stream model. To efficiently model the whole stream sample, we used a fast leapfrog integrator with constant time steps of 1 Myr, which corresponds to the time step used in the VL2 resimulation.

For each model stream created in a test potential, we assess the likelihood of this particular set of parameters by comparing the model stream to the observed one. In this work, the “observed” streams are the streams simulated in the VL2 (Section 3.1) or their analytically evolved analogs (Section 3.2). Naturally, the same method can be applied to streams observed in the Galactic halo and beyond. We assume that there are N member stars observed in each stream with accurate positions and velocities. We call this our data. As a shorthand, we denote this 6D phase space as X and the data points in it as X_n . Within the framework developed by Hogg et al. (2010) and Hogg (2012), the probability of a model point x_k generating a data point X_n is then

$$p(X_n|x_k, \theta, I) = \mathcal{N}(X_n|x_k, \sigma_n^2 + \Sigma_k^2), \quad (5)$$

where I represents any prior information concerning the problem, $\mathcal{N}(x|m, V)$ is a 6-D Gaussian distribution for x given mean m and variance tensor V , σ_n^2 is the observational variance tensor, while Σ_k^2 is a smoothing variance tensor that effectively turns the finite set of model points into a smooth density function in phase space, and θ are the potential parameters. If the progenitor’s orbit and phase information are unknown, they can be included in θ as initial-condition parameters. We treat the smoothing between the model points, Σ_k , as a hyper-parameter in our modeling process and marginalize over it when reporting our results. For simplicity, we assume that the observations are perfect and there are no observational errors. However, this formalism allows for the inclusion of realistic errors by simply setting $\sigma_n \neq 0$.

The likelihood of the data point X_n being represented by the whole of the model stream is calculated by marginalizing out

the model points x_k in Equation (5):

$$p(X_n|\theta, I) = \sum_{k=1}^K P_k p(X_n|k, \theta, I), \quad (6)$$

where we use K model points, equally weighted such that $P_k = 1/K$. The precision of the recovered potential parameters increases with the number of data and model points used, but so does the modeling run time. In order for the likelihood function to be smooth and easily sampled, the number of model points should be several times larger than the number of data points. An empirical study of the combination of these effects on convergence, precision, and run time led to the chosen setup with $N = 100$ data points and $K = 500$ model points. This somewhat arbitrary choice can be easily modified to optimally utilize available computational resources.

Finally, we assume that the data points are independent. The likelihood of all of the data points being drawn from the model distribution is, in this case, a product of their individual likelihoods given by Equation (6):

$$P(\{X_n\}|\theta, I) = \prod_{n=1}^N p(X_n|\theta, I). \quad (7)$$

The natural logarithm of likelihood defined by Equation (7) is used as an input for the affine invariant MCMC algorithm (*emcee*; Foreman-Mackey et al. 2013). The parameter space of θ is explored with 150 walkers. Convergence is obtained after a short burn-in phase of 100 steps. The chains are then restarted around the best-fit value and evolved for another 1000 steps. The convergence was ensured by unchanging values of the mean and standard deviation among the walkers in the third and fourth quartiles. We report the median values and 1σ spreads of the resulting parameter distributions in the following sections.

4.2. N -body Test of the FAST-FORWARD Method

Since both our stream generation and potential recovery are based on the streakline method, it is crucial to establish whether or not our method correctly recovers gravitational potentials using a different stream model. To demonstrate that the FAST-FORWARD method works for non-streakline streams, we test it on an N -body simulation of a Palomar 5-like stream before applying it to the stream data sets described in Section 3.

