

POLARIS: AMPLITUDE, PERIOD CHANGE, AND COMPANIONS

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ABSTRACT

Polaris has presented us with the rare phenomenon of a Cepheid with a pulsation amplitude that has decreased over the last 50 yr. In this study we have used this property to see whether the amplitude decrease during the last 15 yr has had any effect on upper atmosphere heating. We obtained *IUE* high- and low-resolution spectra but found no change in either the Mg II chromospheric emission or the flux at 1800 Å between 1978 and 1993 when the pulsation amplitude dropped by 50% (from 2.8 to 1.6 km s⁻¹). The energy distribution from 1700 Å through *V*, *B*, *R*(KC), and *I*(KC) is like that of a nonvariable supergiant of the same color rather than a full amplitude Cepheid in that it has more flux at 1800 Å than the full amplitude Cepheid δ Cep. Polaris also has a rapidly changing period (3.2 s yr⁻¹), in common with other overtone pulsators. We argue that this is a natural consequence of the different envelope locations that dominate pulsation growth rates in fundamental and overtone pulsation. In fundamental mode pulsators, the deeper envelope is more important in determining growth rates than for overtone pulsators. For fundamental mode pulsators, evolutionary changes in the radius produce approximately linear changes in period. In overtone pulsators, pulsation reacts to small evolutionary changes in a more unstable way because the modes are more sensitive to high envelope features such as opacity bumps, and the growth rates for the many closely spaced overtone modes change easily. Finally, the upper limit to the X-ray flux from an *Einstein* observation implies that the companion in the astrometric orbit is earlier than F4 V. The combination of upper and lower limits on the companion from *IUE* and *Einstein* respectively catch the companion mass between 1.7 and 1.4 M_{\odot} . The X-ray limit is consistent with the more distant companion α UMi B being a physical companion in a hierarchical triple system. However the X-ray limits require that the even more distant companions α UMi C and D are too old to be physically associated with Polaris.

Subject headings: binaries: visual — Cepheids — stars: atmospheres — stars: individual (Polaris, α Ursae Minoris) — stars: oscillations

1. INTRODUCTION

The importance of classical Cepheid variable stars as extragalactic distance indicators has recently been emphasized by the *Hubble Space Telescope* (*HST*) results from the Virgo Cluster (e.g., Freedman et al. 2001). At the same time, increasingly sophisticated hydrodynamic codes have made it possible to simulate the details of the atmosphere with improved accuracy.

As the brightest classical Cepheid, Polaris presents some unique possibilities for observations (notwithstanding its challenging location for many telescopes). On the other hand, the character of its pulsations (low amplitude, amplitude change, period change) also present some exceptional features within the framework of pulsation.

1.1. Pulsation

Feast & Catchpole (1997) have recently determined a distance to Polaris from a *Hipparcos* parallax: 7.56 ± 0.48 mas. Based on this parallax, they concluded that Polaris is pulsating not in the fundamental mode but in the first overtone or possibly even a higher mode. The absolute magnitude from the parallax is -3.63 mag, with the -3.49 mag

resulting from the 1σ limit on the parallax. The absolute magnitude inferred from the period-luminosity-color relation of Feast & Walker (1987) is -2.97 mag. Using a ratio of first overtone to fundamental periods of $P_1/P_0 = 0.70$, the absolute magnitude becomes -3.51 mag if the observed period is the first overtone, which is in much better agreement with its observed luminosity.

This mode identification was confirmed by the measurement of the diameter of Polaris by the Navy Prototype Optical Interferometer (NPOI) team (Nordgren et al. 1999; Nordgren 2000), which is an excellent match to the predictions from the Cepheid period-radius relation if the period is a first overtone.

Polaris is a low-amplitude Cepheid with a period of 4.0 days. It was discovered by Arellano Ferro (1983), however, to have been decreasing in amplitude for the last 50 yr. The decrease in photometric amplitude was confirmed by radial velocity studies (Kamper, Evans, & Lyons 1984; Dinshaw et al. 1989). Fernie, Kamper, & Seager (1993) and Brown & Bochonko (1994) summarize more recent data.

Hatzes & Cochran (2000) have provided additional high-accuracy radial velocity data. Besides the decrease in amplitude, they have also found a residual component in the radial velocities with an amplitude of 400 m s⁻¹ and a period of 40 days. This residual motion is accompanied with

¹ *IUE* Guest Observer.

changes in the line bisector, a measure of atmospheric changes. They argue that this is more likely to be produced by nonradial pulsation than either a low-mass companion or rotational modulation by spots.

There is as yet no explanation for the decrease in pulsation amplitude. Cepheids in general have light curves that repeat identically to within the precision of the observations. V473 Lyr (Burki et al. 1986) is an exception which shows cyclical variations in amplitude with a period of 1200 days. Y Oph may also have a changing amplitude (Fernie, Khoshnevisan, & Seager 1995). In the case of Polaris, one of the early suggestions was that it was at the edge of the instability strip, causing its pulsation to die out as it left the strip. Since it has a steadily increasing period of pulsation, the standard interpretation is that it is evolving to cooler temperatures in the HR diagram. Fernie et al. (1993) have shown that there are a number of Cepheids with substantial amplitude between Polaris and the red edge of the instability strip, making this explanation unlikely. The interference of two pulsation modes is a possible cause of amplitudes that increase and decrease cyclically. For V473 Lyr, radial pulsation does not provide two modes closely enough spaced in the fundamental or low-overtone regime. For Polaris, the interfering mode explanation is possible, but there is so far no evidence that the amplitude of Polaris is cyclic.

However, without understanding the cause of the decrease in amplitude, we can use it to see what atmospheric parameters are affected by the amplitude decrease. Specifically a high-resolution spectrum of Polaris at 2800 Å was obtained with the *International Ultraviolet Explorer* (IUE) satellite in 1979. This spectrum shows that the Mg II *h* and *k* lines have the standard two peaks of emission separated by the dip due to self-absorption. Because the amplitude had fallen to half the 1979 value by 1993, it was decided to obtain another IUE spectrum to see whether there had been any change in the Mg II lines. Although the absolute change in the amplitude was small, since the Mg II emission was clearly present in 1979, either an increase or decrease could have been detected.

As a final observational clue to upper atmosphere heating, Evans & Teays (1996) have shown that there is a dramatic difference in the flux at 1800 Å between the full amplitude Cepheid δ Cep and nonvariable supergiants. Coupled with the low level of X-ray flux in another full amplitude Cepheid, ζ Gem (Sasselov 1994), they suggested that pulsation inhibits some form of chromospheric heating (acoustic heating is a possibility) that results in the absence of a corona. In this paper we will discuss the 1800 Å flux of Polaris to determine the effects of very low-amplitude pulsation on the chromosphere.

