The Astrophysical Journal, 596:67–71, 2003 October 10
© 2003. The American Astronomical Society. All rights reserved. Printed in U.S.A.

DISCOVERY OF A CLUSTERED QUASAR PAIR AT $z \approx 5$: BIASED PEAKS IN EARLY STRUCTURE FORMATION[1]

S. G. Djorgovski
Palomar Observatory 105-24, California Institute of Technology, Pasadena, CA 91125; george@astro.caltech.edu

D. Stern
Jet Propulsion Laboratory, Pasadena, CA 91109; stern@zwolfkinder.jpl.nasa.gov

and

A. A. Mahabal and R. Brunner[2]
Palomar Observatory 105-24, California Institute of Technology, Pasadena, CA 91125;
aam@astro.caltech.edu, rb@astro.caltech.edu
Received 2001 September 24; accepted 2003 June 16

ABSTRACT

We report the discovery of a quasar at $z = 4.96 \pm 0.03$ within a few Mpc of the quasar SDSS 0338+0021 at $z = 5.02 \pm 0.02$. The newly found quasar has SDSS i and z magnitudes of ≈21.2, and an estimated absolute magnitude $M_B \approx -25.2$. The projected separation on the sky is 196″, and the redshift difference $\Delta z = 0.063 \pm 0.008$. The probability of finding this quasar pair by chance in the absence of clustering in this particular volume is $\sim 10^{-4}$ to 10^{-3}. We conclude that the two objects probably mark a large-scale structure, possibly a protocluster, at $z \approx 5$. This is the most distant such structure currently known. Our search in the field of 13 other QSOs at $z \gtrsim 4.8$ so far has not resulted in any detections of comparable luminous QSO pairs, and it is thus not yet clear how representative is this structure at $z \approx 5$. However, along with the other evidence for clustering of quasars and young galaxies at somewhat lower redshifts, the observations are at least qualitatively consistent with a strong biasing of the first luminous and massive objects, in agreement with general predictions of theoretical models. More extensive searches for clustered quasars and luminous galaxies at these redshifts will provide valuable empirical constraints for our understanding of early galaxy and structure formation.

Subject headings: cosmology: observations — galaxies: formation — quasars: general — quasars: individual (SDSS 0338+0021, RD 657)

1. INTRODUCTION

Quasars at large redshifts represent a powerful probe of structure formation in the early universe. Dissipative mergers and tidal interactions during the early stages of galaxy assembly may be fueling both bursts of star formation and early active galactic nucleus (AGN) activity (see, e.g., Silk & Rees 1998; Franceschini et al. 1999; Monaco, Salucci, & Danese 2000; Kauffmann & Haehnelt 2000; Granato et al. 2001). This fundamental connection is supported by the remarkable correlations between the masses of central black holes in nearby galaxies and the velocity dispersions and luminosities (~masses) of their old, metal-rich stellar populations (Magorrian et al. 1998; Ferrarese & Merritt 2000; Gebhardt et al. 2000), and by the high metallicities observed in the high-z quasar spectra (Hamann & Ferland 1999 and references therein). Quasars can also be used directly to probe evolution of large-scale structure out to high redshifts, as demonstrated clearly by the 2dF QSO Redshift (2QZ) survey (Croom et al. 2001; Hoyle et al. 2002).

There should also be a fundamental connection between the formation of galaxies and the large-scale density field in the early universe. The highest density peaks, where presumably the first luminous objects formed, should be strongly clustered, because of biasing (Kaiser 1984). This is a generic and robust prediction for essentially every model of structure formation, independent of any astrophysical details of galaxy formation.

Luminous high-z quasars are likely situated in massive hosts (Turner 1991). Such massive halos should be rare and might be associated with ~4 to 5 σ peaks of the primordial density field (Efstathiou & Rees 1988; Cole & Kaiser 1989; Nusser & Silk 1993). High-z quasars can thus be used as biased tracers of the early large-scale structure, possibly marking the cores of future rich clusters.

A search for protoclusters around known high-z objects such as quasars thus provides an important test of our basic ideas about the biased galaxy formation. We have conducted a search for clustered protogalaxies and AGNs in the fields of selected quasars at $z > 4$. Preliminary results from our program have been described by Djorgovski (1999) and Djorgovski et al. (1999), and a complete account will be presented elsewhere.