The test stream was evolved for 6 Gyr in an analytic Milky Way-like potential using a modified version of the direct N -body code NBODY6 (Aarseth 2003). The potential includes contributions from a disk, a bulge, and a flattened axisymmetric NFW halo. When modeling this stream, we fix the baryonic contribution to the gravitational potential and only fit the dark matter halo, which we represent as a three-parameter NFW potential (Equation (2) with fixed orientation angle $\phi = 90^\circ$ and y-axis flattening $q_1 = 1$). The best-fit model, obtained through the FAST-FORWARD modeling described above, accurately recovers the underlying potential. The recovered values of the potential parameters are listed in Table 1 and compared to their respective true values. The halo circular velocity and flattening are recovered very accurately (less than 1% error), while the errors in the scale radius are on the order of $\lesssim 5\%$. This discrepancy in the recovery of global parameters can be explained by the progenitor’s orbit being inside the scale radius of the halo potential. Since the stream is not probing the region where the potential power-law slope breaks, it is less

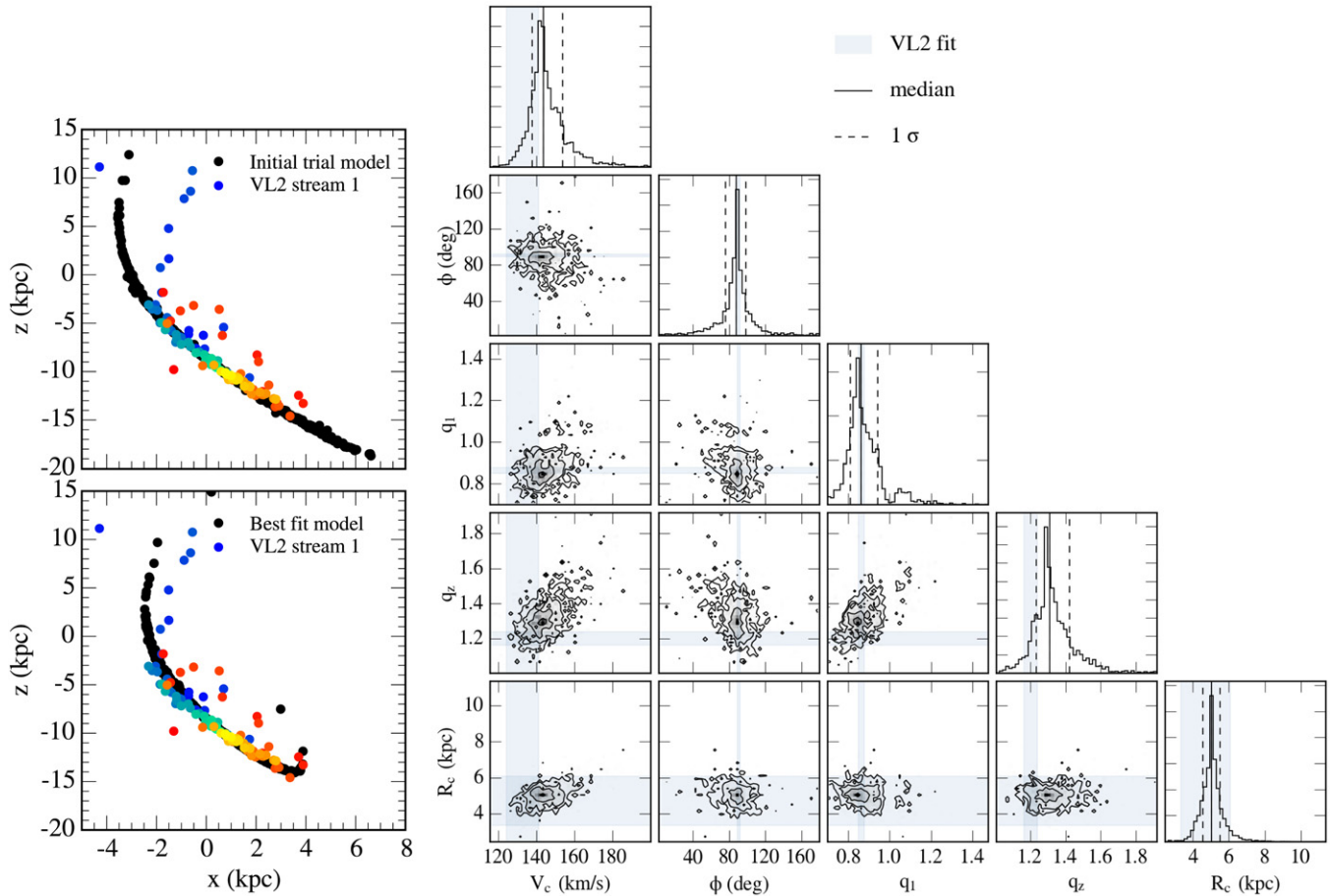


Figure 4. Modeling results for VL2 stream 1 (top of Figure 3). Top left: initially, the trial potential produces an unlikely stream model (black) for the observed stream (colors). Bottom left: the best-fit model is a good match of the observed stream. Right: posterior parameter distributions in one (histograms) and two dimensions (gray contours). The range of values recovered by direct fits to the VL2 simulation is shown in solid blue, while the black solid and dashed lines represent the median and 1σ intervals of the parameter distributions. For this stream, all of the parameter estimates are in 1σ agreement with the expectations from fits to the VL2 data.

(A color version of this figure is available in the online journal.)

Table 1
FAST-FORWARD Potential Recovery Using an N -body Stream

Parameter ^a	True Value	Recovered Value	Ratio ^b
Circular velocity (km s^{-1})	417	418^{+26}_{-17}	1.00
Flattening q_z	0.94	$0.94^{+0.01}_{-0.01}$	1.00