1.2. Companions

Polaris is a member of a multiple system. Orbital motion has been seen both spectroscopically (Roemer 1965) and astrometrically (Wyller 1957). Kamper (1996) revised and rediscussed both orbits based on his additional data. The secondary to this system has never been detected, however using IUE observations from 1200 to 3000 Å, Evans (1988) showed that the companion is later than A8 V. Based on the new information that Polaris is an overtone pulsator, the companion spectral type should be revised to A7 V or later.

Recently Wielen et al. (2000) have added a particularly important piece of information about the orbit, the inclina-

tion. They compared the *Hipparcos* proper motions (which are nearly instantaneous in the context of the 30 yr orbit) with the long-term average proper motion. The inclination they found is 130.2 ± 4.8 , with an alternate (less probable) possibility of a prograde solution. It is interesting to note that the mass they derive with this inclination and an assumed mass of the Cepheid of $6 M_{\odot}$ ($1.54 M_{\odot}$, F0 V), is less than the upper limit set from the IUE spectrum. Even using a slightly smaller mass for the Cepheid based on observed Cepheid masses (Evans et al. 1998) only decreases the spectral type to F1 V.

In addition to the unseen companion in this orbit, there are other companions summarized by Fernie (1966); α UMi B is an F star (8th mag) 19" away, and α UMi C and D are 43" (13th mag) and 83" (12th mag) distant, respectively. Kamper confirmed that the radial velocity and astrometric motion of α UMi B match those of the Cepheid, making it a likely physical companion.

2. ULTRAVIOLET STUDIES

2.1. IUE Observations

Both high- and low-resolution spectra of Polaris were obtained with the IUE satellite shortly after launch. In order to explore whether the reduced pulsation amplitude has had any effect on either the energy distribution or the Mg II emission, new high- and low-resolution spectra were obtained in 1993.

Reductions were done with the SUN version of the *International Ultraviolet Explorer* Regional Data Analysis software at York University in the manner described in Evans et al. (1993) and the references therein. Sensitivity degradation corrections were only available until 1991 for the 1993 data. This results in a 2% to 3% underestimate of the flux in the LWP spectra and half that in the SWP spectra.

At the epochs of the observations (1978 and 1993) the radial velocity amplitudes of Polaris used below are 2.8 km s^{-1} for 1978 (Ferne et al. 1993) and 1.6 km s^{-1} for 1993 (K. W. Kamper 1995, private communication).

2.2. IUE Low-Resolution Spectra: Energy Distributions

The *International Ultraviolet Explorer* low-resolution spectra are listed in Table 1. The phases are taken from the ephemeris of Brown & Bochonko (1994):

$$JD_{\text{maxlight}} = 2431495.91 + 3.96959E + 2.0116 \times 10^{-7}E^2,$$

where E is the epoch. This is a slightly updated version of the quadratic ephemeris of Arellano Ferro (1983). (Spectra from 1978 and 1979 are referred to as 1978 spectra for convenience below.)

As expected, the ultraviolet spectra show small variations to the flux level at different phases. For this reason, in order to make the comparison with nonvariable supergiants below, we will use the spectra at phase 0.70 (LWP 26219 and SWP 48472), which is very close to mean light.

The colors of Polaris at mean light were used. They should be accurate for the IUE observations since the amplitude in 1993 was so small (0.01 mag in V; Fernie et al. 1993). $E(B-V) = 0.00$ mag was used, as has been used in other IUE work (Evans 1988). Mean values of $V-R$ and $V-I$ on the Johnson system were taken from Arellano Ferro (1983) transformed to the Kron Cousins system using the relations from Fernie (1983). The resulting values used

TABLE 1
LOW-RESOLUTION SPECTRA

Long Wavelength Spectrum	Short Wavelength Spectrum	JD 2,400,000	Phase
LWR 4109	SWP 4747	43957.58	0.78
LWP 21396	SWP 42608	48532.84	0.93
LWP 26219	SWP 48472	49226.93	0.70
LWP 26242	SWP 48494	49228.90	0.20

for the colors are $(B - V)_0 = 0.59$ mag, $V - R_0(\text{KC}) = 0.34$ mag, and $V - I_0(\text{KC}) = 0.60$ mag.

In addition to the broad band colors B , V , $R(\text{KC})$, $I(\text{KC})$, J , H , and K , IUE spectra were binned into 100 Å bins between 1800 and 3200 Å. The details, including the calibration of the broadband colors are discussed in Evans et al. (1993) and Evans & Teays (1996).

The resulting energy distribution of Polaris is shown by the plus signs in Figure 1. For comparison, the energy distribution of the F7 Ib nonvariable supergiant 45 Dra is shown. It has a virtually identical $(B - V)_0$ (0.60 mag) to that of Polaris (0.59 mag). The spectra used to generate Figures 1 and 2 are listed by Evans et al. (1993) and Evans & Teays (1996). Figure 1 shows that Polaris has an identical energy distribution to the nonvariable supergiant of the same color from 1800 Å through the near infrared.

The fact that the energy distribution of Polaris matches that of nonvariable supergiants is further demonstrated by Figure 2. Here the energy distribution of Polaris is compared with those of two supergiants that bracket it in $(B - V)_0$; α Per [$(B - V)_0 = 0.44$ mag] and β Aql [$(B - V)_0 = 0.81$ mag] are used in Figure 2. The dashed line shows the energy distribution interpolated to the $(B - V)_0$ of Polaris using the supergiants. The spectrum of Polaris is intermediate between the two supergiants, and more precisely, the interpolated energy distribution is a good match for that of Polaris.

The SWP spectra near mean light at the two epochs (SWP 4747 and SWP 48472) have the same flux level. As discussed below, this wavelength region is sensitive to high atmosphere heating, which apparently is unchanged by the change in amplitude.

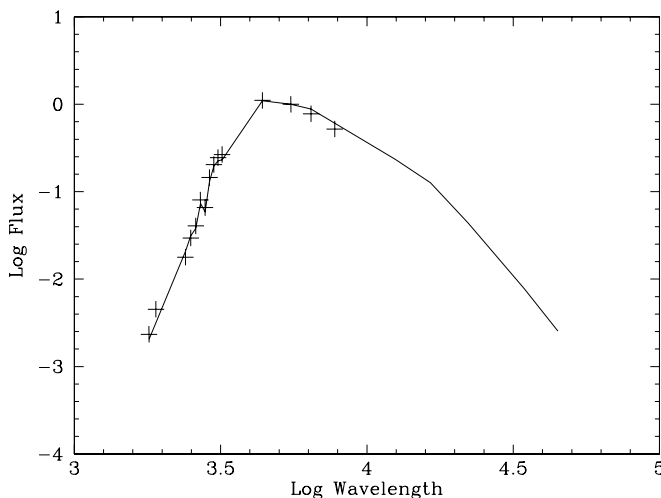


FIG. 1.—Energy distributions of Polaris and 45 Dra. The plus signs are Polaris; the solid line connects the points for the nonvariable supergiant 45 Dra. In all figures, the wavelength is in angstroms and the flux is in $\text{ergs s}^{-1} \text{Å}^{-1} \text{cm}^{-2}$.