Here we present the discovery of a clustered quasar pair at $z \approx 5$, which we interpret as a signature of a primordial large-scale structure, possibly a core of a forming rich cluster. This is the most distant large-scale structure currently known. One of the quasars, SDSS 0338+0021, was

---

[1] Based on observations obtained at the W. M. Keck Observatory, which is operated by the California Association for Research in Astronomy, a scientific partnership among California Institute of Technology, the University of California and the National Aeronautics and Space Administration.

[2] Current address: Department of Astronomy, University of Illinois, Urbana, IL 61801.

discovered by Fan et al. (1999); the other was found in our much deeper search in its vicinity.

2. OBSERVATIONS AND DATA REDUCTIONS

Deep images of the field were obtained at the Palomar 200 inch Hale telescope using the prime-focus Cosmic imager (Kells et al. 1998). The instrument field of view (FOV) is $9'.7$ square, with $0''.286$ pixels. Multiple dithered exposures totalling 1800, 3000, and 7200 s were obtained in Gunn r, i, and z filters respectively on 2000 November 29 and 30 UT, and of 3600, 1800, and 2400 s in Gunn g, r, and z filters respectively on 2000 December 31 UT, all in good conditions. The data were reduced using standard procedures. Limiting magnitudes (3 σ in a $3''$ aperture) are $r_{\rm lim} \approx 25.9$, $i_{\rm lim} \approx 25.5$, and $z_{\rm lim} \approx 23.1$ mag in the SDSS ($\sim$AB$_\nu$) photometric system. Multicolor selection was used to identify candidates for objects at $z \gtrsim 5$, as illustrated in Figure 1. Details of the data reduction and candidate selection procedure will be presented elsewhere.

The first set of imaging data was used to select candidates for multislit spectroscopy, and spectra of several objects, including the known QSO (SDSS 0338+0021), were obtained at the Keck I 10 m telescope on 2000 December 29 UT in good conditions. We used the LRIS instrument (Oke et al. 1995) with a 400 line mm^{-1} grating ($\lambda_{\rm blaze} = 8500$ Å) and a GG495 order-sorting filter, through $1''.2$ wide slitlets, with a mean dispersion in the wavelength region of interest of $\approx$1.86 Å pixel^{-1}. Two exposures of 1800 s each were obtained through a single slitmask at a P.A. = 340°, close to the parallactic angle at the time. Exposures of arc lamps were used for wavelength calibration, with the flexure shifts corrected using the measurements of selected night sky lines. An average of archival response curves for this grating was used for the flux calibration.

Only one high-priority color-selected candidate, which we designated RD 657 (for "red dropout" and a serial number in our CCD object catalog), could be accommodated on this slit mask. It turned out to be a quasar at $z \approx 5$. The position of this object (J2000) is

$$\alpha = 03^{\rm h}38^{\rm m}30^{\rm s}.03\ , \quad \delta = +00^\circ18'40''.4\ ,$$

giving the projected separation from SDSS 0338+0021 of $196''$. Setting the photometric zero points on the magnitudes of SDSS 0338+0021 published by Fan et al. (1999), i.e., $r = 21.70$, $i = 19.96$, and $z = 19.74$, we find that the magnitudes of the new QSO in the same SDSS system are $r = 23.01$, $i = 21.24$, and $z = 21.16$ mag, with estimated uncertainties of $\sim$0.1 mag, making it about 3.5 times fainter than the SDSS QSO. Figure 2 shows the finding chart for the field, with both quasars marked.