Scale radius r_h (kpc)	36.54	38^{+7}_{-4}	1.04

Notes.

^a Parameters of an NFW halo (as defined by Equation (2), assuming a fixed orientation angle $\phi = 90^\circ$ and y -axis flattening $q_1 = 1$).

^b Recovered/True.

sensitive to the value of the scale radius. This test shows that the FAST-FORWARD method can be used to reliably recover potential parameters, but also indicates that some streams are less well suited for recovering halo parameters than others.

5. RESULTS

In this section, we present the results from applying the FAST-FORWARD method for potential recovery to our stream samples. We start by illustrating the fitting procedure and posterior parameter distributions obtained using a single VL2 stream in Section 5.1. The same procedure is then repeated for all of the streams and the summary results are presented in

Section 5.2 for analytic streams and Section 5.3 for the VL2 stream sample.

5.1. Potential Recovery Example Using a Single Stream

We first apply the FAST-FORWARD method to a single VL2 stream, recover the underlying potential assuming a logarithmic form, and show detailed constraints on the potential parameters. To illustrate how streams can discriminate between the different logarithmic potentials, we show a VL2 stream observed in the x - z plane in the left panel of Figure 4. On top, we compare the stream (colors) to a model (black) created in a trial potential. This model is not a good representation of the observed VL2 stream because it extends past the observed range. More precisely, many of the model stars are far away from the observed stars, so this model has a low likelihood (calculated using Equation (6)). The MCMC sampler searches the parameter space for a potential that maximizes the likelihood, and such a best-fit model is shown on the bottom of Figure 4. It matches the observed stream well.

Once the best-fitting potential parameters are located, the MCMC sampler explores the surrounding parameter space to recover the probability distributions of parameters. On the right of Figure 4, we show the posterior probability distributions of potential parameters obtained by modeling the VL2 stream shown on the left. Panels on the diagonal show the