2.3. Ultraviolet Colors

Energy distributions like those shown in the previous section have also been obtained for a series of phases of δ Cep in the coasting portion of the pulsation cycle (descending branch of the light curve), as discussed in Evans & Teays (1996) and references cited there. They present the same data for nonvariable supergiants covering the spectral type range covered in Cepheid pulsation cycles (F2 Ib to G8 Ib). Ultraviolet colors were created from the flux in bands of 100 Å centered around 1800 and 2900 Å, $m(1800) - m(2900)$ (converted to magnitudes). As was shown by Evans et al. (1990), the sequence formed by the δ Cep series has proportionately less flux at 1800 Å than the nonvariable sequence at the same $(B - V)_0$. Polaris fits very well on the nonvariable sequence, as would be expected from the detailed comparison of energy distributions in the previous section.

The extra flux at 1800 Å in Polaris could either be from the companion, or because the atmosphere of such a low-amplitude Cepheid is more like that of a nonvariable than that of a Cepheid. This has previously been discussed by Evans (1988) and Evans et al. (1990). In order to explore this question further, we present two additional color-color diagrams, Figures 3 and 4. In both, we have created an ultraviolet-visual color from the one of the flux bands used in Evans et al. (1990) and V . Figure 3 shows the $m(1800) - V$ color as a function of $(B - V)_0$; Figure 4 shows $m(2900) - V$. Figure 3 again shows that Polaris is in better agreement with the nonvariable sequence in the proportion of flux at 1800 Å than with the δ Cep sequence. Figure 4 shows that the same is true of Polaris at 2900 Å. Figures 3 and 4 show

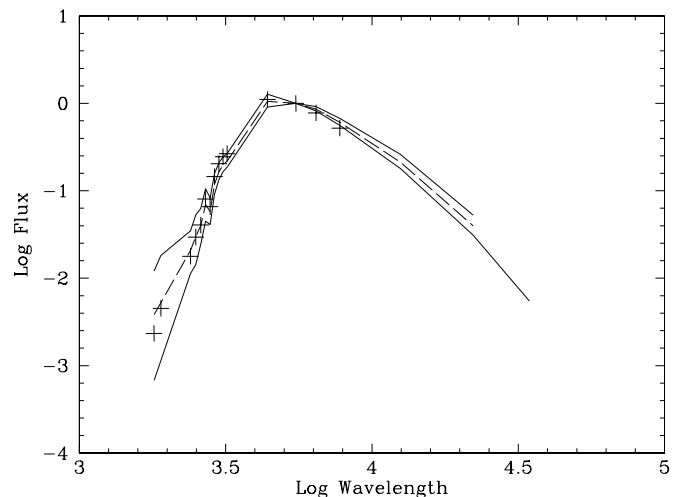


FIG. 2.—Energy distributions of Polaris and the nonvariable supergiants α Per and β Aql. The plus signs are Polaris. The solid lines connect the points for α Per and β Aql with α Per being the heavier line. The dashed line is the energy distribution interpolated between α Per and β Aql to the $(B - V)_0$ of Polaris.

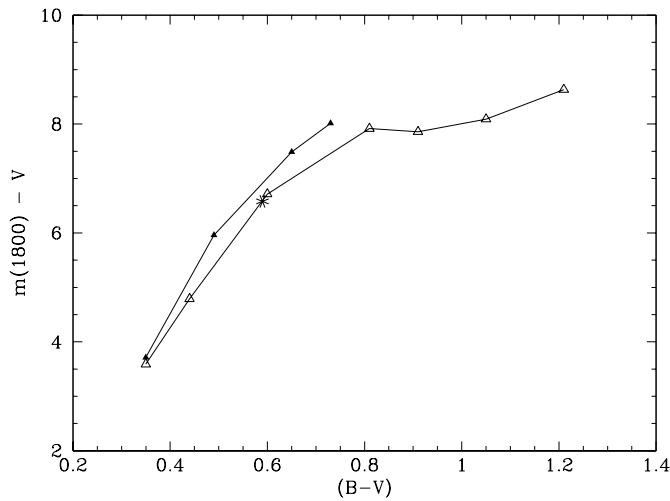


FIG. 3.—Color $m(1800) - V$ as a function of $B - V_0$. The filled triangles are δ Cep, the open triangles are nonvariable supergiants, and the asterisk is Polaris. Both colors are in magnitudes.

that the fraction of the flux drops markedly between 2900 and 1800 Å in yellow supergiants. Since the flux of a companion to Polaris that could be detected (say an A or early F main-sequence star) is much flatter in this region, it would make a much larger relative contribution at 1800 Å than at 2900 Å. Since this is not the case in Figures 3 and 4, the implication is that the difference between the Polaris energy distribution and the δ Cep energy distribution is due to a difference in the atmospheres of the two stars rather than the detection of the companion of Polaris.

2.4. Energy Distributions

The fact that the energy distribution of Polaris matches those of nonvariable supergiants with the same $(B - V)_0$ (Figs. 1 and 2) is not surprising in and of itself. However, when similar comparisons were made for the full amplitude Cepheid δ Cep (Evans & Teays 1996), it was found that δ Cep has significantly less flux at short wavelengths (for which we use the shorthand reference 1800 Å flux below). This was taken to be an extension of the discovery by Morossi et al. (1993) that the flux at short wavelengths for

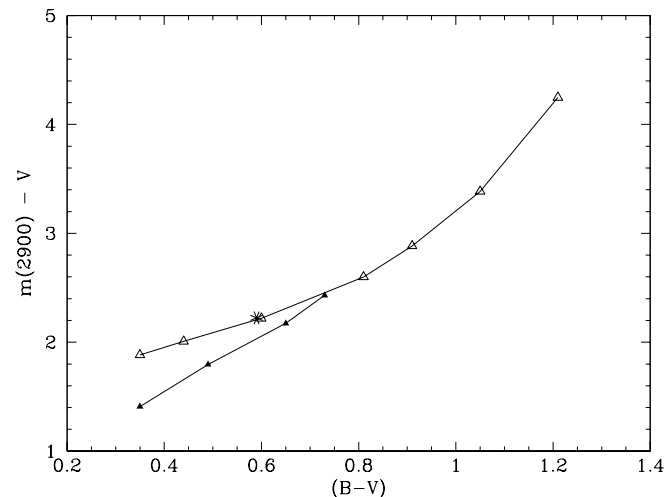


FIG. 4.—Color $m(2900) - V$ as a function of $B - V_0$. The symbols are the same as Fig. 3. Both colors are in magnitudes.

giants exceeds that predicted by radiative models. Evans & Teays found that, like the giants, supergiants have flux at 1800 Å in excess of that predicted by radiative models. Morossi et al. interpret this as extra heating to a high level of the atmosphere. (One possibility for the heating is acoustic heating.) The results in δ Cep from Evans & Teays show that this extra flux at 1800 Å is not present in the full amplitude Cepheid. This suggests that pulsation inhibits the upper atmosphere heating process. Polaris, on the other hand matches the energy distributions of the nonvariables, suggesting that its upper atmosphere is determined by the same heating processes as the nonvariables. The pulsation motion of Polaris is very small, smaller than microturbulent motions found in supergiant atmospheres, and so presumably the pulsation is unimportant compared with these motions and its upper atmosphere is the same as if it were not pulsating. The global radial pulsation found in Polaris' atmosphere does provide a repeated nonrandom perturbation to the atmosphere which could lead to resonance and amplification. However, this apparently does not happen in any significant way.