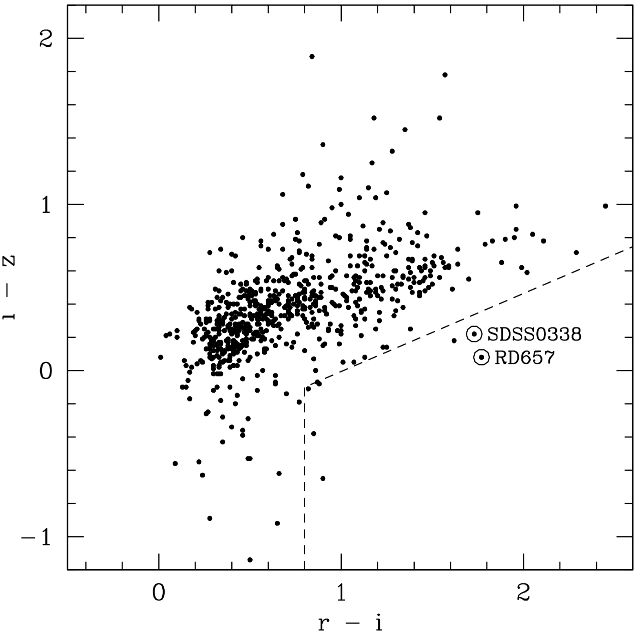

FIG. 1.—Color-color diagram for all objects detected in the P200 imaging field, with the two quasars labeled. The dashed line shows our color selection boundary. The two quasars are labeled.

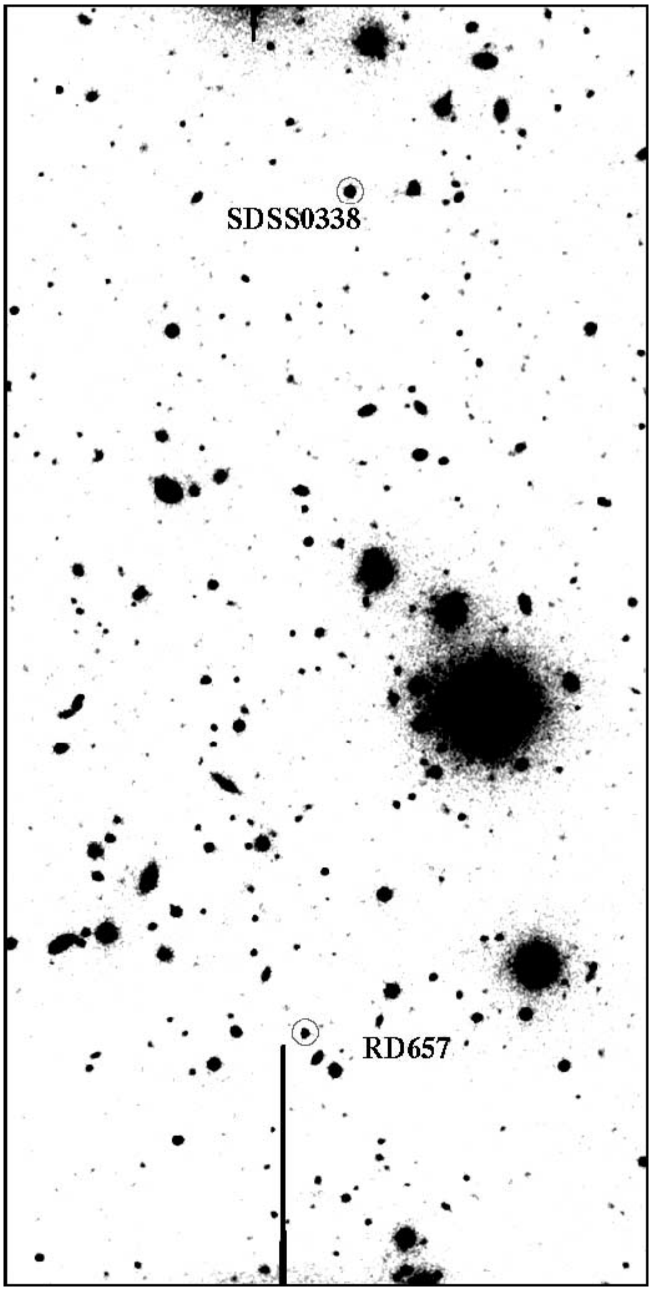

FIG. 2.—Finding chart for the field, from the P200 i-band images. The field size is $150'' \times 300''$, with north up and east to the left. The two quasars are labeled.

Additional long-slit spectra of both objects were obtained using LRIS on 2000 December 31 UT in good conditions. We used a 600 line mm^{-1} grating ($\lambda_{\rm blaze} = 7500$ Å) through a 1."0 wide slit, covering the wavelength range $\lambda \sim 4900$–7450 Å, with a mean dispersion in the wavelength region of interest of $\approx$1.255 Å pixel^{-1}. Two exposures of 1200 s were obtained for SDSS 0338+0021, and four exposures of 1200 s for RD 657, with the object dithered on the slit between the exposures. The slit P.A. was always close to the parallactic angle. Arc lamps were used for wavelength calibration, with the flexure shifts corrected using the measurements of selected night sky lines, and exposures of standard star Hiltner 600 were used for the flux calibration.