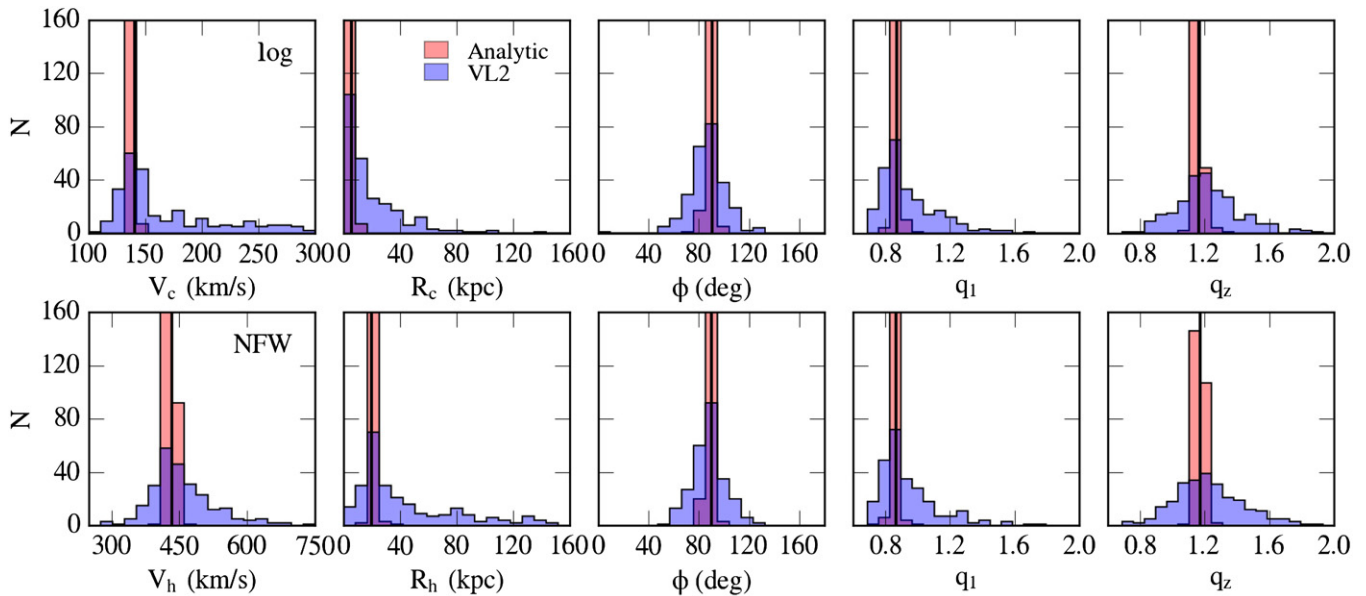


Figure 5. Distributions of best-fit parameter values across a sample of 256 streams, logarithmic on top and NFW on bottom. Analytic streams are shown in red while the VL2 streams are blue. The true parameter values are marked with vertical black lines. The distributions of both the analytic and the VL2 streams peak at the true values, with the VL2 streams having a larger spread around the mean.

(A color version of this figure is available in the online journal.)

one-dimensional parameter distributions as histograms overplotted with the median (solid vertical line) and 68% uncertainty limits (dashed vertical lines). Subsequently, we quote the median as our best estimate of a parameter.

The range of parameters expected from fitting the present day VL2 potential directly (full radial range in Figure 2, Section 2.2) is shown as the shaded blue regions in Figure 4. For this stream, all of the recovered parameters are within 1σ of the expected values. Additional information about the covariance between the potential parameters is presented on 2D probability distributions (lower left panels). The color of the contours corresponds to the probability density, with darker gray corresponding to more probable regions of the parameter space. All the walkers have converged to a single solution and there are no indications of multimodality in the parameter distributions. There is some correlation between circular velocity and core radius, while other parameters are mostly uncorrelated for this stream. In the next sections, we present our results for the whole sample of analytic and VL2 streams.

5.2. Streams Evolved in Analytic Potentials

We proceed to apply the FAST-FORWARD potential recovery method on our analytic stream samples, where the potential is smooth and static and its form is known. One set of streams was created in a logarithmic potential and another in an NFW potential (Section 3.2). Each stream from these samples was modeled individually in the appropriate potential as described above, and here we report the recovered potential parameters for the whole sample.

The distribution of median potential parameters recovered using all analytic streams individually is shown with the red histograms in Figure 5. The streams created in a logarithmic potential are on top, while those formed in an NFW potential are on the bottom. The input parameter values are marked on each panel with vertical solid lines. In both potentials, most of the streams recover the true values. The distributions' modes coincide with the true values while the medians are within 3%

for all of the parameters. The largest discrepancies are seen in core radius for the logarithmic potential and in scale radius for the NFW potential. Fractional errors in other parameters are smaller than 10% for more than 98% of the streams in both potentials.

We conclude that true potential parameters are accurately recovered using the FAST-FORWARD method on streams evolved in an analytic potential. In addition, the method provides uncertainties for the recovered parameters. The typical 68% uncertainty range in a parameter's distribution function, which we also call the 1σ error, is small, on the order of $\lesssim 10\%$. These errors are only slightly overestimated, since $\sim 90\%$ of streams in our sample recover true parameter values on the 1σ level. Combined, these results reveal no biases inherent to the FAST-FORWARD potential recovery method. Its estimates of potential parameters using analytic streams on a variety of orbits are both accurate and precise.