The IUE spectra, particularly the short wavelength spectra from 1700 to 2000 Å can also be used to see whether there is any change in the flux corresponding to the change in amplitude. For the two spectra close to mean light, no difference in the flux level was seen between 1978 and 1993. Again, presumably this is because the absolute change in amplitude is small, though the amplitude dropped by 50% in this time.

2.5. IUE High-Resolution Spectra: Mg II Emission

As for the low-resolution IUE spectra, high-resolution spectra of Polaris had been obtained early in its program (1978). In order to see whether the drop by half of the pulsation amplitude in the intervening 14 yr has had an effect on the emission, new spectra were obtained in 1993. Table 2 lists the spectra, the dates of observation and the phases computed as above. As Table 2 shows, at both epochs, spectra were available at several pulsation phases.

Figure 5 shows the comparison between the Mg II lines at the two epochs. The spectrum for 1993 is a composite spectrum combining the stronger spectrum (LWP 26248) in the line cores, but showing the weaker spectrum (LWP 26243) further in the wings, since the stronger spectrum is saturated there. Because the 1978 spectrum (LWR 3170) is also saturated in the wings, only the unsaturated part is shown. As can be seen, the shape of the central emission and self-absorption is the same at the two epochs. Examination of the line cores at all the pulsation phases leads to the conclusion that no changes are seen in the lines which are larger than the noise. The LWP camera has a reseau located at the center of the violet emission in LWP 26248. However, com-

TABLE 2
HIGH-RESOLUTION SPECTRA

Long Wavelength Spectrum	JD 2,400,000	Phase
LWR 3163	43856.72	0.38
LWR 3166	43856.90	0.43
LWR 3170	43857.56	0.60
LWR 3184	43858.90	0.93
LWP 26218	49226.90	0.70
LWP 26243	49228.93	0.21
LWP 26248	49229.18	0.27

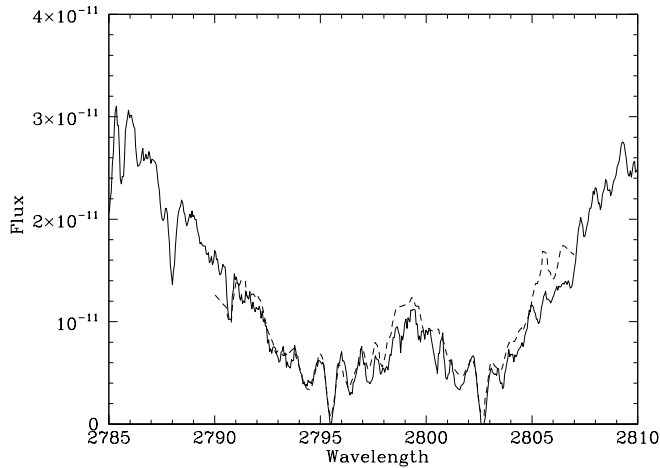


FIG. 5.—Comparison between the observed Mg II profiles of Polaris at different pulsation amplitudes. The dashed line is from 1978, when the pulsation amplitude was 2.8 km s^{-1} . The solid line is the 1993 profile when the amplitude had fallen to 1.58 km s^{-1} .

parison with the earlier spectrum that is taken with the other camera, which does not have a reseau at that position, shows that the violet flux in the LWP spectrum cannot be much affected by the reseau.

3. X-RAY STUDIES

The *Einstein* X-ray satellite (IPC) observed Polaris for 8 ks in 1979 March. No signal was detected, which provides an upper limit of $4 \times 10^{-14} \text{ ergs s}^{-1} \text{ cm}^{-2}$ in the 0.5 to 4 keV band (F. D. Seward 1995, private communication).

This limit was determined by standard procedures in The *Einstein Observatory* Catalog of IPC X-Ray Sources (Harris et al. 1990). Because the upper limit is for a single predefined location, the confidence level was set at 2.5σ , resulting in a 1% chance of missing a source that is present. To this flux limit, there is no evidence of a corona for Polaris. Using the distance to Polaris from the *Hipparcos* parallax (Feast & Catchpole 1997) of 132 pc, this corresponds to a $\log L_X$ of less than 28.9 ergs s^{-1} .

To put this upper limit in context, we have compiled a list of X-ray observations of supergiants from the literature (Table 3). The luminosities are taken directly from the sources cited. Because the stellar sources are relatively soft, the fact that the *Einstein* IPC includes energy up to 4.5 keV but *ROSAT* PSPC is only sensitive to energies less than 2.5 keV makes a relatively small difference in the X-ray flux measured. To quantify this statement, we used the HEASARC W3PIMMS. We used a Raymond-Smith model with solar abundance and the temperature used by Maggio et al. (1990) of $\log T = 6.5$ to compute their counts to flux conversion factor. Using PIMMS we got the relative count rates between *ROSAT* PSPC and *Einstein* IPC. To go from count rate to flux, Maggio et al. used a constant conversion factor ($2.0 \times 10^{-11} \text{ ergs count}^{-1} \text{ cm}^{-2}$) for *Einstein* IPC. Reimers et al. (1996) used a conversion factor depending on the observed hardness ratio for *ROSAT* PSPC observations. However, the same group (Hünsch et al. 1996) used a constant conversion factor ($4.0 \times 10^{-12} \text{ ergs count}^{-1} \text{ cm}^{-2}$) when no hardness ratio was available, which we adopted. Using this approach, the fluxes from the two instruments differed by less than 50% or 0.3 in $\log L_X$ for a very low extinction ($N_H = 1 \times 10^{-19}$).