We rebinned both grating data sets to a common sampling grid of 2 Å pixel^{-1}, smoothed with Gaussians with $\sigma = 2$ Å (lower than the optimal smoothing filter for these data, thus resulting in no loss of information). Figure 3 shows the combined spectra of the two objects. The redshifts determined from the emission lines (taking into account the absorption of the blue side of Lyα) are $z_e = 5.02 \pm 0.02$ for SDSS 0338+0021 and $z_e = 4.96 \pm 0.03$ for RD 657, with the large uncertainties due to the intrinsic difficulty of centering of broad emission lines. We also note the presence of a Lyman limit system at $z_{\rm LLS} = 4.99$ in the spectrum of the brighter QSO.

While absolute values of the redshifts cannot be measured very precisely, we used a simple cross-correlation to evaluate the redshift difference between the two objects. We excluded the portions of the spectra blueward of the centroids of the Lyα lines, since the differences in the IGM absorption between the two lines of sight could significantly affect the results. We obtain $\Delta z = 0.063 \pm 0.008$, which corresponds to the rest-frame velocity difference of $\Delta v = 3150 \pm 400$ km s^{-1} (these are the formal errors; possible systematic errors due to the mismatch of the spectra are hard to estimate precisely, but could be of the same order). Looking for correlated absorption systems in the two spectra would require data with a higher S/N and a higher resolution.

We obtained preliminary spectra of a number of other, fainter, color-selected candidates in the field. While the results are still inconclusive, none of them are luminous AGNs.

3. DISCUSSION AND CONCLUSIONS

In what follows, for the sake of consistency with the previous work we will use the "standard quasar cosmology" with $H_0 = 50$ km s^{-1} Mpc^{-1}, $\Omega_0 = 1$, and $\Lambda_0 = 0$.

In this cosmology, the observed angular separation of 196" corresponds to a projected spatial separation of