5.3. Streams Evolved in the VL2 Potential

In this section, we present potential recovery results for streams evolved in a clumpy and evolving VL2 potential, but modeled in smooth and static potentials. The distributions of potential parameters are overplotted as blue histograms in Figure 5 (logarithmic potential on top, NFW on bottom). The parameter values marked with vertical solid lines were initially obtained by fitting the logarithmic/NFW forms directly to the current VL2 potential inside 150 kpc.

Similar to the analytic streams, the modes of the parameter distributions for the VL2 streams are centered on the true values of the global, present day VL2 potential. In this case, however, the distributions are not as symmetric, and so their medians are within $\sim 10\%$ of the true values. The largest deviation is seen in the scale radius for the NFW profile, driven by overestimates coming from streams on orbits far beyond the true scale radius, and thus less sensitive to this parameter. In general, the distributions are wide and only 40%–60% of the VL2 streams recover parameters within 10% of the true values

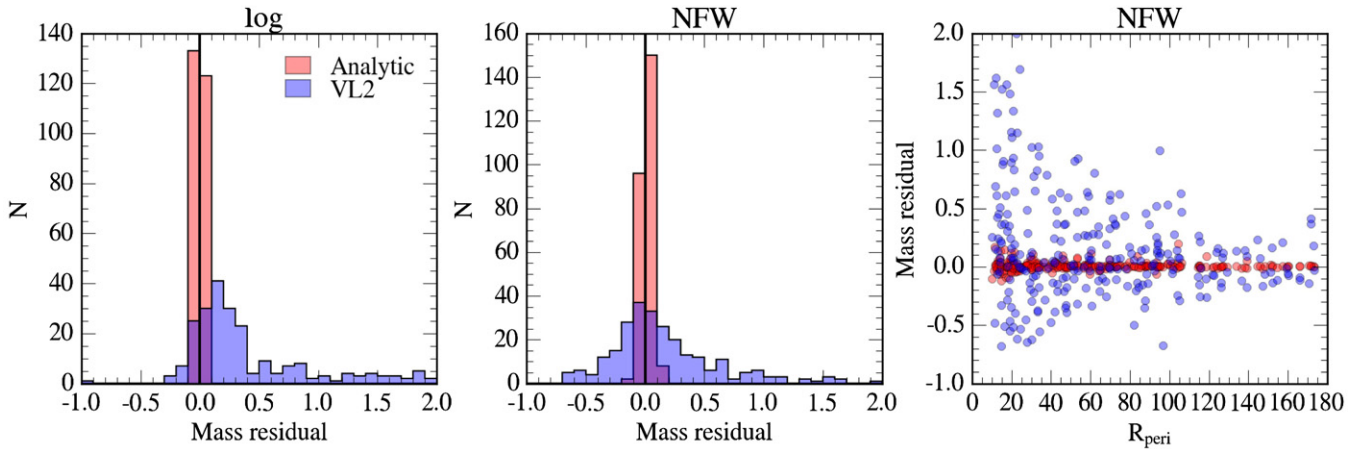


Figure 6. Distributions of residuals in mass within 150 kpc when assuming the logarithmic (left) and NFW potentials (middle). Analytic streams (red) recover the true mass very accurately in both potential forms, and their residuals are centered on zero (black vertical line). VL2 streams (blue) are biased toward more massive halos in the logarithmic form, and lower mass halos in the NFW potential. Large residuals are more likely for streams with smaller pericentric distances in both potentials; to reduce clutter, we present only NFW results (right).

(A color version of this figure is available in the online journal.)

(compared to 90% of analytic streams). Correspondingly, the uncertainties in the recovered parameters are also larger for VL2 streams, on average 15%. However, we find that our errors are an accurate assessment of measurement uncertainties, as $\sim 50\%$ of the streams recover true parameters within 1σ .