TABLE 3
F AND G SUPERGIANTS OBSERVED IN X-RAYS

Supergiant	Spectral Type	V_r^*	L_X (ergs s^{-1})	Instrument	Reference
Nonvariables					
Alp Per	F5 Ib	B	30.12	<i>ROSAT</i> PSPC	Prosser et al. 1996
Bet Aqr	GO Ib	B, M	30.00	<i>ROSAT</i> PSPC	Reimers et al. 1996
Alp Aqr	G2 Ib	B	<29.73	<i>ROSAT</i> PSPC	Reimers et al. 1996
Del TrA	G2 II		<29.18	<i>ROSAT</i> PSPC	Reimers et al. 1996
9 Peg	G5 Ib	B	<29.34	<i>ROSAT</i> PSPC	Reimers et al. 1996
Bet Cam	GO Ib	B	31.52	<i>ROSAT</i> PSPC	Reimers et al. 1996
Bet Dra	G2 II	B	31.50	<i>ROSAT</i> PSPC	Reimers et al. 1996
Alp Car	F0 II		29.8	<i>Einstein</i> IPC	Maggio et al. 1990
11 Pup	F7II		<29.0	<i>Einstein</i> IPC	Maggio et al. 1990
Bet Lep	G5 II	M	29.0	<i>Einstein</i> IPC	Maggio et al. 1990
Bet Crv	G5 II	M	<28.7	<i>Einstein</i> IPC	Maggio et al. 1990
Eps Gem	G8 Ib		<29.0	<i>Einstein</i> IPC	Maggio et al. 1990
Rho Pup	F6 II		28.7	<i>Einstein</i> IPC	Maggio et al. 1990
	(DelSct)				
HD74180	F3 Ia		<28.9	<i>Einstein</i> IPC	Maggio et al. 1990
HD102350	G5 Ib-II		<29.6	<i>Einstein</i> IPC	Maggio et al. 1990
Iot ¹ Sco	F2 Iae		<28.9	<i>Einstein</i> IPC	Maggio et al. 1990
Eps Sct	G8 II		<28.8	<i>Einstein</i> IPC	Maggio et al. 1990
Cepheids					
Zet Gem			<29.0	<i>ROSAT</i> PSPC	Sasselov 1994
Polaris			< 28.9	<i>Einstein</i> IPC	This paper
Bet Dor			<29.4	<i>Einstein</i> IPC	Maggio et al. 1990
Del Cep			<29.1	<i>Einstein</i> IPC	Maggio et al. 1990

NOTE.— B = Butler 1998, and M = Murdoch et al. 1993.

In the previous analyses, extinction was typically ignored. Because the stars observed are extremely bright, this does not cause large errors. The same tests we made to compare instruments can also be used to investigate the effect of reddening. For $N_H = 5 \times 10^{20}$, the observed *Einstein* flux is 2.6 times the *ROSAT* flux. This corresponds to an $E(B-V) = 0.1$ mag. Most of the stars in Table 3 have an extinction smaller than this.

Figure 6 shows X-ray detections and nondetections of yellow (F and G) supergiants as a function of spectral type. Out of the 17 non-Cepheid supergiants in Table 3, seven have been detected in X-rays.

For comparison, Hünsch, Schmitt, & Voges (1998) recently investigated how many giants and supergiants were detected in the *ROSAT* all-sky survey (RASS). In their catalogue, there are 19 yellow supergiants (F and G stars with luminosity classes II, II-Ib, or Ib). Eight of these are found in Table 3, generally with approximately the same X-ray flux. In two cases, they detected a star for which only an upper limit was found in previous studies (HD 102350 and β Crv). If the flux from this paper had been used by the previous investigators, the luminosities (log) would be 29.3 and 28.1 ergs s^{-1} , respectively. This would only change Figure 6 and the detection rate slightly. However, the fraction and level of detection of the supergiants in Figure 6 is in contrast to the level found by Schröder, Hünsch, & Schmitt (1998) for giants. Using the RASS data for giants with known distances, they find five of 36 giants are detected at a level greater than $\log L_X = 28$ ergs s^{-1} . This is a much smaller detection rate than for supergiants in the Hünsch et al. study.

Perhaps the most important question to ask in assessing the result that roughly half the yellow supergiants observed have been detected in X-rays, is whether it is the supergiants themselves that produce the X-rays or whether they could come from a fainter companion. While it is nearly impossible to prove that a star is single, we have looked into information about the X-ray detected supergiants to see whether they seem to be binary systems. Two important new studies have recently been made which include several of these

stars. Murdoch, Hearnshaw, & Clark (1993) and Butler (1998) have observed the stars indicated in Table 3 with very high radial velocity accuracy (55 m s^{-1} and 30 m s^{-1} , respectively). They find velocity variations at this level for many of the supergiants. However, with one exception (β Aqr), the stars in Table 3 show some sort of atmospheric pulsations rather than orbital motion. This statement is based on the periodicities of Butler. The longest period he finds is 230 days for 9 Peg but for all the rest, periods are less than 100 days. Since the supergiants are at least largely blue loop (central He burning stars), they will have been red giants, and only orbits large enough to accommodate a red giant (typical mass $5 M_\odot$) will have survived that stage. For the Population I classical Cepheid subset of blue loop stars, we have a nice observational confirmation of the approximate size of the red giant phase in that orbital periods are all a year or larger. In fact, the only Cepheid with a circular orbit indicating interaction between the stars, Z Lac (Sugars & Evans 1996), is the shortest orbital period known. Hence, shorter periods seen in the Murdoch et al. and Butler radial velocity studies are incompatible with orbital motion in blue loop stars. The velocity variations do, however, make very clear that it would be very difficult to detect orbital motion at this level.

β Aqr is the exception to the lack of orbital motion detected in the supergiants. Both Murdoch et al. and Butler find velocity variations in observational projects lasting several years. Butler discusses two possible periods, 203 days and 1024 days. While he leans toward the shorter period, in light of the discussion above, a longer period is a more likely orbital period. Figure 7 in Murdoch et al. show that for reasonable orbital parameters, both a very small inclination and also a very small companion mass are probably required by the observations, if they are in fact due to orbital motion.

Thus, all the supergiants detected in X-rays (α Per, β Aqr, β Cam, β Dra, α Car, β Lep, and ρ Pup) have been searched very carefully for orbital motion except α Car. ρ Pup does have a variable velocity, however that is due to δ Scuti pulsations (Yang, Walker, & Bennett 1987).

Because the point-spread function (PSF) of both *Einstein* IPC and *ROSAT* PSPC observations are relatively large, a second star along the same line of sight could be the source of the X-rays. For the *Einstein* study, the criterion was that a companion be closer than $2'$; for the *ROSAT* study, the optical and X-ray positions were required to coincide within $30''$. However, the criterion for suggesting that the X-ray flux may be produced by a companion is rather more stringent than simply that there is another star nearby. The companion would also have to be quite young to produce X-rays at the level of Table 3. For instance, Figure 6 in Gagné, Caillault, & Stauffer (1995) shows that almost no field stars have an X-ray luminosity of $\log L_X = 29$ ergs s^{-1} . This means that only physical companions, not optical ones are likely sources of X-rays. Using the notes in the *Bright Star Catalogue*, α Per, β Cam, β Aqr, and α Car do not have line of sight companions likely to be responsible for the X-rays. We note that the remaining 3 stars may have candidate companions, but whether they are young enough to produce the observed X-rays is not known.