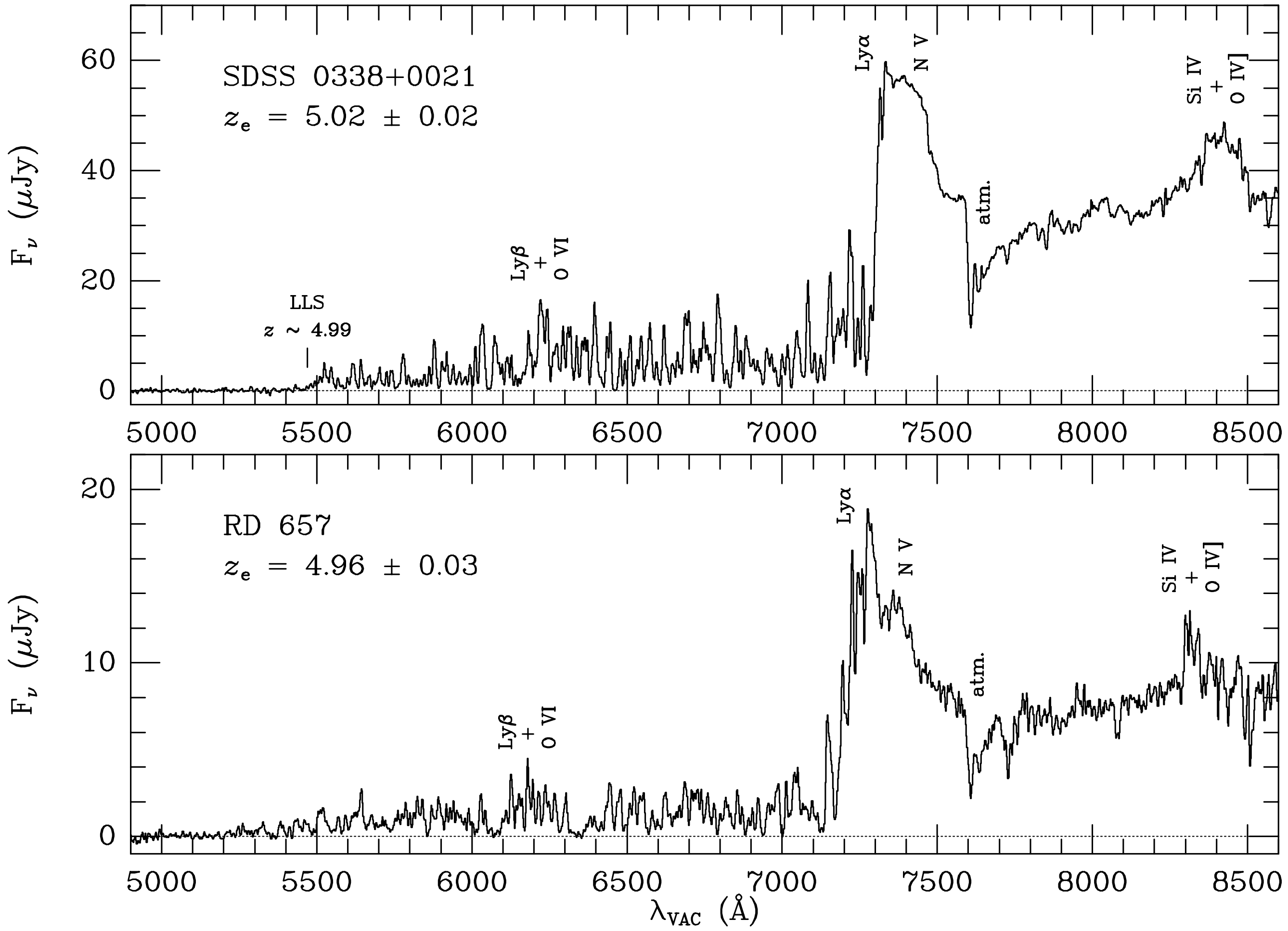

FIG. 3.—Combined Keck/LRIS spectra of the two quasars, with the main features labeled

1.126 proper Mpc, or 6.744 comoving Mpc at $\langle z \rangle = 4.99$. If we assume that the observed $\Delta z = 0.063 \pm 0.008$ is due entirely to cosmological expansion, the radial separation is 4.27 ± 0.56 proper Mpc (25.6 ± 3.6 comoving). The quadratic sum of the two suggests a spatial separation of the two quasars of 4.4 ± 0.6 proper Mpc (26.5 ± 3.6 comoving). However, this interpretation of the observed redshift difference as being due only to the Hubble expansion is uncertain. If we did not have any radial separation information, we could estimate the spatial separation from the projected separation alone: statistically, for a pair randomly oriented in space, the ratio of the two is $\pi/2$, so that the most probable spatial separation corresponding to our projected separation is 1.77 proper Mpc (10.6 comoving). In the discussion below (which is similar to that by Schneider et al. 2000) we consider both of these possibilities, i.e., comoving separations of 26.5 Mpc (the cosmological expansion model) and 10.6 Mpc (the deprojection model).

Individual quasars represent rare events in the general population of galaxies at any redshift; how likely is it to have two of them so close together? In order to estimate the probability of finding such a close quasar pair by chance at this redshift, we use the evolving QSO luminosity function (QLF) by Fan et al. (2001). For SDSS 0338+0021, we adopt the absolute blue magnitude $M_B = -26.56$ from Fan et al. (1999). Using the mean i and z magnitude difference of $\approx$1.35 mag, we estimate $M_B = -25.2$ for RD 657. The Fan et al. (2001) QLF gives a number density of 4.35×10^{-8} Mpc^{-3} for quasars with $M_B \leq -25.2$ at this redshift. The comoving volumes enclosed by spheres with radii equivalent to the physical separations of the quasars of 10.6 and 25.6 Mpc are 4.97×10^3 Mpc^3 and 7.01×10^4 Mpc^3, respectively.

The first question we can ask is, what is the a priori probability of finding such a close pair of QSOs at this redshift, regardless of the particular survey strategy? Assuming a Poissonian distribution of quasars, the probabilities of finding two QSOs at these luminosities in these volumes are 2.3×10^{-8} and 4.6×10^{-6}, respectively. A similar reasoning was applied to the two serendipitously discovered quasar pairs at $z > 4$ (Schneider, Schmidt, & Gunn 1994a, 2000).