The outliers in the recovered parameters could be caused by an inadequate potential form used to model those streams. For such a stream, no good fit would be found, and all of our models would have low likelihood values. Since we used the same setup for all of the streams, it is straightforward to compare the maximum likelihoods among the stream samples. However, we find no indication that the outliers are due to low likelihood fits. By excluding the 20% of the lowest-likelihood streams from the sample, the recovered parameter distributions retain the modes of the full samples, while the changes to the distributions themselves are minimal. The large outliers are truly biased streams, and not recognizable as such from the fitting procedure.

Although some streams are individually biased, collectively they recover global properties of the present day VL2 halo (blue histograms on Figure 5), even when we assume a smooth and static analytic potential in their modeling. This collective behavior makes it reasonable to assume that the potential constraints obtained by modeling multiple streams simultaneously will narrow down the parameter distributions. Indeed, Deg & Widrow (2014) have shown that potential recovery with two streams improves on the individual results for the halo shape parameters. The Milky Way streams have so far been modeled only individually, yielding different estimates for the dark matter halo shape (e.g., Law & Majewski 2010; Koposov et al. 2010). Joint stream constraints will be crucial in measuring such a complex potential.

5.4. Mass Estimates Using Streams

Mass is the most basic property of dark matter halos. So far, we have demonstrated how tidal streams can constrain the gravitational potential, and in the following we will use these constraints to determine the mass of a halo. For brevity, we focus on mass within 150 kpc, but similar conclusions can be reached for any of the global halo parameters. To get the halo mass, we follow the procedure outlined in Section 2.2; first, we calculate the density field from the potential using the Poisson equation

and then integrate it out to 150 kpc. This limit was chosen to match the radial extent of our stream sample.

Analytic streams accurately measure the halo mass. Their mass residuals, defined as the difference between the stream estimate of mass and the actual value, normalized to the actual mass, are shown in red in Figure 6 (left panel for logarithmic potential, middle for NFW). The distributions have the mode at 0% error in both the logarithmic and NFW potentials. These distributions are also very narrow, with 100% of the logarithmic and 96% of the NFW streams having less than 10% error in mass. An accurate mass determination with analytic streams is not surprising, given their accurate recovery of individual potential parameters. However, it is encouraging that the mass, a calculated global quantity, is recovered with the same precision as the parameters being solved for by stream modeling.

Residuals in mass calculated using the VL2 streams are shown in blue histograms in Figure 6. These distributions are wider than the corresponding analytic ones, as has already been seen in distributions of potential parameters. Unlike the analytic stream sample, VL2 mass estimates are biased to higher masses when assuming a logarithmic potential (mode at 20%), and to slightly lower masses when assuming an NFW potential (mode at -5%). These values are comparable to discrepancies in mass predicted by logarithmic and NFW fits to the VL2 potential directly (18% and -6% , respectively). Therefore, assuming an analytic potential limits the dark matter halo mass measurement to an accuracy of 5%–20%, depending on the choice of analytic parameterization.

Apart from the distribution median displacement in the VL2 sample, there is also a tail of high-mass outliers. These correspond to streams with high recovered circular velocities and scale radii. We explore the origin of these outliers by showing mass residuals for VL2 streams, modeled in an NFW potential as a function of the closest pericentric passage of their progenitors in the right panel of Figure 6. All of the high-mass outliers originate from streams with small pericentric radii, while the streams that never enter the inner ~ 70 kpc have mass residuals smaller than 50%, motivating the search for streams at large galactocentric radii. VL2 streams modeled in a logarithmic potential show a similar trend, and so we omit them from this panel to reduce clutter. The fraction of streams with extremely large mass estimates is small; most streams in

the inner region of VL2 produce errors in mass smaller than 30%. This is encouraging for measuring the mass of the Milky Way, since all of the Galactic cold streams have so far been discovered within 30 kpc.

6. DISCUSSION

6.1. Comparison of VL2 and Analytic Streams

The presence of substructure in realistic dark matter halos is the main qualitative difference between the VL2 and analytic stream models. Accretion of subhalos over cosmic time causes the potential of the host halo to evolve, and their subsequent motion within the halo adds to a time dependent, clumpy potential, which is only approximately described by the analytic forms. In potential recovery using streams, the effects of (1) substructure, (2) potential evolution, and (3) inadequate potential form in stream modeling result in wider parameter distributions for the VL2 sample than the analytic case. Below, we look into each separately.