To complete the discussion of whether the observed X-rays are produced by the supergiants, we note that for β Aqr, the Murdoch et al. results imply that it is highly likely that the companion in the putative orbit is a very low mass

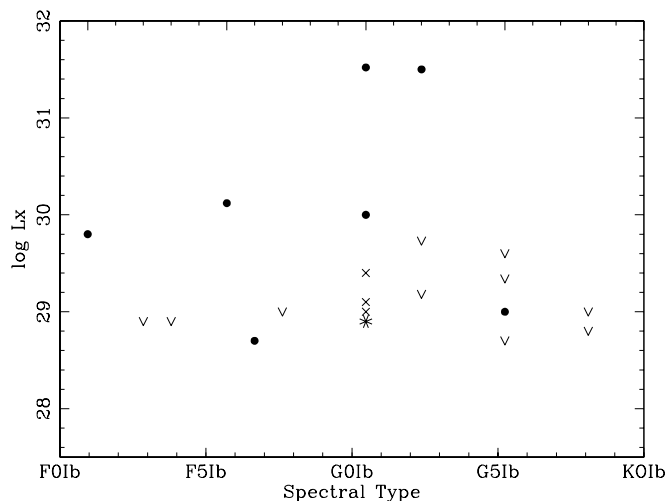


FIG. 6.—X-ray observations of supergiants. Filled circles are nonvariable supergiants that have been detected; down arrows are the upper limits of nonvariable supergiants which have not been detected. Cepheids which have not been detected are crosses, arbitrarily plotted at G0 Ib; the asterisk is Polaris, which is also an upper limit. L_X is in ergs s^{-1} .

star. In the same way that field stars are not expected to produce the X-ray flux observed, very low mass stars are not either. The best illustration of the X-ray flux for stars with a range of masses is the α Per cluster (Randich et al. 1996). These stars are approximately the same age as the Cepheids and other blue loop supergiants (such as α Per itself). Figures 4, 5, and 6 in that study show that virtually all G and K main-sequence stars have an X-ray luminosity of $\log L_x = 29$ ergs s^{-1} or greater. Many M stars are not detected or have a luminosity smaller than this. This means that even if β Aqr has a low-mass binary companion, it might not be responsible for the observed X-rays.

In summary, the X-ray flux for *some* of the F and G supergiants may come from hidden companions. However, it would require a large number of quite improbable circumstances to account for all of the supergiant detections in this way. Our working model is that at least some of the X-rays are intrinsic to the supergiants themselves. Figure 6 shows that some supergiants have been detected, but some have not to the level of approximately $\log L_x = 29$ ergs s^{-1} . The investigation of Hünsch et al. (1996) seems relevant to this working model. In a volume limited survey of late type giants, they found that essentially all giants with $B - V < 1.2$ were detected at a level of $\log L_x = 27$ ergs s^{-1} , but only a small fraction were brighter than that.

Figure 6 also shows that although 4 classical Cepheids have been observed, to the same sensitivity as the nonvariable supergiants, none have been detected in X-rays.

4. PERIOD CHANGE

The direct determination of overtone pulsation from the *Hipparcos* parallax (Feast & Catchpole 1997) and the radius measurement (Nordgren 2000) is corroborated by several additional pieces of evidence. An important approach to identifying pulsation mode in Cepheids is the use of Fourier diagnostics, which have been shown to separate singly excited fundamental and first overtone pulsators (see for instance the discussion of Antonello, Poretti, & Reduzzi (1990) and sources cited therein). An important result of variable star studies from microlensing projects in the Magellanic Clouds has been the confirmation that Fourier diagnostics do indeed identify overtone pulsators. Unfortunately, the very low amplitude of Polaris precludes a rigorous use of this test, although Fernie et al. (1993) have found no measurable departures from a sine curve in either light or velocity data. The low amplitude of pulsation, itself, however, is characteristic of overtone pulsators. In fact so-called s Cepheids were recognized as a class on the basis of low amplitude and symmetric light curves before it was realized that they are overtone pulsators. Polaris certainly exhibits these features.

There is one further characteristic of overtone or s Cepheids that Polaris possesses. Parenago (1956) did a systematic study of period changes for 42 Cepheids. Later, in a series of publications, Szabados compiled periods and period changes for an extensive list of Cepheids over many years. In the first study in the series (Szabados 1977), he concentrated on stars with periods less than 5 days. When he found a changed period, he used the difference between the old and new periods to form fractional period changes $|\Delta P| / P$. (These period changes take place over many cycles, so the fractional period change is not the change per cycle.) The fractional period changes had an average value of 0.00011 for small amplitude Cepheids but a value of 0.000007 for full

amplitude Cepheids. In his full discussion of period changes for all periods (Szabados 1983) he tabulates a slightly different parameter,

$$\frac{|\Delta P|}{P \Delta E},$$

the percentage change in period per cycle (where E is the epoch). For full amplitude (fundamental mode) pulsators, he shows that the size of the period change increases as luminosity increases. However, s Cepheids have an order of magnitude larger period changes than full amplitude Cepheids of the same period.

The simplest interpretation of period changes is that they are due to evolution of the star through the instability strip at approximately constant luminosity. This has been discussed, for instance, by Fernie (1984). He finds that the period changes are in general agreement with the predictions of the evolutionary tracks of Becker, Iben, & Tuggle (1977). Although there has been some discussion about whether the data and predictions match exactly, the agreement in scale and trend over a large luminosity range is strong evidence that we are seeing period changes due to evolution.

Polaris shares the rapid period change of other s Cepheids. The ephemeris from Dinshaw et al. (1989) has a period change of 3.2 s yr^{-1} . This is an order of magnitude larger than the typical period change for Cepheids of similar period as listed by Fernie (1984). Furthermore, such a list only provides stars with measurable period changes. A sizable fraction of Cepheids do not show period changes, presumably because they are smaller than can be detected from the available data. Hence the average period change of fundamental mode Cepheids is even smaller and the period change of Polaris is more unusual.

Because we have unusually complete data for Polaris, including a direct determination of the distance, we can draw some other inferences about its pulsation and evolutionary properties. Figure 7 shows the HR diagram of Cepheids with independently determined distances. Data are derived from two sources: cluster members (Feast & Walker 1987) and binary companions (Evans 1991, 1992a, 1992b, 1992c). Overtone pulsators (SU Cas, SZ Tau) and double mode pulsators (V367 Sct, Y Car) are indicated. The luminosity from the parallax distance for Polaris is shown. The lines show the approximate red and blue edges of the instability strip from Fernie (1990) from field Cepheids with luminosities inferred from the PLC.