However, the volume in which we found this pair was not selected at random: it is centered on a previously known QSO. We can thus ask an alternative question, which is specific for our experiment, namely, what is the probability that another QSO is found at random in this particular volume? (We note that the same answer would apply whether or not there is an already known QSO in its center.) The probabilities then become 2.2×10^{-4} and 3.0×10^{-3}, respectively. Thus, it is still unlikely that this pair represents a chance occurrence, suggesting that there is some physical clustering present.

We note that as of early 2003, we observed a total of 14 fields of QSOs at $z \gtrsim 4.8$ covering the FOV $\sim 25'$ diameter ($\sim$52 comoving Mpc in the cosmology used here) to a comparable depth. Our spectroscopic follow-up is still incomplete, and thus it would be premature to include this additional volume in the present computation, but to date no other cases of comparably bright QSO pairs have been found, suggesting that this system must be a relatively rare event. A full analysis and estimates of the QSO clustering and bias will be presented in a future paper, once the survey is complete.

We also note that in a deeper Keck survey of $\sim$20 QSOs at $z \sim 4$–4.7, but covering a smaller FOV, $\sim 6 \times 8$ arcmin^2 ($\sim 12 \times 16$ comoving Mpc^2) we found at least two AGN companions to the known, bright QSOs, with sub-Mpc separations (not gravitational lenses), as well as a number of clustered faint galaxy companions (Djorgovski 1999; Djorgovski et al. 1999; S. G. Djorgovski et al., in preparation).

The clustering strength cannot be meaningfully measured from a single pair of objects in a survey of as yet poorly defined coverage. With this caveat in mind, the small probability of a random occurrence of such a pair implies an effective r_0 that could be considerably greater than the observed pair separation, i.e., $r_0 \gg 10$ comoving Mpc. At low redshifts, there is some spread of results between different groups (see, e.g., Boyle & Mo 1993; Croom & Shanks 1996; Sabbey et al. 2001), but most authors find that the observed clustering length of quasars is comparable to that of galaxy groups, $r_0 \sim 10$–20 Mpc (Bahcall & Chokshi 1991; see Hartwick & Schade 1990 for a review and references). A standard parameterization of the evolution of clustering in comoving coordinates is given by the formula

$$\xi(r, z) = \left(\frac{r}{r_0}\right)^{-\gamma} (1+z)^{-(3-\gamma+\epsilon)} ,$$

where $\gamma \approx 1.8$ and ϵ is the evolution parameter. For our chosen cosmology, the expected value for the CDM cosmogony is $\epsilon \approx 0.8$, and this is consistent with observations of the evolution of galaxy clustering at $z < 1$ (see, e.g., Carlberg et al. 2000). Thus, one expects a strong *decrease* in the clustering strength at higher redshifts, and in any model gravitational clustering is always expected to grow in time. How do we then explain the apparent *increase* in the strength of quasar clustering at high redshifts?

The most natural explanation is that quasars represent highly biased peaks of the density field, and that the bias itself evolves in time. Ever since the first detections of QSO clustering (e.g., Shaver 1984; Shanks et al. 1987; Iovino & Shaver 1988; Mo & Fang 1993) it was considered possible that QSOs represent biased tracers of the density field, but the evolution of such bias was not clear. La Franca, Andreani, & Cristiani (1998) found a turn-up in the clustering strength of quasars even at redshifts as low as $z \sim 2$, but this was not confirmed in a much larger sample by Croom et al. (2001). Outram et al. (2003) find no evidence for an increase in the QSO clustering power spectrum amplitude out to $z \sim 2.2$.

The first hints of such an effect at high redshifts were provided by the three few-Mpc quasar pairs at $z > 3$, found in the statistically complete survey by Schneider et al. (1994b), as first pointed out by Djorgovski, Thompson, & Smith (1993), and subsequently confirmed by more detailed analysis (Kundic 1997; Stephens et al. 1997). Intriguingly, the frequency of the few-Mpc separation quasar pairs at lower redshifts is roughly what may be expected from normal galaxy clustering (Djorgovski 1991; see also Zhdanov & Surdej 2001). There is even a hint of a possible superclustering of quasars at $z > 4$, on scales $\sim 100\ h^{-1}$ comoving Mpc (Djorgovski 1998), comparable to the scales of the first Doppler peak in CMBR fluctuations. Observations of large numbers of field galaxies at $z \sim 3$–3.5 also show a relatively strong clustering, with $r_0 \sim 5$–10 Mpc, comparable to the galaxy clustering at $z \sim 0$ (Steidel et al. 1998; Adelberger et al. 1998); this is also almost certainly a manifestation of biasing.