Substructure. Although they contribute about 10% of the dark matter in Milky Way-mass halos, subhalos occupy only a small fraction of the total halo volume. To assess their impact on the streams, we count the total number of their encounters in the 6 Gyr of evolution. Assuming that an encounter happens when a subhalo passes within ~ 1.5 kpc of a stream, the streams in our sample have had on average 2.4 encounters with dark subhalos more massive than $10^5 M_\odot$. Although no clear trend between the error in mass estimate and the number of stream-subhalo encounters is observed, streams that experienced encounters with massive subhalos ($M_{\text{sub}} \gtrsim 10^7 M_\odot$) are more likely to result in overestimates of halo mass. These massive encounters can disrupt parts of the stream, making the coherent part appear shorter. In general, if all the other parameters are kept fixed, then globular clusters evolved in a more massive halo have smaller tidal radii and consequently develop shorter tidal tails. This explains the tendency to overestimate halo mass using streams which have encountered massive subhalos. The stream length also depends on its age, which is unknown in a realistic case, so this effect of substructure might be even less apparent. In summary, while encounters with subhalos can significantly alter the stream and affect subsequent potential recovery, this only happens for a small subset of streams ($\lesssim 15\%$ have had more than five encounters). Hence, halo substructure only marginally contributes to the spread in recovered potential parameters across the VL2 stream sample.

Evolution. The continuing accretion of substructure on dark matter halos produces potential evolution that is rarely modeled when attempting to recover gravitational potentials (however, see Buist & Helmi 2014 for a recent formulation that takes into account growth of the scale mass and radius). As shown in Section 2.3, the shape of the VL2 halo has experienced little change in the last 6 Gyr, but both mass and scale radius have increased by 5%–10%. Although all of the streams have been exposed to the growth of the host dark matter halo, most recover present day values of scale mass and radius. Similarly, Peñarrubia et al. (2006) found that streams adapt to adiabatic potential changes, and so in the case of smooth and slow potential evolution, the present day information on streams only yields constraints on the present day potential. However, the change of the potential parameters in the last 6 Gyr makes up for only $\sim 20\%$ of the spread in potential recovery results using the VL2 stream sample. The spread in obtained parameters is only in part due to potential evolution.

Potential form. Another aspect of simulated dark matter halos that is poorly captured by analytic models is the radial changes in flattenings and orientation angles (e.g., Allgood et al. 2006). When fitting such a halo with a potential that only has global parameters for flattening and orientation, the output depends on the radial extent of the available data, as shown in Section 2.2. The radial variations in the VL2 shape parameters are on the order of 20%–50% of the FWHM spread in recovered shape parameters across our stream samples. Thus, an inadequate potential form used in modeling streams can also only partly explain the observed spread in recovered parameters.

Each of these effects contributes to the spread in recovered potential parameters. We estimate that the role of direct encounters with subhalos is minor, but potential evolution and a simplistic potential form individually contribute to the spread on a 20% level. Combined, radial variations in the VL2 shape parameters in the last 6 Gyr make up for more than $\sim 70\%$ of the spread in recovered parameters. The spread in the recovered distributions of potential parameters appears to be driven by the potential evolution and by streams probing different radii in the host halo. Studying these effects in dedicated simulations with only substructure or only smooth halo growth will provide a clearer picture on the impact each has on potential recovery.

The combined effects of potential clumpiness, evolution, and incorrect form result in biased total mass estimates when replacing the realistic potential with an analytic approximation. The measurement is equally biased when constraining the potential by applying the FAST-FORWARD method on our VL2 stream sample, or directly fitting analytic potentials to the VL2 particle data. We note that many different classes of potential recovery methods using tidal streams have been proposed recently (e.g., Sanders & Binney 2013; Price-Whelan & Johnston 2013; Deg & Widrow 2014; Bovy 2014; Sanders 2014). While the details of potential recovery using these other methods may be differently affected by the assumption of a smooth and static potential, our conclusions regarding the mass bias should remain valid for all potential recovery methods based on this assumption.