As shown by Figure 7, the overtone and double mode Cepheids are found on the blue side of the strip. There appear to be some fundamental mode pulsators in this area also, although they also occupy the redder portion of the instability strip where no overtone pulsators are found. The new absolute magnitude of Polaris puts it within the region of the instability strip occupied by overtone pulsators near the blue edge.

The evidence of distance, amplitude, period change, color, and perhaps Fourier diagnostics all agree with overtone pulsation for Polaris. This leads to the following situation for the period change. Polaris has an increasing period. Interpreted in evolutionary terms, it is moving toward cooler temperatures. This is one reason for the original idea that the decrease in amplitude might be due to evolution out of the instability strip. Now, however, it seems clear that Polaris is near the blue edge of the strip. Inter-

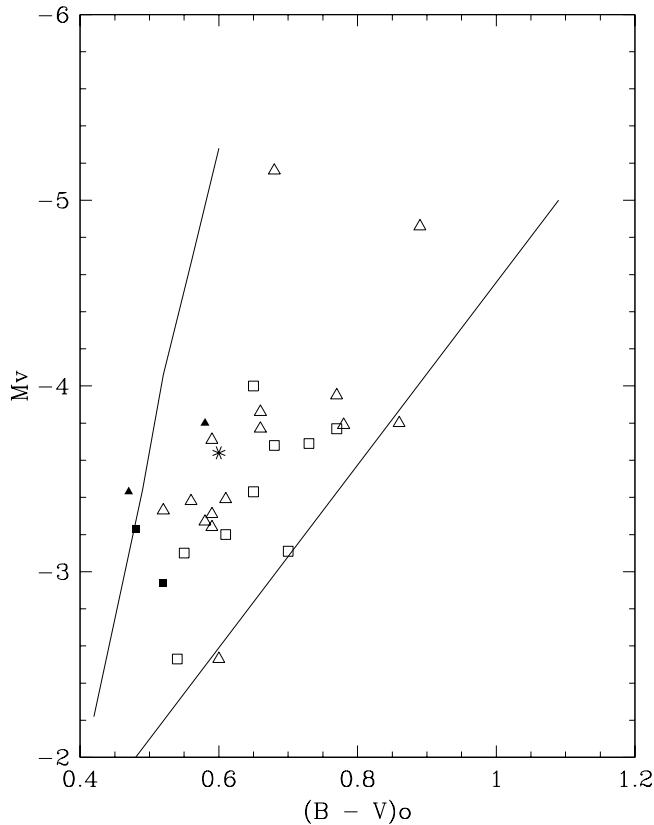


FIG. 7.—HR diagram for Cepheids with direct distance determinations. Squares are from binary companions (Evans); triangles are for clusters (Feast & Walker). For all stars, open symbols are fundamental mode pulsators; filled symbols are for overtone and double mode Cepheids. Polaris is marked with an asterisk.

preting the period change as evolutionary would mean that Polaris is moving into the instability strip. This would not explain the decrease in pulsation amplitude.

Thus in the case of Polaris, there is strong evidence that the large size of the period change, its sense, and the decrease in amplitude are not consistent with an evolutionary explanation. This is coupled with the fact that *s* Cepheids as a group have period changes an order of magnitude larger than would be expected from fundamental mode pulsators with the same period on observational and evolutionary grounds. There is another piece of corroborative evidence that the large period changes of *s* Cepheids do not match evolutionary expectations. Overtone pulsators are probably found at the tips of the blue loops (though existing evolutionary tracks in general do not have blue loops that penetrate the instability strip for low masses). This is a location of slower evolution than the cooler portions of the evolutionary tracks before or afterward, although the period change predicted depends on both the speed of evolution and the orientation with respect to lines of constant period. Hence the unusually rapid period changes are probably not expected.

Our conclusion is that period changes in Cepheids derive from two causes. For fundamental mode pulsators, they are in reasonable agreement with evolutionary predictions. Polaris, however, clearly does not fit into this scheme and the other *s* Cepheids apparently do not either. We propose a different explanation for its period change below.

The differences in period (and amplitude) stability between fundamental and overtone pulsators can be understood through the following considerations.

Evolutionary changes.—In the Cepheid instability strip (CIS) evolutionary changes (such as from evolutionary tracks such as those of Schaller et al. 1992) proceed at rates that can be approximated to be constant for a fixed (M, Z) :

$$\frac{dT_{\text{eff}}}{dt}, \frac{dR}{dt}, \frac{dL}{dt} \approx \text{const.}$$

Acoustic properties.—Growth rates (η_i for mode i)

$$\frac{d\eta_i}{dt}$$

can change very dramatically with small evolutionary changes. This causes the following differences in period change behavior between fundamental and overtone pulsation.

Fundamental pulsators.—

$$\frac{dP}{dt} \propto \frac{dT_{\text{eff}}}{dt} \approx \text{const.}$$

Small changes in period and temperature produce only very small changes in the growth rates (see Fig. 11 of Buchler, Yecko, & Kollath 1997). Resonances with higher overtones affect only light curve shape and amplitude.

High-order overtones.—Small changes in period and temperature can produce large changes in the growth rates. Modes “pile-up” near the F blue edge of the CIS. In addition they interact more. As surface modes, they are more sensitive to the opacity bumps in the envelope and the much thinner (and temperature sensitive) convection zone. It is these bumps that “pile up” against the surface at high T_{eff} . Small changes in radius, as an overtone pulsator evolves through the instability strip, will result in large changes in growth rates, probably for several modes. It seems highly likely that changes in the mixture of modes excited result in comparatively rapid period changes as well as amplitude changes. While predicting the changes in the dominant frequency clearly requires nonlinear pulsation calculations, there is a simple linear effect that is in the right direction (though too small to account for the full period change).

It follows from the linear adiabatic wave equation for any radial mode that the angular pulsation frequency ($\omega = 2\pi/P$) is

$$\omega^2 = (3\Gamma_1 - 4) \frac{GM}{R^3},$$

where Γ_1 is the adiabatic exponent:

$$\Gamma_1 = \frac{d \ln P_s}{d \ln \rho},$$

where R is the radius, M is the mass, P_s is the pressure, and ρ is the density.

This leads easily to the function for P ,

$$P = fR^{3/2}M^{-1/2}.$$

This expression will give us the rate of period change with $R(t)$ for the fundamental (F) radial mode of a Cepheid of mass M . Here the coefficient f is a function of the pulsation constant. For instance from linear adiabatic calculations (King et al. 1975) $f_1 = 0.71f_0$, where f_1 and f_0 are the coeffi-

cients for the first overtone and the fundamental, respectively. That is, from this simple linear theory, the period change of first overtone dP_0/dt is more rapid than that of the fundamental. We stress that the difference in period change from this simple estimate is smaller than that observed between Cepheids and fundamental mode pulsators, and full nonlinear studies are certainly needed. However, it is in the right sense.