However, the bias should be even stronger at higher redshifts, and what is observed at $z \sim 3$ should be even more pronounced at $z \sim 5$: the earliest massive galaxies, including quasar hosts, should be strongly clustered. An example may be the possible grouping of Lyα emitters at $z \approx 4.86$ in the Subaru Deep Field (Ouchi et al. 2003; Shimasaku et al. 2003). Strong increase in biasing at high redshifts is also indicated in numerous theoretical studies, e.g., by Brainerd & Villumsen (1994), Matarrese et al. (1997), Moscardini et al. (1998), Blanton et al. (2000), Magliocchetti et al. (2000), Valageas, Silk, & Schaefer (2001), Basilakos & Plionis (2001). What these studies show is that a simple (r_0, ϵ) parameterization of the clustering evolution is inadequate, and that the evolution of the bias factor, b, plays a key role. The effective bias factor generally increases with the redshift and the object mass (e.g., especially for the more luminous Lyman-break galaxies or the quasar hosts). For example, Croom et al. (2002) find a marginally stronger clustering for the brighter QSOs, which might be residing in more massive hosts, and thus be more biased.

The chief uncertainty in our current understanding and interpretation of the structure evolution at high redshift, as indicated by luminous objects we can observe, is the evolution of bias. Observations of the clustering of quasars and galaxies around them at $z \sim 4$–5 and beyond can provide valuable empirical constraints in this endeavor. A better understanding of the primordial clustering of luminous sources is also important for models of the cosmic reionization (see Djorgovski et al. 2001 and references therein), and thus for the interpretation of CMBR fluctuations at high angular frequencies. The quasar pair described here may be indicative of the biased clustering at $z \sim 5$, and more extensive and deeper surveys will provide additional observational input for the models of galaxy and large-scale structure formation.

We thank the anonymous referee for the insightful comments and suggestions. We wish to acknowledge the expert assistance of the staff of the Palomar and W. M. Keck observatories. The LRIS data were obtained in the course of a collaborative project with F. Harrison and P. Mao. S. G. D. acknowledges partial funding from the Ajax Foundation. The work of D. S. was carried out at the Jet Propulsion Laboratory, Caltech, under a contract with NASA.