6.2. Implications for Known Milky Way Streams

In this section, we discuss the error in mass estimates induced by the assumption of a smooth and static potential when modeling real Milky Way streams. While Galactic streams are influenced by both dark matter and baryons, the streams in our sample were evolved in dark matter-only potentials. Hence, we can only assess the errors in the dark matter component, which will make up only part of the error budget in the estimates of the total galaxy mass.

The most prominent streams originating from globular clusters in the Milky Way are the Palomar 5 tidal tails and GD-1. Based on kinematic observations of the cluster and the position of its tails, Odenkirchen et al. (2001) estimated the Pal 5 pericenter and apocenter distances of 7 kpc and 19 kpc, respectively. Similarly, Koposov et al. (2010) used 6D information on the GD-1 stream to obtain the pericenter and apocenter of 14 kpc and 26 kpc, respectively. These values are uncertain due to observational errors and dependent on the choice of the Galactic potential. We constructed a conservative sample of Palomar 5 and GD-1 analogs from our VL2 sample by requiring that their peri- and apocenters are matched within 5 kpc. The median mass residual of these samples is small ($\lesssim 5\%$), but the dispersion within the sample is $\sim 50\%$. In other words, while these streams collectively provide an accurate measurement of

the host halo mass, results for the individual streams can be heavily biased, even when the streams are observed in full 6D phase space and the observational errors are negligible. Missing data dimensions (e.g., distances or proper motions) and larger errors are likely to further relax constraints on the gravitational potential and possibly increase the bias.

Total halo mass measurements using tidal streams evolved in a complex, dark matter-only potential are more accurate when distant ($d > 70$ kpc) streams are employed (see Figure 6). In a realistic Galactic potential, dark matter halo potential recovery using streams with small pericenters is further impeded by the disk. Although the effect of dark matter subhalos on streams at small galactocentric radii ($d < 20$ kpc) is expected to be lower due to their depletion by the disk (D’Onghia et al. 2010), the potential is dominated by the disk, and so the streams at those distances might not be very sensitive to the global halo shape (Koposov et al. 2010). Consequently, distant streams are also expected to provide a better handle on the global halo potential in a galactic potential. While there are no distant cold streams known at the moment, the outer Milky Way halo will be mapped down to faint magnitudes in the LSST era (Ivezic et al. 2008). Any streams discovered in upcoming deep surveys will provide valuable anchors for the total mass, and combined with the nearby streams, they will improve constraints of the entire Galactic potential.

7. SUMMARY

In this study, we revisited gravitational potential recovery using tidal streams. In particular, we tested the performance of analytic potentials for representing a realistic dark matter halo. In doing so, we developed a novel method to forward model streams originating from globular clusters, with realistic stream morphologies and stream–orbit offsets. We applied this method to two samples of streams, one evolved in an analytic and the other in an N -body potential. We modeled these streams in a smooth and static potential, and produced estimates of potential parameters and total mass within 150 kpc.

The true potential parameters are accurately recovered by our analytic stream samples. In a realistic, N -body halo, these parameters change with time and radius. While the streams evolved in such a halo collectively recover the present day, global values, the distributions of individual estimates are wide. The width of these distributions is comparable to the parameter variations in the radial range probed by the streams during their evolution. We estimate that the impact of substructure on this widening is marginal due to scarcity of streams that have been significantly influenced by subhalos.

The global halo mass is also accurately measured using streams in the analytic case but biased in a realistic halo. This bias stems from the analytic potential only approximating the true potential form, and is independent of the potential recovery method. The amount of bias depends on the choice of such a potential, and is lower for the NFW ($\sim 5\%$) than for the logarithmic potential ($\sim 20\%$). In addition to the overall bias, streams passing through small galactocentric distances are shown to have larger errors in mass estimates. Streams beyond 70 kpc typically measure the total mass with 30% accuracy.

We are entering an exciting age for Galactic studies. Near-future missions like *Gaia*, DES, and LSST will accurately map a large volume of the Milky Way, providing a detailed look into streams populating a large range of galactocentric distances. Using the methods outlined in this paper, we will be ready to model these streams and accurately pin down the Milky Way

mass. This will have profound implications for placing the Milky Way in a cosmological context.

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