We note that a mixture of modes suggested by the observations of Hatzes & Cochran (though interpreted as non-radial modes) is another aspect of Polaris' pulsation which is in agreement with the suggestion that a complex mode structure is an important component in the variations of Polaris.

It would clearly be desirable to investigate the dichotomy between period changes in fundamental and overtone pulsators further. One possibility comes from photometry from microlensing programs. Inspection of the period change ($O-C$) diagram of Fernie et al. (1993) shows that departure from a straight line for the rapid period change in Polaris begins to become apparent in about 2000 cycles or 20 yr. This is from photometry from a random series of studies for a very small amplitude star. For a star with an amplitude of half a magnitude and photometry of the quantity and quality microlensing programs, a changing period would reveal itself much more quickly, though a longer time span is necessary to fully characterize the period change. In the Magellanic Clouds, for instance, this would create a data set which could be used to investigate the interdependence of period change, luminosity, color, and metallicity.

5. DISCUSSION

5.1. Atmospheric Heating

One of the main aims of this study is to compare upper atmospheric diagnostics of full amplitude Cepheids, a low-amplitude Cepheid (Polaris), and nonvariable supergiants. The ultraviolet diagnostics (Figs. 1–4) indicate that Polaris has an energy distribution more similar to a nonvariable than a full amplitude Cepheid. We interpret the reduction of ultraviolet flux in a full amplitude Cepheid as indicating that there is some source of heating in the upper atmosphere of nonvariables which does not exist in a pulsating atmosphere. A naive expectation might be that pulsation might heat the atmosphere, but the opposite appears to be true. A possible explanation is that pulsation has disrupted magnetic activity, which creates the high temperatures in the upper atmosphere of the nonvariables. This disruption could be either in the dynamo in the envelope or in the loop structures in the upper atmosphere.

The X-ray data currently available (Fig. 6) do not contradict this model in that a number of nonvariables have been detected in X-rays but so far there are only upper limits to the Cepheid fluxes.

5.2. Companions

The recent derivation of an inclination for the astrometric orbit of Polaris using *Hipparcos* data (Wielen et al. 2000) takes us a step closer to the exciting possibility of measuring the mass of Polaris. An important consideration in planning future observations is estimating the characteristics of the as yet unseen companion. Paradoxically, the upper limit to the X-ray flux provides important positive information about the companion.

The investigation of Randich et al. (1996) of the X-ray properties of the α Per cluster is again useful in providing a sample of stars comparable in age to Polaris. They detect essentially all the main-sequence stars from late F through early M. Specifically the dividing line falls at $(B-V)_0 = 0.40$ mag, which corresponds to spectral type F4 V with a mass of $1.4 M_\odot$ (Harmanec 1988). Virtually all stars later than this were detected at an X-ray luminosity greater than $\log L_x = 29.0$ ergs s^{-1} , most with L_x greater than 29.5 ergs s^{-1} . The upper limit to the X-ray flux of Polaris is $L_x < 28.9$ ergs s^{-1} (Fig. 6). Thus, a companion less massive than $1.4 M_\odot$ would have been detected in the *Einstein* observation. Combined with the mass upper limit from *IUE*, this catches the mass of the astrometric companion of Polaris between 1.4 and $1.7 M_\odot$, a very small range. The mass for the companion that Wielen et al. (2000) estimated from the astrometric orbit assuming a mass of $6 M_\odot$ for the Cepheid is $1.54 M_\odot$, which is nicely within these limits. (The companion mass is slightly smaller but still within the limits if a smaller Cepheid mass is used, as indicated by recent *HST* results from Evans et al. 1998)

In addition to placing a tight limit on the mass of the companion in the astrometric orbit, the X-ray upper limit also provides information about the three more distant companions. The closest companion α UMi B is an F3 V star (Fernie 1966), with appropriate magnitude and colors to be at the same distance as the Cepheid. Kamper (1996) derived both radial and tangential velocities and found that they agree with the velocities of the Cepheid, and concluded that it is a likely physical companion. The fact that it was not detected in X-rays does not contradict this. It is hotter (slightly) than stars where X-rays were overwhelmingly found in the α Per cluster, which means the lack of X-rays does not rule out a young star, which a companion to Polaris must be. Wielen et al. (2000) have estimated an orbital period of approximately 50,000 yr if it and Polaris are in an orbit. This would be an appropriate orbit for the outer member of a hierarchical system containing the 30 yr astrometric system. We note that this provides another triple system to add to the statistics of intermediate mass multiple systems derived from Cepheid studies.

The two more distant companions C and D have separations of $43''$ and $83''$, respectively. The observed magnitudes are $V = 13.1$ and 12.1 mag. At the distance of Polaris they would have absolute magnitudes of 7.5 and 6.5 mag, and as suggested by Fernie would still be somewhat above the main sequence. Again, the α Per cluster is a good comparison. Young stars with a comparable magnitude in the cluster are overwhelmingly X-ray producers with a typical luminosity closer to $\log L_x = 29.5$ ergs s^{-1} than 29.0 . The fact that C and D are not detected in X-rays implies that they are older than Polaris, and not physical companions.

6. SUMMARY

We have discussed several characteristics about Polaris and its companions:

Energy distribution.—Although Polaris is a first overtone radial pulsator, the energy distribution is more similar to that of a nonvariable than a full amplitude Cepheid. We interpret this to show that low-amplitude pulsation has little effect on upper atmosphere heating. The decrease in pulsation amplitude has not produced measurable changes in either the energy distribution or the Mg II h and k profiles.

Period change.—Polaris and other Cepheids pulsating in an overtone mode have much larger period changes than fundamental mode pulsators. We attribute this to growth rates for shallower overtone modes, which are more sensitive to small changes in temperature and radius as the star evolves through the instability strip. This is in contrast to period changes for fundamental mode pulsators, which are dominated by period changes linearly related to evolutionary changes in the radius.

Companions.—The upper limit to the X-ray luminosity for the Polaris system is $\log L_X < 28.9$ ergs s^{-1} from an *Einstein* observation. This is not unusual for the Cepheid, whether its atmosphere behaves more like a pulsator or a nonvariable. It does provide a lower limit to the mass of the companion in the astrometric orbit, since a companion as young as Polaris would have a larger X-ray luminosity if it were later in spectral type than F4 V. The combined mass limits from the *Einstein* and *IUE* observations catch the mass of the astrometric companion between 1.4 and 1.7

M_\odot . The X-ray upper limit is consistent with the closest visual companion, Polaris B, being physically associated with Polaris, since it is hotter than this X-ray limit. This makes the system a hierarchical triple system. The X-ray limit, however, implies two more distant companions, Polaris C and D are not physically associated with Polaris. Their magnitudes are faint enough that if they were young and at the distance of Polaris, they should have been detected in X-rays.

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