REFERENCES

Adelberger, K., Steidel, C., Giavalisco, M., Dickinson, M., Pettini, M., & Kellogg, M. 1998, ApJ, 505, 18
Bahcall, N., & Chokshi, A. 1991, ApJ, 380, L9
Basilakos, S., & Plionis, M. 2001, ApJ, 550, 522
Blanton, M., Cen, R., Ostriker, J., Strauss, M., & Tegmark, M. 2000, ApJ, 531, 1
Boyle, B., & Mo, H. J. 1993, MNRAS, 260, 925
Brainerd, T., & Villumsen, J. 1994, ApJ, 431, 477
Carlberg, R., Yee, H., Morris, S., Lin, H., Hall, P., Patton, D., Sawicki, M., & Shepherd, C. 2000, ApJ, 542, 57
Cole, S., & Kaiser, N. 1989, MNRAS, 237, 1127
Croom, S., Boyle, B., Loaring, N., Miller, L., Outram, P., Shanks, T., & Smith, R. 2002, MNRAS, 335, 459
Croom, S., & Shanks, T. 1996, MNRAS, 281, 893
Croom, S., Shanks, T., Boyle, B., Smith, R., Miller, L., Loaring, N., & Hoyle, F. 2001, MNRAS, 325, 483
Djorgovski, S. G. 1991, in ASP Conf. Ser. 21, The Space Distribution of Quasars, ed. D. Crampton (San Francisco: ASP), 349
———. 1998, in Fundamental Parameters in Cosmology, ed. Y. Giraud-Heraud et al. (Gif sur Yvette: Eds. Frontières), 313
———. 1999, in ASP Conf. Ser. 193, The Hy-Redshift Universe: Galaxy Formation and Evolution at High Redshift, ed. A. Bunker & W. van Breugel (San Francisco: ASP), 397
Djorgovski, S. G., Castro, S., Stern, D., & Mahabal, A. 2001, ApJ, 560, L5
Djorgovski, S. G., Odewahn, S. C., Gal, R. R., Brunner, R., & de Carvalho, R. 1999, in ASP Conf. Ser. 191, Photometric Redshifts and the Detection of High Redshift Galaxies, ed. R. Weymann et al. (San Francisco: ASP), 179
Djorgovski, S., Thompson, D., & Smith, J. 1993, in First Light in the Universe, ed. B. Rocca-Volmerange et al. (Gif sur Yvette: Eds. Frontières), 67
Efstathiou, G., & Rees, M. 1988, MNRAS, 230, P5
Fan, X., et al. 1999, AJ, 118, 1
———. 2001, AJ, 121, 54
Ferrarese, L., & Merritt, D. 2000, ApJ, 539, L9
Franceschini, A., Hasinger, G., Miyaji, T., & Malquori, D. 1999, MNRAS, 310, L5
Gebhardt, K., et al. 2000, ApJ, 539, L13
Granato, G., Silva, L., Monaco, P., Salucci, P., De Zotti, G., & Danese, L. 2001, MNRAS, 324, 757
Hamann, F., & Ferland, G. 1999, ARA&A, 37, 487
Hartwick, F. D. A., & Schade, D. 1990, ARA&A, 28, 437
Hoyle, F., Outram, P., Shanks, T., Croom, S., Boyle, B., Loaring, N., Miller, L., & Smith, R. 2002, MNRAS, 329, 336
Iovino, A., & Shaver, P. 1988, ApJ, 330, L13
Kaiser, N. 1984, ApJ, 284, L9
Kauffmann, G., & Haehnelt, M. 2000, MNRAS, 311, 576
Kells, W., Dressler, A., Sivaramakrishnan, A., Carr, D., Koch, E., Epps, H., Hilyard, D., & Pardeilhan, G. 1998, PASP, 110, 1487
Kundic, T. 1997, ApJ, 482, 631
La Franca, F., Andreani, P., & Cristiani, S. 1998, ApJ, 497, 529
Magliocchetti, M., Bagla, J., Maddox, S., & Lahav, O. 2000, MNRAS, 314, 546
Magorrian, J., et al. 1998, AJ, 115, 2285
Matarrese, S., Coles, P., Lucchin, F., & Moscardini, L. 1997, MNRAS, 286, 115
Mo, H. J., & Fang, L. Z. 1993, ApJ, 410, 493
Monaco, P., Salucci, P., & Danese, L. 2000, MNRAS, 311, 279
Moscardini, L., Coles, P., Lucchin, F., & Matarrese, S. 1998, MNRAS, 299, 95
Nusser, A., & Silk, J. 1993, ApJ, 411, L1
Oke, J. B., et al. 1995, PASP, 107, 375
Ouchi, M., et al. 2003, ApJ, 582, 60
Outram, P., Hoyle, F., Shanks, T., Croom, S., Boyle, B., Miller, L., Smith, R., & Myers, A. 2003, MNRAS, 342, 483
Sabbey, C., et al. 2001, ApJ, 548, 585
Schneider, D., Schmidt, M., & Gunn, J. 1994, AJ, 107, 880
———. 1994, AJ, 107, 1245
Schneider, D., et al. 2000, AJ, 120, 2183
Shanks, T., Fong, R., Boyle, B., & Peterson, B. 1987, MNRAS, 227, 739
Shaver, P. 1984, A&A, 136, L9
Shimasaku, K., et al. 2003, ApJ, 586, L111
Silk, J., & Rees, M. 1998, A&A, 331, L1
Steidel, C., Adelberger, K., Dickinson, M., Pettini, M., & Kellogg, M. 1998, ApJ, 492, 428
Stephens, A., Schneider, D., Schmidt, M., Gunn, J., & Weinberg, D. 1997, AJ, 114, 41
Turner, E. 1991, AJ, 101, 5
Valageas, P., Silk, J., & Schaefer, R. 2001, A&A, 366, 363
Zhdanov, V., & Surdej, J. 2001, A&A, 372, 1